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# THE EFFECT OF BORON UPON THE MAGNETIC AND OTHER PROPERTIES OF ELECTROLYTIC IRON MELTED IN VACUO

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# THE EFFECT OF BORON UPON THE MAGNETIC AND OTHER PROPERTIES OF ELECTROLYTIC IRON MELTED IN VACUO.

## I. Introduction.

1. *Nature of the Work Presented.*—In Bulletin No. 72, published by the Engineering Experiment Station in 1914, the results of an investigation of the magnetic, electrical, and mechanical properties of electrolytic iron melted in vacuo were presented. It was shown that the magnetic properties of such iron, as far as permeability and hysteresis loss are concerned, are far superior to any grade of iron known at that time. The electrical resistance of the iron, however, was shown to be comparatively low, making it unsuitable for use where eddy-current losses must be kept down. Experiments indicated further that if carbon is added to such iron, the resultant product, after melting in vacuo, contains a less amount of carbon than was added. The conclusion was reached that there is present in the iron a sufficient amount of oxygen in the form of iron oxide to oxidize about 0.3 per cent carbon to CO, or about 0.4 per cent oxygen. The object of this investigation is, therefore, twofold; first, to reduce this iron oxide by means of suitable deoxidizing agents with the view still further to improve the magnetic quality of the iron, and second, to determine the effects of these reagents, when alloyed with the iron, upon the magnetic and electrical properties. Some of the best reducing agents for this purpose are carbon, boron, silicon, and aluminum.

It has been shown* that boron as a deoxidizer has a very beneficial effect upon copper castings, and it was thought that it might possibly have a similar effect upon iron. The present bulletin gives the results obtained by the addition of boron to electrolytic iron melted in vacuo. The results obtained with the other elements mentioned above will be published later.

2. *Previous Researches on Iron-Boron Alloys.*—Iron-boron alloys have, thus far, received only slight attention, and as far as can be found, practically nothing has been done to determine the mag-

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*E. Weintraub: Cast Copper of High Electrical Conductivity. Trans. Am. Electro-Chem Soc., XVIII, p. 207, 1910.

netic properties of such alloys. The latest research in this field is that of G. Hannesen* who has made a thorough study of iron-boron alloys from a chemical and metallurgical point of view. The alloys were produced by melting Goldschmidt ferro-boron, containing 21.13 per cent boron, with Krupp iron, in an atmosphere of hydrogen, and alloys were obtained containing from 0 to 8.5 per cent boron. Alloys containing more than 7 per cent boron were so brittle that it was almost impossible to produce specimens from them. Hannesen determined all the critical temperature points for the iron-boron alloys from the freezing temperature down to 500° C. (see Fig. 1). The one that is of particular interest here is the point at which the $\gamma$ iron changes into $\beta$ iron. This transformation point was found to decrease from about 895° for pure iron to 710° C for all alloys containing more than 0.8 per cent boron. With regard to the crystallization of iron-boron alloys he found that $\delta$-iron crystallizes out first, until the solution contains 1.38 per cent boron, then $\gamma$-iron until the concentration has reached 4 per cent boron, and finally the compound $Fe_5B_2$, completing the freezing. The latter is the eutectic of the iron-boron alloys, named boron-pearlite, and its freezing point was found to be 1164° ∓ 6° C. By means of a compass needle Hannesen found that all the alloys tested were ferro-magnetic both in the annealed and quenched state.

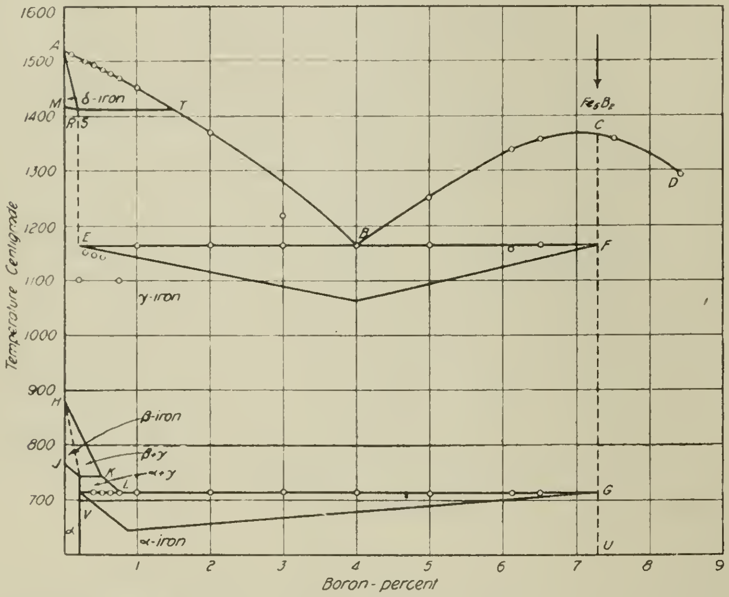

FIG. 1. EQUILIBRIUM DIAGRAM FOR IRON-BORON ALLOYS (HANNESEN).

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*Zeitschr. Anorg. Chemie. 89, p. 257, Nov. 5, 1914. Gives history of researches on iron-boron alloys, also a number of references.

This valuable article was received after the present bulletin had gone to press, which accounts for the few references that have been made to it.

3. *Review of Experiments.*—Doubly refined electrolytic iron, containing .006 per cent carbon and 0.1 per cent silicon, was used in the investigation. Before melting it was thoroughly cleaned by means of HCl, distilled water, and alcohol. The crucibles, holding 500 to 600 grams of crushed iron, were made from fused magnesia. The melting was done in a vacuum furnace of the Arsem type, in which a pressure of from 2 to 5 mm. of mercury was maintained during the melting of the iron. After the iron had cooled in the furnace it was removed from the crucible, reheated in a forge to ordinary forging temperature, and forged under a steam hammer into rods ½ in. (1.25 cm.) in diameter and 20 in. (50 cm.) long. From these rods the following test pieces were prepared: one magnetic test piece .392 in (.996 cm.) in diameter and 14 in. (35.5 cm.) long, two mechanical test pieces, one specimen for critical temperature investigation, and one for the study of the microstructure. The annealing was done in a Hoskins type resistor furnace. A platinum–platinum + 10% rhodium thermo-couple was used to measure the temperature. The temperature-time cooling curves were straight lines connecting 900° and 200° C., and the cooling between these temperatures was accomplished usually in 24 hours although in some cases it extended over 48 hours. The magnetic tests were made by the compensated double bar and yoke method, recommended by the Bureau of Standards, in connection with a Grassot fluxmeter. Measurements made with this apparatus checked very well with the measurements made upon bars submitted to the Bureau of Standards for calibration, and also with measurements made by the calibration laboratories of some of the largest manufacturing companies. The electrical resistance was measured on a Thompson double bridge. Photomicrographs were taken after the various heat treatments on an inverted metallurgical microscope, and critical temperature determinations were made for most of the specimens by uniformly cooling a small specimen from 1000° C. in a Hoskins muffle furnace and observing the time elapsing for each 10° temperature interval.

4. *Summary and Conclusions Drawn from the Previous Investigation.*—The investigation recorded in the earlier bulletin was summarized as follows:

1. Pure iron melted in an atmosphere of carbon monoxide under atmospheric pressure will absorb both carbon and oxygen with the result that the iron thus produced is of an inferior magnetic quality.

2. Low carbon iron, containing from 0.1 to 0.5 per cent carbon, melted in vacuo will lose 50 to 90 per cent of the original carbon content.

3. The magnetic quality of electrolytic iron melted in vacuo is decidedly superior to any grade of iron thus far produced, the maximum permeability obtained being 19,000 at a flux density of 9,500 gausses. The average hysteresis loss obtained is less than 50 per cent of that found in the best grades of commercial transformer iron.

4. The specific electrical resistance of pure iron melted in vacuo is 9.96 microhms.

5. Swedish charcoal iron melted in vacuo has a magnetic quality approximating that of electrolytic iron melted in vacuo, due chiefly to the reduction of the carbon content.

From these facts it appears that a superior quality of iron for magnetic purposes may be obtained by melting electrolytic iron in vacuo. While the electrical resistance of the iron thus obtained is very low, this defect may be remedied by the addition of such alloying elements as silicon or aluminum, elements that are known to increase the electrical resistance very materially without affecting the magnetic quality to any large extent. Experiments are now under way for determining the effect of such alloying elements, and the results will be published at some later date.

## II. Alloying Materials.

The boron was added to the iron in either of two forms: one known commercially as "boron suboxide" of the formula $B_7O$, and the other as "boron suboxide flux" containing about 8 per cent $B_7O$, the remainder being magnesium borate and boric anhydride. These boron products were developed by Dr. E. Weintraub* and the boron suboxide used in this investigation was obtained through his courtesy.

The chemical analysis of the suboxide shows that it contains 82.68 per cent boron, giving 99.86 per cent $B_7O$, so that for the present purpose it may be assumed that the material is pure $B_7O$. If it be further assumed that when this substance is heated it will decompose as follows:

$$3\,B_7O = B_2O_3 + 19B,$$

then 75 per cent active boron remains for the reduction of iron oxide.

The boron suboxide flux was found to contain 20.32 per cent boron, but as already stated, part of this is in an inactive form. It was not analysed for $B_7O$, but using Dr. Weintraub's figure, namely 8 per cent, this material should contain 6 per cent active boron.

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*Trans. Am. Electro-Chem. Soc. XVI, p. 165 (1909).
Trans. Am. Electro-Chem. Soc. XVIII, p. 207 (1910).

A special investigation was made to determine the most suitable method of chemical analysis for such small quantities of boron as existed in the specimens. A description of this investigation and the method finally adopted will be found in the Appendix.

## III. Results.

The results of the investigation are presented in the following figures and tables.

The magnetic and electrical properties for the $B_7O$ series and the $B_7O$-flux series are shown graphically in Figs. 2a, 2b, and 3. The mechanical properties of the $B_7O$ series are shown in Fig. 4. Table I gives the chemical analysis for boron for all of the specimens, and Figs. 5 to 24 inclusive are photomicrographs of a few of the specimens after various heat treatments.

Table I.
Boron Analysis.

| Specimen No. | $B_7O$ Added per cent | $B_7O$ Flux Added per cent | Active Boron Added per cent | Boron Content as per Chem. Anal. per cent | Boron Lost per cent | Oxygen Absorbed per cent |
|---|---|---|---|---|---|---|
| 3B 59 | .01 | | .0075 | Trace* | .0000 | .0000 |
| 3B 45 | .02 | | .0150 | " | .0050 | .0110 |
| 3B 60 | .03 | | .0225 | " | .0125 | .0275 |
| 3B 46 | .04 | | .0300 | " | .0200 | .0440 |
| 3B 61 | .05 | | .0375 | " | .0275 | .0605 |
| 3B 47 | .06 | | .0450 | " | .0350 | .0770 |
| 3B 62 | .07 | | .0525 | " | .0425 | .0935 |
| 3B 48 | .08 | | .0600 | " | .0500 | .1100 |
| 3B 49 | .10 | | .0750 | " | .0650 | .1340 |
| 3B 50 | .12 | | .0900 | .020 | .0700 | .1540 |
| 3B 51 | .14 | | .1050 | .032 | .0730 | .1600 |
| 3B 52 | .16 | | .1200 | .039 | .0810 | .1780 |
| 3B 53 | .18 | | .1350 | .037 | .0980 | .2160 |
| 3B 54 | .20 | | .1500 | .041 | .1090 | .2400 |
| 3B 55 | .25 | | .1875 | .056 | .1315 | .2900 |
| 3B 56 | .30 | | .2250 | .080 | .1450 | .3200 |
| 3B 57 | .40 | | .3000 | .102 | .1980 | .4350 |
| 3B 58 | .50 | | .3750 | .180 | .1950 | .4300 |
| 3B 34 | | .10 | .006 | Trace* | .0000 | .0000 |
| 3B 33 | | .25 | .015 | " | .0050 | .0110 |
| 3B 35 | | .75 | .045 | .020 | .0250 | .0550 |
| 3B 36 | | 1.00 | .060 | .020 | .0400 | .0880 |
| 3B 37 | | 1.50 | .090 | .070 | .0200 | .0440 |
| 3B 38 | | 2.00 | .120 | .076 | .0440 | .0970 |
| 3B 39 | | 2.50 | .150 | .086 | .0640 | .1400 |
| 3B 40 | | 3.00 | .180 | .080 | .1000 | .2200 |
| 3B 41 | | 4.00 | .240 | .095 | .1450 | .3200 |
| 3B 42 | | 5.00 | .300 | .130 | .1700 | .3750 |
| 3B 43 | | 6.00 | .360 | .250 | .1100 | .2400 |
| 3B 44 | | 10.00 | .600 | .450 | .1500 | .3300 |

*Trace is considered as .01% in the computations for the next column.

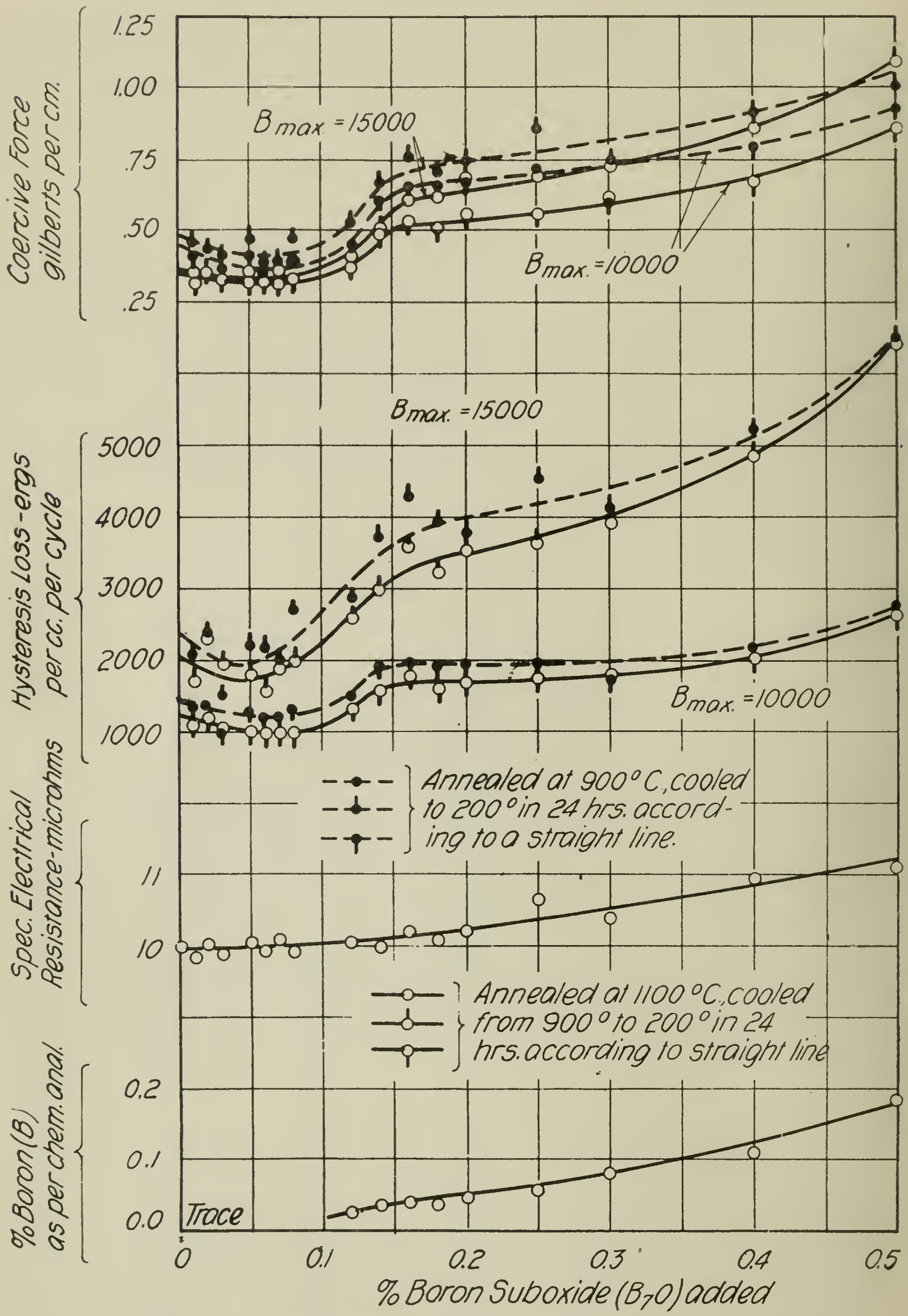

FIG. 2a. MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-BORON ALLOYS. $B_7O$ SERIES.

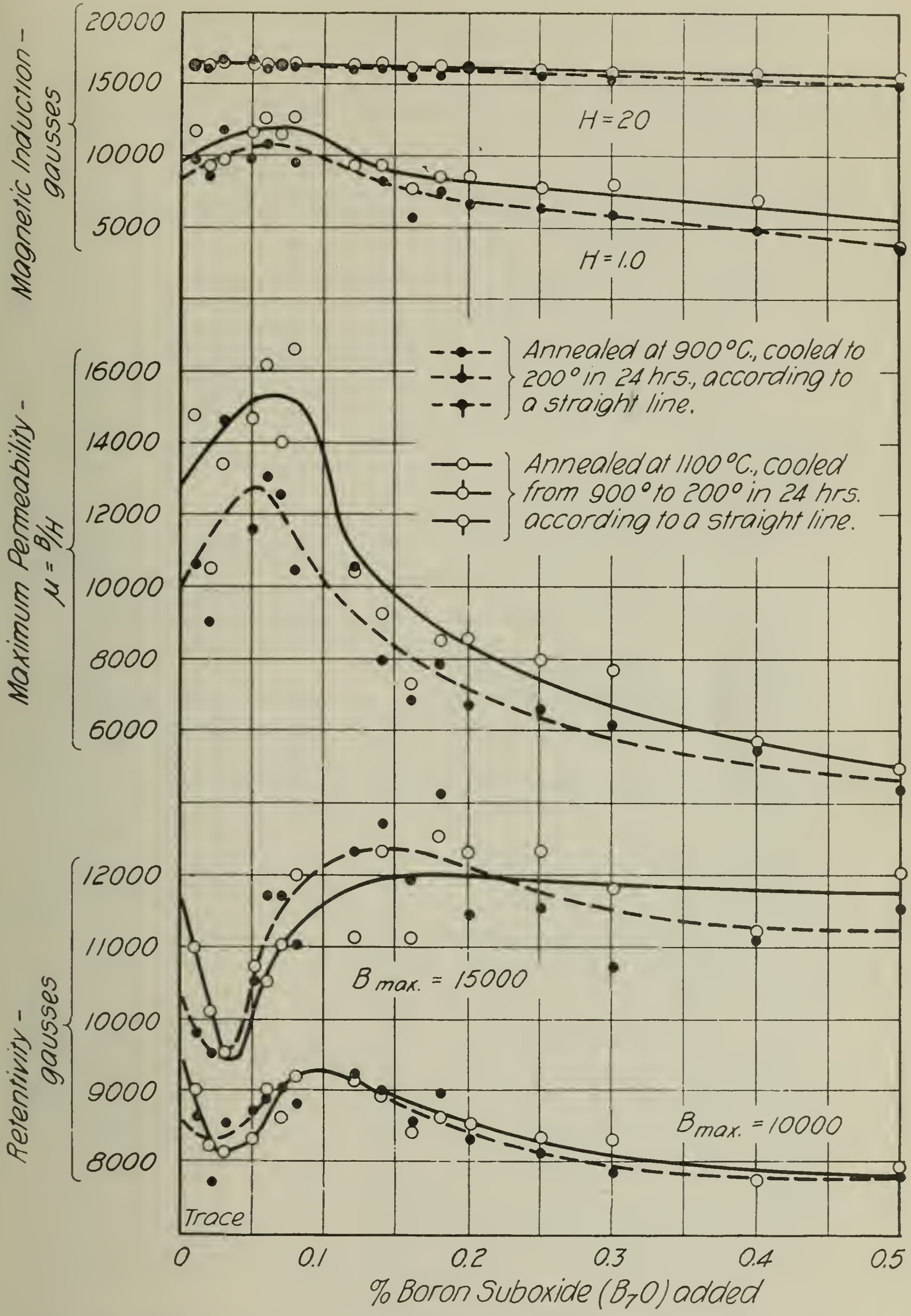

FIG. 2b. MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-BORON ALLOYS. $B_7O$ SERIES.

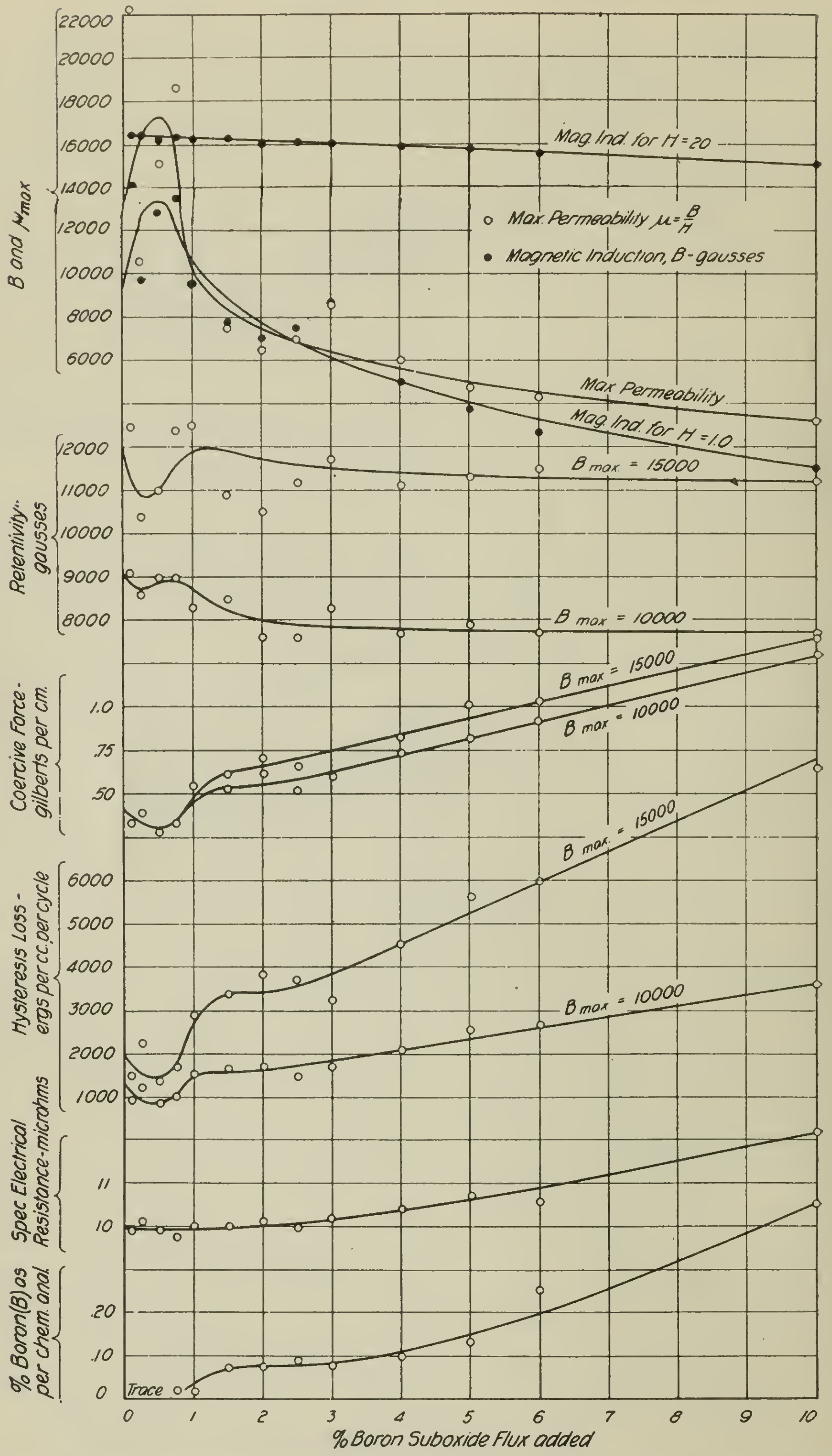

FIG. 3. MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-BORON ALLOYS. $B_7O$-FLUX SERIES.

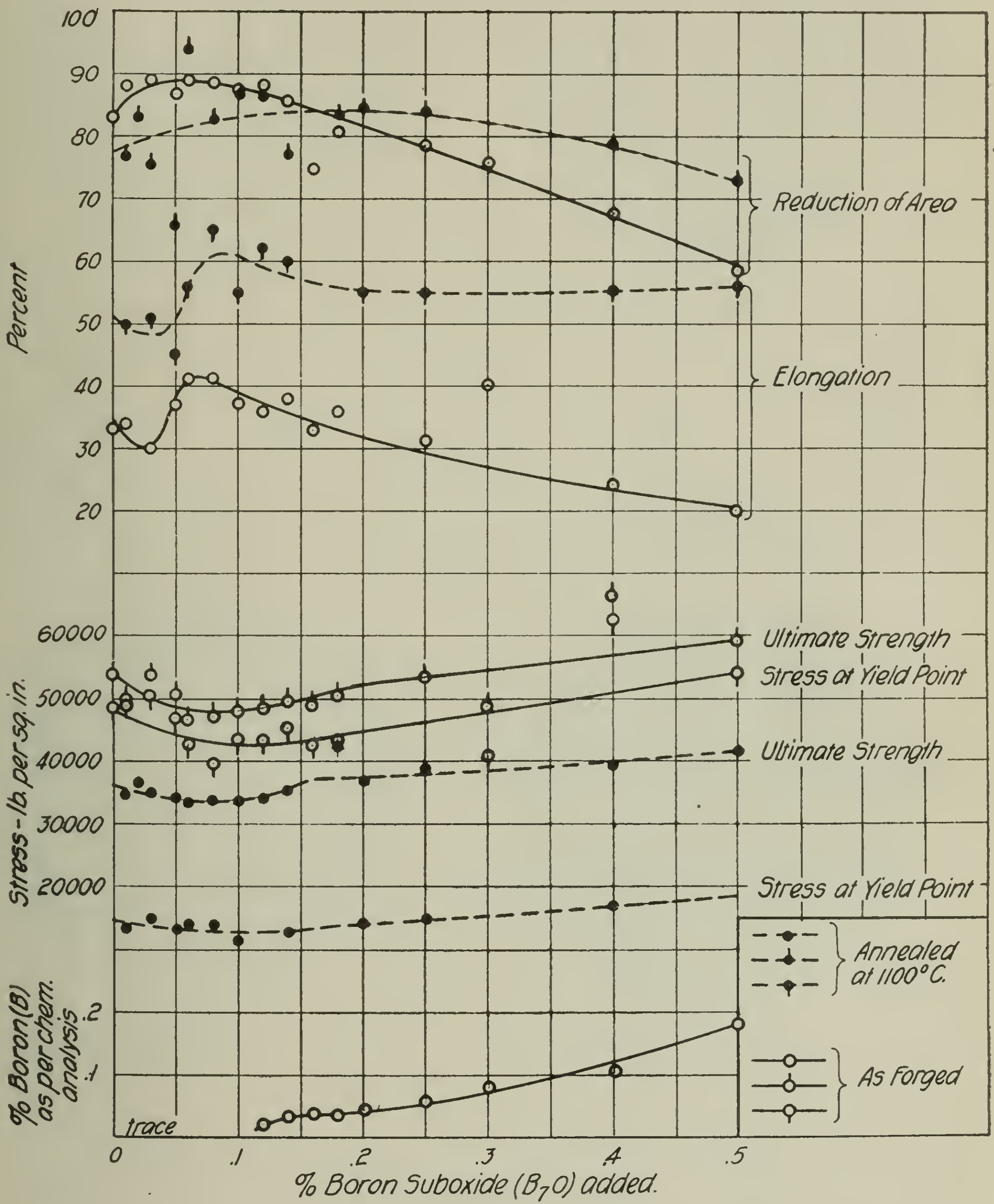

FIG. 4. MECHANICAL PROPERTIES, $B_7O$ SERIES.

## IV. DISCUSSION OF RESULTS.

It was mentioned in the introduction that in spite of the careful cleaning, the iron evidently contained some iron oxide that alloyed with the iron upon melting, and that a certain amount of a reducing substance had to be added to rid the product of the oxide. Table 2, taken from Bulletin No. 72, shows the results obtained when carbon was used as a reducing agent.

TABLE 2.

CARBON ANYALYSIS.

| Specimen No. | Carbon Added per cent | Carbon Content as per Chem. Anal. per cent | Carbon Lost per cent | Equiv. Amount of Oxygen Absorbed per cent |
|---|---|---|---|---|
| 3C01 | .05 | .013 | .037 | .049 |
| 3C02 | .10 | .012 | .088 | .117 |
| 3C03 | .50 | .181 | .319 | .420 |

The equivalent amount of oxygen absorbed is calculated on the basis of the reaction.

$$C + FeO \rightleftarrows CO + Fe \qquad (1)$$

Table 2 shows that the reaction, when the carbon content is above .01 per cent, takes place from left to right as long as there is any FeO left to oxidize the carbon. As was shown in Bulletin No. 72, the final carbon content depends not only upon the relative amounts of carbon and FeO present, but also upon the pressure under which the iron is melted. If the amount of carbon added is more than sufficient to reduce the FeO present, the remainder will combine with the iron. Thus when 0.5 per cent carbon was added, the carbon content of the finished product was found to be .181 per cent. In that case .319 per cent carbon was oxidized, corresponding to .42 per cent oxygen. If .319 per cent carbon had been added the result would probably have been a product containing approximately .01 per cent carbon and free from FeO.

Table 1 shows that similar deductions can be made from the present investigation. The amount of boron lost increases with the amount added up to about 0.3 per cent of active boron added. In the last case—considering the $B_7O$ series—the amount of boron lost is about 0.2 per cent. If it be assumed that the boron is oxidized according to the reaction

$$2B + 3FeO = B_2O_3 + 3Fe \qquad (2)$$

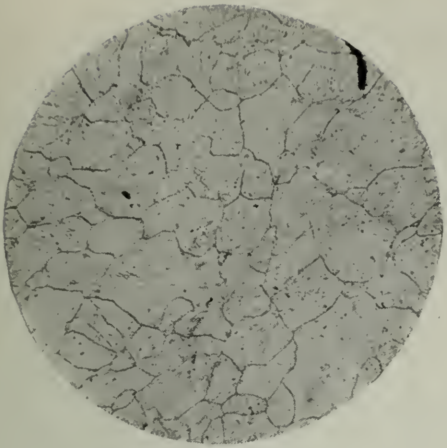
Fig. 5. 3-39A x 40.
As Forged.

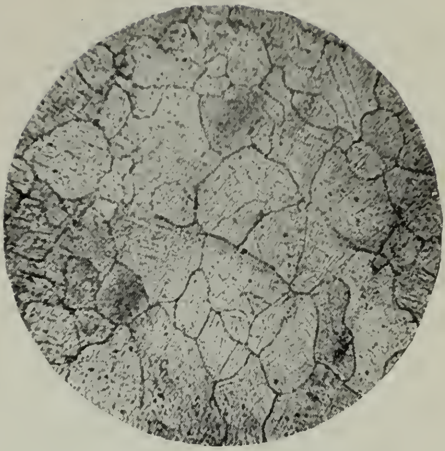
Fig. 6. 3-39B x 40.
Annealed at 680° C.

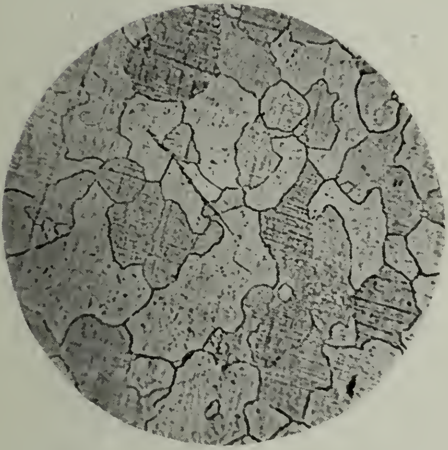
Fig. 7. 3-39 D and E, x 40.
Annealed at 900 and 950° C.

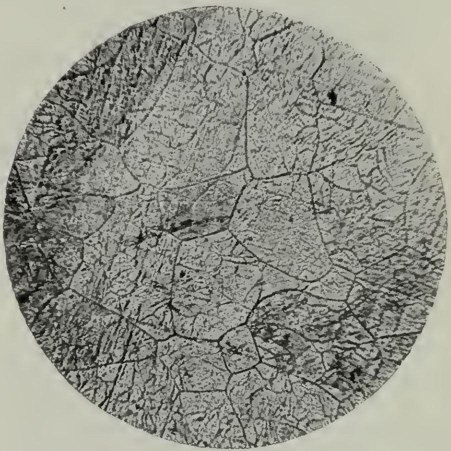
Fig. 8. 3-39S x 40.
Annealed at 900° C.
Cooled in 48 Hours.

Photomicrographs of Electrolytic Iron Melted in Vacuo, Showing Structure After Various Heat Treatments. Etched by Picric Acid.

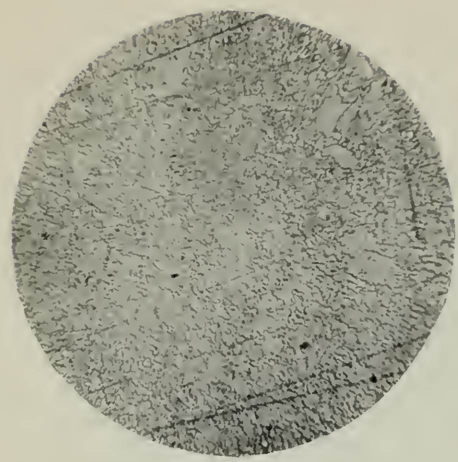

Fig. 9. 3B49A x 40.
As Forged.

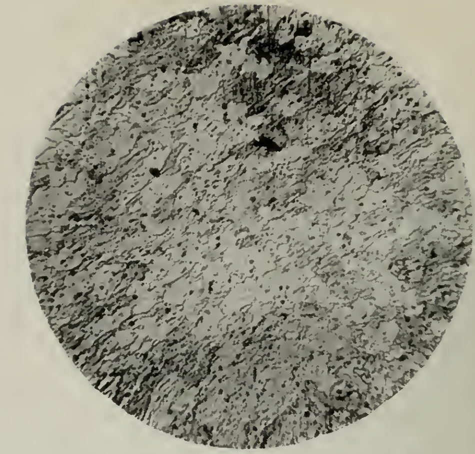

Fig. 10. 3B49 B and C, x 40.
Annealed at 530° and 700° C.

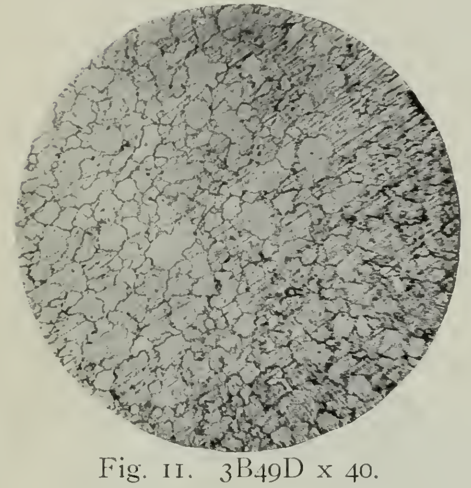

Fig. 11. 3B49D x 40.
Annealed at 800° C.

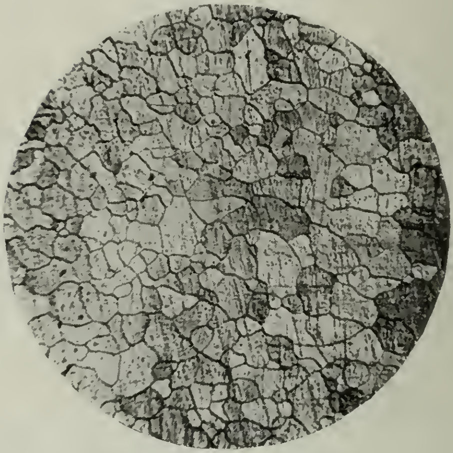

Fig. 12. 3B49E x 40.
Annealed at 906° C.

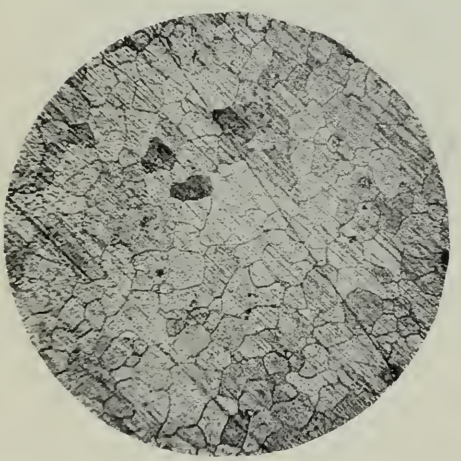

Fig. 13. 3B49F x 40.
Annealed at 953° C.

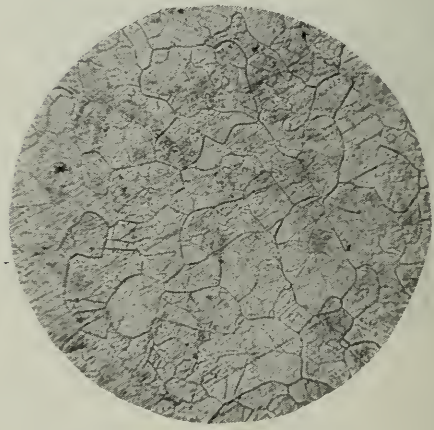

Fig. 14. 3B49J x 40.
Annealed at 900° C.
Cooled in 24 Hours.

Structure after Various Heat Treatments. Electrolytic Iron with 0.1% $B_7O$ Added. Melted in Vacuo. Chemical Analysis Gave a Boron Content of Less than .02%. Etched by Picric Acid.

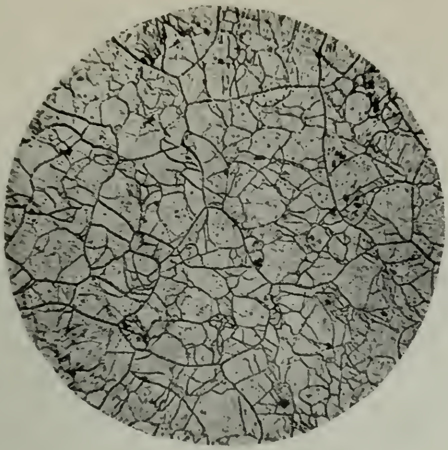

Fig. 15. 3B33 x 40.
.25% $B_2O$-Flux Added.
Chemical Analysis Gives: B-Trace.

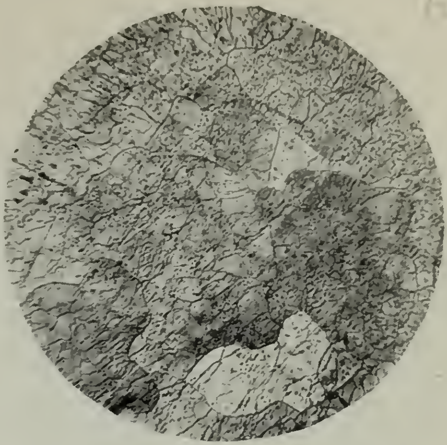

Fig. 16. 3B38 x 40.
2% $B_2O$-Flux Added.
Chemical Analysis Gives: B-.076%.

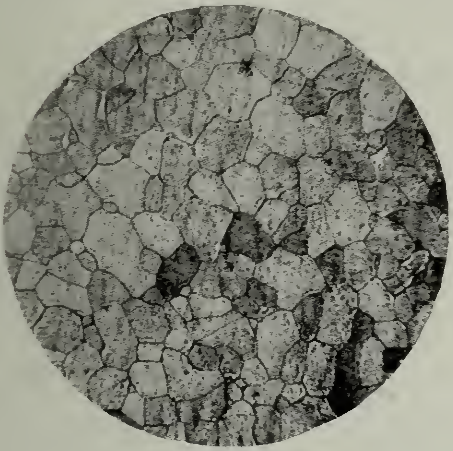

Fig. 17. 3B59 x 40.
.01% $B_2O$ Added.
Chemical Analysis Gives: B-Trace.

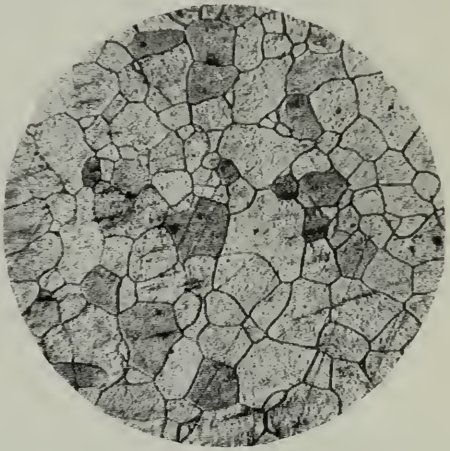

Fig. 18. 3B61 x 40.
.05% $B_2O$ Added.
Chemical Analysis Gives: B-Trace.

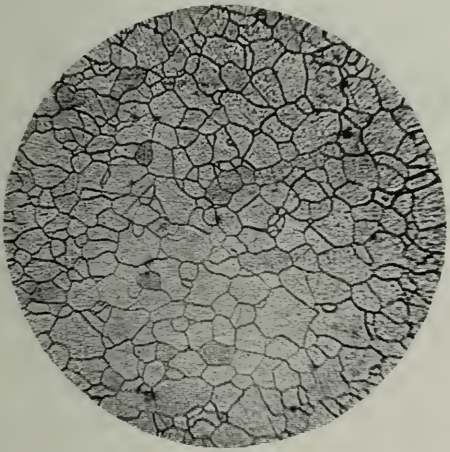

Fig. 19. 3B48 x 40.
.08% $B_2O$ Added.
Chemical Analysis Gives: B-Trace.

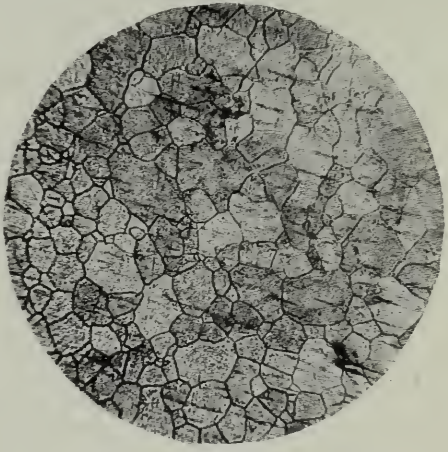

Fig. 20. 3B57 x 40.
0.4% $B_2O$ Added.
Chemical Analysis Gives: B-.102%.

Structure of Electrolytic Iron with $B_2O$ or $B_2O$-Flux Added. Melted in Vacuo. Annealed at 900° C. Cooled to 200° C in 24 Hours According to a Straight Line. Etched by Picric Acid.

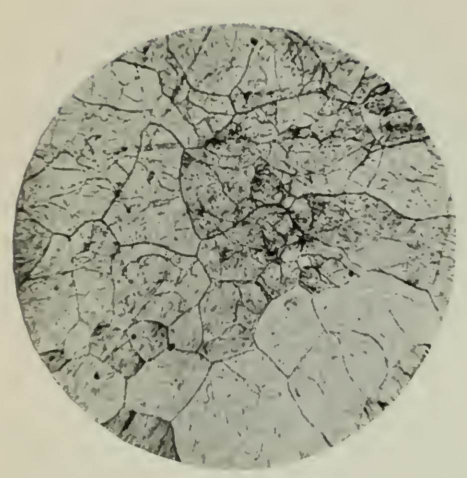

Fig. 21. 3B59 x 40.
.01% $B_7O$ Added.
Chemical Analysis Gives: B-Trace.

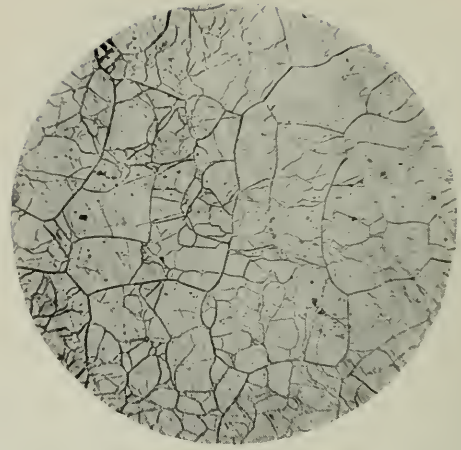

Fig. 22. 3B61 x 40.
.05% $B_7O$ Added.
Chemical Analysis Gives: B-Trace.

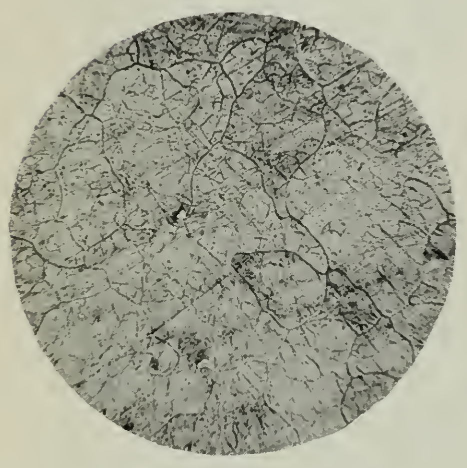

Fig. 23. 3B56 x 40.
0.3% $B_7O$ Added.
Chemical Analysis Gives: B-.08%.

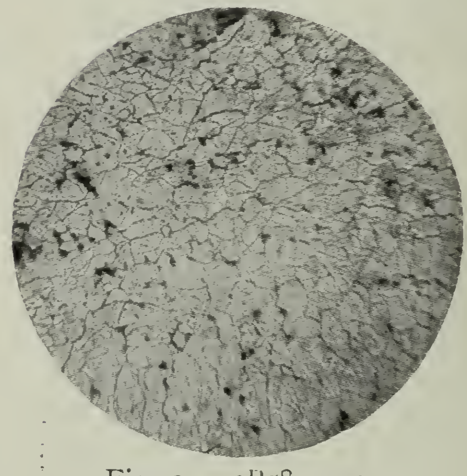

Fig. 24. 3B58 x 40.
0.5% $B_7O$ Added.
Chemical Analysis Gives: B-.180%.

STRUCTURE OF ELECTROLYTIC IRON WITH $B_7O$ ADDED. MELTED IN VACUO. ANNEALED AT 1100° C. COOLED FROM 900° TO 200° C. IN 24 HOURS ACCORDING TO A STRAIGHT LINE. ETCHED BY PICRIC ACID.

the amount of oxygen absorbed corresponding to 0.2 per cent boron is .44 per cent. This result agrees very well with that shown in Table 2, namely .42 per cent oxygen. In the case of the $B_7O$-flux series the agreement is not as close, the calculated oxygen content being from .33 to .375 per cent. It should be remembered, however, that the latter values are based upon the assumption that the $B_7O$ content of the flux is 8 per cent, and this may or may not be exact. By comparing Table 1 with Table 2 another interesting deduction may be made. In the case of carbon, practically none of this element is absorbed by the iron until the FeO has been reduced. In the case of boron, however, it is seen that while a large portion of it is oxidized, some of it combines with the iron, even though all the FeO is not reduced. Boron can consequently not be used as a reducing agent for iron without having a portion of it combine with the iron. The question then arises as to the effect of the combined boron upon the properties of the iron.

The effect of boron upon the magnetic and electrical properties of iron is shown in Figs. 2a, 2b, and 3. It is clearly seen that with an increase of the amount of boron added—whether in the form of $B_7O$ or $B_7O$-flux—the electrical resistance, the hysteresis loss, the coercive force, and the retentivity, all decrease slightly, until the amount of combined boron becomes measurable, when all these quantities begin to increase. Evidently then, boron has a deteriorating effect upon the magnetic quality of the iron. The beneficial effect for very small additions of boron must be attributed to the slight reduction of FeO offsetting the effect of the boron absorbed by the iron. While the increase in the electrical resistance of the iron is highly desirable for magnetic purposes, the increase obtained is altogether insufficient to offset the decreased magnetic quality.

The effect of boron upon the mechanical properties of iron is shown in Fig. 4. The curves have in general the same shape as those shown in Figs. 2a, 2b, and 3 and serve to confirm the deduction already made. They show, furthermore, that annealing has a marked softening effect upon the alloys.

In general, it may be said that boron, when entering into combination with pure iron, has a similar effect upon the properties of the iron as has carbon. Both elements increase the electrical resistance and the hysteresis loss, and decrease the permeability; furthermore, they both increase the mechanical strength of the iron. Their dissimilarity appears in their relative affinity for oxygen and molten iron in vacuo. While the iron will not combine with carbon until all the iron oxide is reduced, boron will be absorbed by iron and oxygen in the ratio of approximately 1 to 2. Consequently boron is not as suitable as a reducing agent for iron as is carbon.

With regard to the increase in the electrical resistance of iron due to boron—and to carbon as well—it is interesting to note that the results obtained in this investigation do not agree with the law suggested by Barrett*, and referred to in Bulletin No. 72, that "1 per cent of any element added to pure iron increases its specific resistance by an amount approximately proportional to the specific heat or inversely proportional to the atomic weight of the alloying element". To facilitate comparison, the table due to Barrett is repeated here.

---

*Proc. Royal Soc. 69, p. 480, 1902.

### TABLE 3.

### EFFECT OF DIFFERENT ELEMENTS UPON THE ELECTRICAL RESISTANCE OF PURE IRON. (FROM BARRETT.)

| Alloy of Iron with | Increase in Spec. Resist. for 1% Added of Element of Col. 1 microhms | Spec. Heat of Alloying Element | Atomic Weight of Alloying Element |
|---|---|---|---|
| Tungsten | 2.0 | .035 | 184 |
| Cobalt | 3.0 | .107 | 59 |
| Nickel | 3.5 | .109 | 59 |
| Cromium | 5.0 | .100(?) | 52 |
| Carbon | 5.0 | .160* | 12 |
| Manganese | 8.0 | .122 | 55 |
| Silicon | 13.0 | .183 | 28 |
| Aluminum | 14.0 | .212 | 27 |

*For graphite.

The present investigation gives the following results:

### TABLE 4.

### EFFECT OF BORON AND CARBON UPON THE ELECTRICAL RESISTANCE OF PURE IRON.

| Alloy of Iron with | Increase in Spec. Resist. for 1 Per Cent of Element of Col. 1 combined with the Iron microhms | Specific Heat of Alloying Element | Atomic Weight of Alloying Element |
|---|---|---|---|
| Boron | 6.2 | .238* | 11.0 |
| Carbon (Bull. No. 72) | 13.3 | .132† | 12.0 |

*At 30° C. †Graphite.

According to Barrett's law boron should increase the resistance of iron by 20 microhms or more. However, it is well known that boron and carbon are exceptions to Dulong and Petit's law, stating that "the product of the specific heat and atomic weight of solid metals is nearly constant," so that it is not surprising to find that these same elements can not be included in laws based upon the specific heat and atomic weight.

The results of the magnetic tests of the iron in the forged state, that is, before annealing, have not been shown in this bulletin, for the reason that the iron in such a state is not comparable in quality with the same iron after it has been annealed. This was shown in Bulletin No. 72. As to the best annealing temperature it was also shown that this lies at or above 900° C. Some uncertainty was expressed as to the desirability of annealing at 1100° C. Referring to Figs. 2a and 2b it will be seen that this uncertainty has now been removed. It is seen that the magnetic quality of the iron after having been annealed at 1100° C. is superior in every respect to what it was after having been annealed at 900° C. The induction at high densities does not appear to have suffered from the treatment, as was the case with some of the iron described in the previous bulletin.

The best results, magnetically, appear to have been obtained with additions of about .05 per cent $B_7O$ or 0.4 per cent $B_7O$-flux. In Table 5 these results are tabulated together with some exceptional values obtained, which fall outside the curves of Figs. 2a, 2b, and 3. For the sake of comparison the corresponding values for a commercial 4 per cent silicon steel have been included. This steel was obtained

TABLE 5.
IRON—BORON ALLOYS.
BEST RESULTS OBTAINED COMPARED WITH COMMERCIAL IRON.

| | .05% $B_7O$ added | .4% $B_7O$-Flux added | Exceptional Values obt'd. | Commercial 4% Silicon Steel |
|---|---|---|---|---|
| Spec. Elec. Resist. at 20°C.—microhms | 9.98 | 9.92 | | 51.15 |
| Hyst. Loss for $B_{max}$=10000—ergs per cc. per cycle | 970 | 870 | | 2260 |
| Hyst. Loss for $B_{max}$=15000—ergs per cc. per cycle | 1680 | 1450 | 1380 | 3030 |
| Coercive Force for $B_{max}$=10000—gilberts per cm. | .31 | .31 | .28 | .88 |
| Coercive Force for $B_{max}$=15000—gilberts per cm. | .33 | .31 | .28 | .88 |
| Retentivity for $B_{max}$=10000—gausses | 8100 | 8700 | 7700 | 5400 |
| Retentivity for $B_{max}$=15000—gausses | 9400 | 10800 | | 5400 |
| Max. Permeability, $\mu=\frac{B}{H}$ | 15300 | 17300 | 22300 | 3400 |
| Densitv for max. Permeability—gausses | 7000 | 9000 | 10500 | 4300 |
| Induction, B, for H=1—gausses | 12000 | 13200 | 14020 | 3200 |
| Induction, B, for H=20—gausses | 16200 | 16300 | 16500 | 15600 |

from the manufacturer in the form of bars suitable for the permeameter, and had received the manufacturer's standard heat treatment. It is supposed to represent at the present time high grade iron for magnetic purposes.

The photomicrographs, Figs. 5 to 24 inclusive, show how boron affects the structure of the iron. Comparing first Figs. 5 to 8, representing a pure electrolytic iron sample, with Figs. 9 to 14, representing iron with a trace of boron, both having been treated approximately alike, the differences are very evident. As forged, the boron iron has a decidedly finer structure than the pure iron; but as the annealing temperature increases, the crystals grow in size, so that after having been annealed at 900° C. they are about half the size of the pure iron crystals. The size of the latter is practically unaffected by the heat treatment until the iron is annealed above 900°. Above 900° both the pure iron and the boron-iron crystals appear to break up into smaller crystals, as may also be seen by comparing Figs. 15 to 20, representing boron-iron annealed at 900°, with Figs. 21 to 24, representing boron-iron, annealed at 1100° C. The amount of boron does not seem to affect the size of the crystals to any considerable extent after the iron has been annealed at 900° or above, unless the boron content is above 0.1 per cent. Fig. 24, representing an alloy containing .18 per cent boron, shows a much smaller crystalline structure than the rest of the figures.

Fig. 24 is also the only one that indicates definitely the presence of boron. Hannesen has shown, in the article referred to in the introduction, that boron in iron-boron alloys appears in a eutectic, containing 4 per cent boron, as dull crystals of the compound $Fe_5B_2$ against a background of bright ferrite crystals. The case is similar to that of iron-carbon alloys in which carbon appears in a eutectoid, containing 0.9 per cent carbon, as dark strata of $Fe_3C$ among strata of ferrite. His lowest boron alloy, containing .25 per cent boron, has the same appearance as Fig. 24, representing an alloy containing .18 per cent boron, and it is probable that the dark spots, under a high magnification, would reveal strata of $Fe_5B_2$ and ferrite.

## V. SUMMARY AND CONCLUSIONS.

The results recorded in this bulletin may be summarized as follows:

1. Small percentages of boron added to pure iron and melted in vacuo have a double effect upon the iron, since part of it reduces the iron oxide present and the remainder combines with the iron. In this respect boron differs from carbon, as the latter does not combine with

the iron unless added in larger quantities than are needed to reduce all the iron oxide present.

2. Small additions of boron to iron have a slightly beneficial effect upon the magnetic properties, probably on account of the reduction of iron oxide which more than offsets the effect of the combined boron. The beneficial effect in the present case reaches a maximum with additions of .05 per cent $B_7O$ or 0.4 per cent $B_7O$-flux, and ceases as soon as the amount of combined boron becomes measurable, which occurs with the addition of 0.1 per cent $B_7O$ or .75 per cent $B_7O$-flux.

3. Boron added in a sufficient amount to leave a measurable quantity combined with the iron has a decidedly detrimental effect upon the magnetic properties of the iron.

4. Boron increases the specific electrical resistance of iron by .62 microhms per 0.1 per cent boron combined with the iron.

5. The effect of boron upon the mechanical properties of iron is in general similar to that of carbon. It raises the elastic limit and increases the ultimate strength, at the same time decreasing the toughness.

Since the autumn of 1914 the investigations of the properties of iron and iron alloys have been carried on under somewhat modified conditions. In particular it may be noted that the pressure under which the iron is molten has been reduced from 2-5 mm. of mercury to about 0.5 mm. and that the annealing is being carried on in a vacuum of 0.2 mm. Furthermore, the apparatus for measuring the magnetic and electrical properties has been improved, so that a higher accuracy is obtained than has been possible heretofore. The results of the investigations now being carried on will probably be ready for publication before the end of the year.

In conclusion the author wishes to express his appreciation of the conscientious work done by Mr. J. M. Lindgren in connection with the chemical analysis, and by Mr. Rudolph McDermet in connection with the metallurgical analysis. Mr. J. H. Belt has also rendered valuable assistance.

# APPENDIX.

## THE DETERMINATION OF BORON IN IRON.

(BY J. M. LINDGREN.)

The present work has had to deal with the determination of small percentages of boron in iron. Two procedures have been tried. First, the well known method of Gooch*, which consists in the formation of methyl borate and distilling the same in a properly arranged apparatus, recovering the boron in the form of calcium borate. The complications involved in this method are accentuated by reason of the large quantity of iron oxide and the difficulty of complete distillation, requiring a repetition of that part of the process at least six times.

The other method made use of is that proposed by Wherry†, which involves the precipitation of the iron, filtering the boric acid and titrating the same with a standard solution of alkali. The application of this method to the present situation made it necessary to determine whether it were possible to recover the relatively small amounts of boric acid in the presence of such heavy precipitations of ferric hydrate. After considerable experimentation this was found to be entirely feasible without departing in any marked degree from the principles involved in the method as set forth by Wherry. The details of the procedure are as follows:

Two to three grams of iron are dissolved in a round bottom Jena flask of about 800 cc. capacity using 10 cc. each of nitric acid sp.g. 1.4, hydrochloric acid sp.g. 1.2, and water. When the solution is complete it is cooled to room temperature and the iron precipitated by use of c.p. calcium carbonate. If not properly carried out this precipitation may result in the formation of colloidal hydrate. It will be noted that the solution consists of approximately 30 cc. If to this there is added all at one time approximately double the amount of dry calcium carbonate necessary for complete neutralization, stirring the solution by rapid rotation of the flask or otherwise vigorously agitating the mixture, a pasty or solid mass results which, upon the addition of hot water, yields a ferric hydroxide in the form of a dark brown and very granular precipitate. Two hundred and fifty to three hundred cc. of carbon dioxide free water are added and boiled with a reflux condenser for at least 30 minutes to assure entire absence of carbon dioxide or bicarbonate.

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*American Chemical Journal, 9, 23 (1887).

†Journal American Chemical Society, 30, 1687 (1908).

Before filtering and in order to still further open up the precipitate to admit of thorough washing, there is now added a considerable quantity of asbestos fiber which has been well washed. A good method for preparing the asbestos is to digest over night on a steam bath a considerable quantity in dilute hydrocholric acid. This should be well washed by suction and the mass returned to the beaker and covered with water which should be kept at the boiling temperature while in use. The wet fibre is added to the precipitated iron. The amount needed in each process would be approximately 50 cc. in bulk, sufficient to thoroughly impregnate the precipitated iron. After boiling for a few minutes the precipitate is filtered at once by means of suction and a splash trap onto a Büchner funnel about 15 cm. in diameter having a double thickness of filter paper carefully fitted over the bottom. It should be washed eight or ten times with well boiled distilled water. By using boiling hot water in the washing, upon the removal of the Büchner funnel and replacing a solid stopper in the neck of the flask a continuation of the suction would reduce the pressure thus producing ebulition resulting in a renewal of the boiling without the addition of further heat. When the boiling has ceased drain back the splash trap and cool under the tap. When cooled to room temperature titrate with standard hydroxide solution of approximately N/10 strength. This solution should also be free from carbon dioxide. Use 4 or 5 drops of phenolphthalein as indicator, titrate with the alkali to a pink color, then add about one gram of mannite. Stir well and continue the titration with the further addition of mannite until a permanent pink color is obtained. Familiarity with the end point is attained by practice. A blank should be carried through using pure iron and following the process exactly as described.

In testing the accuracy of the method, standard solutions were prepared of c.p. boric acid, $B_2O_3$, which had been fused. Known quantities of this material were added to a solution of iron free from boron and carried out as indicated. Contrary to the experience of Wherry when the precipitation by use of calcium carbonate was properly carried out it was found possible to wash the iron entirely free from boric acid and recover the full amount in the filtrate. The use of the boric acid solution made up of the pure fused material as above described also served admirably as a medium for standardizing the alkali used in the titrations.

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 78 APRIL, 1915

# A STUDY OF BOILER LOSSES

BY A. P. KRATZ, ASSISTANT IN MECHANICAL ENGINEERING,
ENGINEERING EXPERIMENT STATION

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# A STUDY OF BOILER LOSSES.

## I. Introduction.

1. *Purpose of the Tests.*—The experiments herein described were undertaken to determine the conditions prerequisite for the continuous operation of the boilers in the new power plant at the University of Illinois, and to permit a detailed study of the boiler and furnace losses under varying conditions of load, depth of fuel bed, and draft. It was hoped that an analysis of the data obtained might render it possible to isolate and determine the amount of the several losses in the boiler, furnace, and setting, thus indicating where improvements could be made in order to increase the over-all operating efficiency.

No attempt was made to obtain record tests and in every case ordinary operating conditions prevailed, with the exception of the maintenance of approximately constant load on the boiler under test. The boiler was operated as one of a battery of two boilers which delivered steam directly into the mains connected with the old plant. The pressure maintained was thus governed by conditions prevailing in the system as a whole.

While a majority of the trials were made with fresh coal from the Mission Field Mine No. 3, at Danville, Illinois, a series was also made using weathered coals from mines in Sangamon, Williamson, and Vermilion counties, to permit a comparison with the tests previously made, and, if possible, to determine the effect of weathering upon the steaming qualities of the coal.

2. *Acknowledgments.*—The experiments described in this bulletin were undertaken with the approval of C. R. Richards, Professor of Mechanical Engineering and Acting Dean of the College of Engineering and Acting Director of the Engineering Experiment Station of the University of Illinois, and were carried out under his supervision. To him the writer is indebted for many valuable suggestions and also for his assistance in criticizing and revising the manuscript. Acknowledgment is also due to Mr. H. F. Geist for his assistance in running the tests and computing the results, and to Mr. W. E. Alley for his help in installing and maintaining the apparatus and running the tests.

## II. THE PLANT.

3. *Boilers.*—The boiler on which these tests were run is one of a battery of two Babcock and Wilcox boilers in the new power plant of the University of Illinois. It is designated as boiler No. 1. The second unit of the battery is placed between boiler No. 1 and the chimney. The battery, as a whole, has 10,160 sq. ft. of heating surface, equally distributed between the two boilers, and is designed to carry a working pressure of 160 lb. per sq. in. Each boiler has two 42 in. by 20.33 ft. drums, and 18 sections of 4-in. tubes 18 ft. long, each section containing 14 tubes. They are set so that the distance from the floor to the first row of tubes is 8½ ft. at the front end.

A cross section through the plant is shown in Fig. 1 and a view of the boiler fronts in Fig. 2. From Fig. 1 it may be seen that a style of baffling different from the standard Babcock and Wilcox type is in use. This construction provides a tile roof over the furnace and makes the attainment of a long combustion chamber possible.

The first pass for the gases is thus at the rear of the setting and

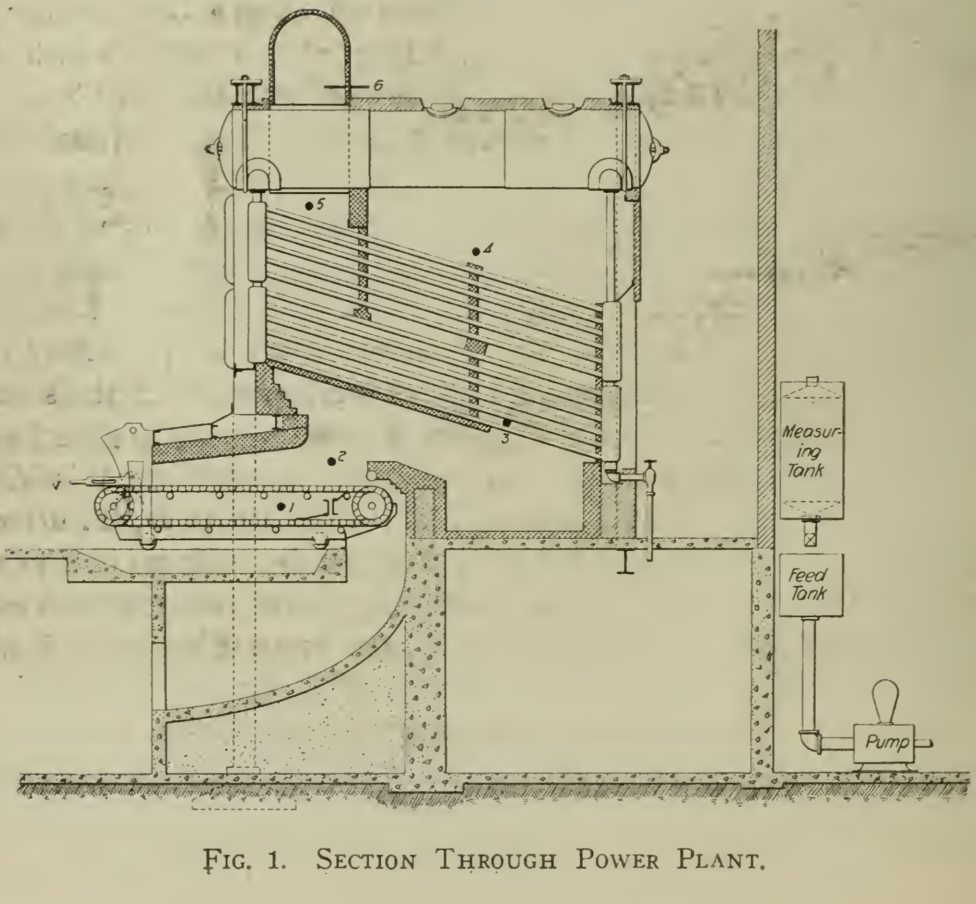

FIG. 1. SECTION THROUGH POWER PLANT.

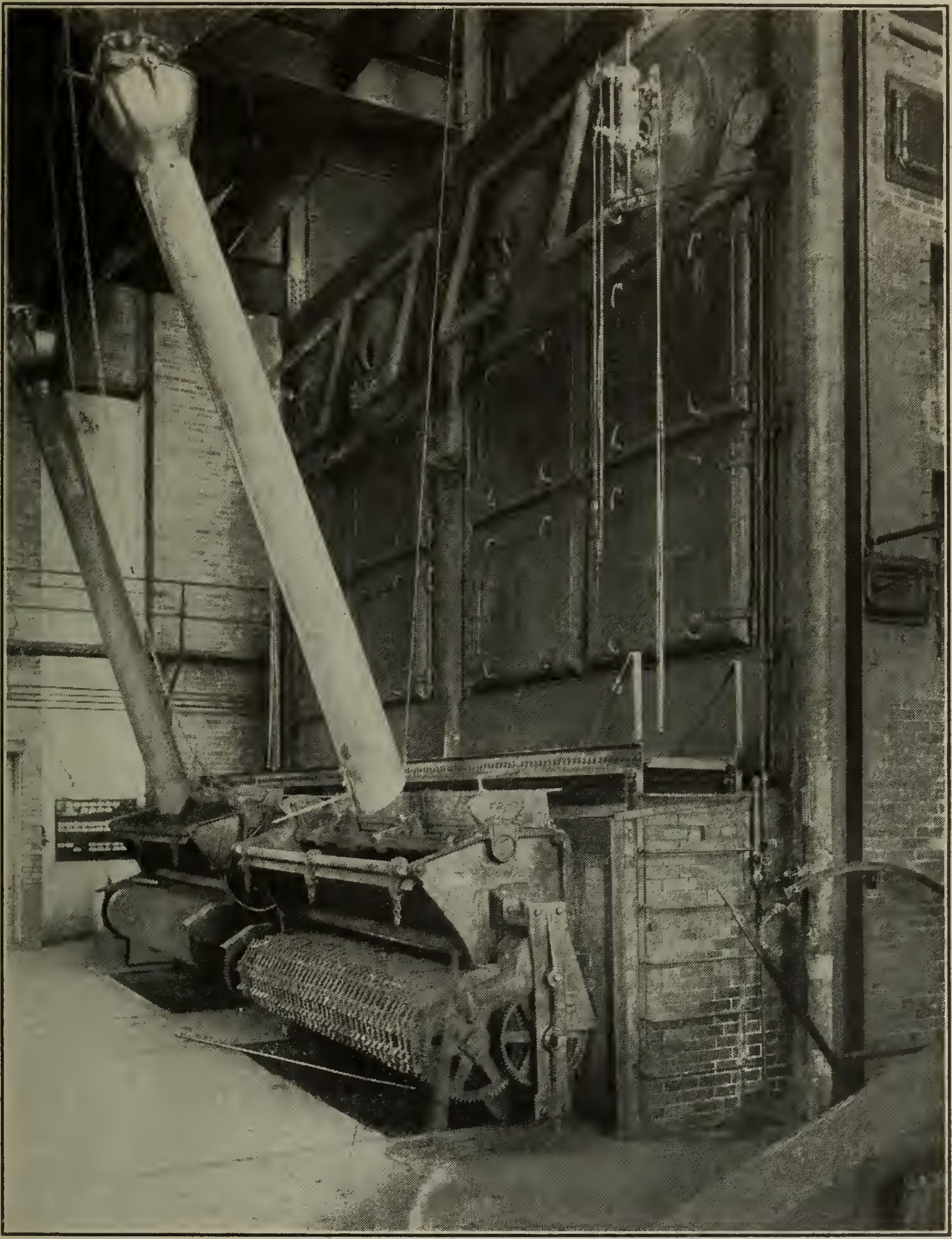

FIG. 2. BOILER FRONTS.

they are discharged into a breeching at the front instead of at the back of the boiler, as in the standard type.

4. *Stokers.*—The stokers are of the chain grate type, having an active grate surface of 90 sq. ft., and were built by the Green Engineering Company. At the time these tests were made, each stoker had a flat fire arch 6 ft. long, set 10 in. above the grates at the front end and 18½ in. at the back. This arch has since been changed so that at the present time it is 15 in. above the grates at the front and 33 in. at the back.

The draft is produced by means of a brick chimney, 175 ft. high, having an internal diameter of 10 ft. This was designed with the expectation that it was to serve an additional battery similar to the one under discussion, but at the time the tests were run the second battery had not been installed.

The feed water is delivered by means of two 14 in. by 8½ in. by 12 in. American steam pumps, the feed mains being arranged so that either pump can feed both boilers as a unit, or so that each can feed its corresponding boiler independently. The feed water consists of condensed steam from the heating system, with a small quantity of "make-up" water, supplied principally through the water backs of the stokers. The water is heated by means of a Webster "Star-Vacuum" heater before it reaches the pump.

5. *General Dimensions.*—The following is a table of the principal dimensions.

| | |
|---|---|
| Number of steam drums | 2 |
| Length of steam drums, feet | 20.33 |
| Diameter of drums, inches | 42 |
| Number of tubes | 252 |
| Outside diameter of tubes, inches | 4 |
| Inside diameter of tubes, inches | 3.75 |
| Mean length of tubes exposed to gases, feet | 18 |
| Width of grate, feet | 9.46 |
| Effective length of grate, feet | 9.50 |
| Area of grate surface, square feet | 90 |
| Area of air space in grate, square feet | 19.6 |
| Ratio of air space to grate area | 0.218 |
| Height of ignition arch at front, inches | 10 |
| Height of ignition arch at back, inches | 18.5 |
| Length of ignition arch, feet | 6 |
| Height of chimney above grate, feet | 175 |

| | |
|---|---|
| Sectional area of chimney at top, square feet | 78.5 |
| Height of gas passage over bridge wall, inches | 33 |
| Area of gas passage over bridge wall, square feet | 26.8 |
| Minimum area through 1st pass, square feet | 27.7 |
| Minimum area through 2nd pass, square feet | 18.1 |
| Area between front baffle and tile on lower tubes, square feet | 23.2 |
| Minimum area through 3rd pass, square feet | 19.0 |
| Total effective area of openings into breeching, square feet | 15.2 |
| Total water heating surface, square feet | 5120 |
| Ratio of water heating surface to grate surface | 57.0 |

## III. Methods of Conducting Tests.

With a few modifications, the methods employed in these experiments were those set forth in the A. S. M. E. code for conducting boiler trials.

6. *Starting and Stopping.*—As previously stated, the boiler tested forms part of the plant equipment, and as such was in continuous operation. Before starting each test, however, the gates were set for the predetermined thickness of fire, and the boiler operated for a period of from 1½ to 2 hours, carrying the load decided upon for that test. The coal bunker supplying the boiler under test was kept filled with Mission Field coal during the entire period over which tests were being made, thus obviating the necessity for emptying it before each run. In the case of the weathered coal, however, the fuel was piled on the floor of the boiler room and before starting the test the coal hopper on the stoker was allowed to empty. It was then filled with the fuel under test and the grates run until they were completely covered with the new fuel. This took from 45 minutes to one hour. It may, therefore, be seen that at the start of each test the grates were covered with the required fuel and the boiler had been running under the predetermined conditions of load and thickness of fuel bed for from 1½ to 2 hours.

On starting a trial the condition of the fuel bed was observed and recorded and an effort was made to have this condition duplicated at the close of the test. The start was made with the ash pit and drip pan clean and the fuel in the hopper just level with the top. The test was closed under the same conditions.

The condition of the fuel bed was maintained as nearly uniform as possible throughout a test and no adjustment was necessary at the end.

7. *Coal.*—A small steel car resting on platform scales was used to weigh the coal. From this car it was shoveled into the hopper. The tests were started with a full car of coal on the scales, the drip pan and ash pit clean and with the fuel in the hopper just level with the top. At the end of each hour the drip pan and ash pit were again cleaned, the hopper leveled, and the coal remaining in the car weighed. Thus the coal fed to the grates was the total of the coal weighed during the hour minus that remaining on the scales at the end.

Before weighing a car of coal two samples were taken from it and placed in tight cans. At the end of the test one sample was quartered down in the usual manner to a weight of about 3,000 grams, placed in a shallow pan, weighed on a Troemer balance and set on top of the boiler to dry. After drying it was weighed a second time to determine the moisture loss, then quartered down to fill a quart jar and sent to the chemist for analysis. Proximate analyses were made on all of the samples. Ultimate analyses also were made on four samples of the Mission Field coal from the average of which ultimate analyses were calculated for the rest of the samples by making use of the individual proximate analyses. In the case of the weathered coal, however, ultimate analyses were necessarily made from each sample.

The second sample referred to above was spread out and allowed to dry, after which it was weighed and screened. The remainder was then weighed a second time and from the two weighings the percentage of dust was calculated. The screen used had ¼-in. round openings and what passed through it has been designated as "dust."

8. *Ash.*—The "ash and refuse" was raked from the pit into wheelbarrows and weighed on platform scales. A sample was taken from each load and placed in a tight can. At the end of the test this was quartered down until it could be contained in a quart jar and was then sent to the chemist for analysis. This analysis was used in determining the carbon loss in the ash. Analyses of both coal and ash were made in the Chemical Laboratory at the University of Illinois.

9. *Water.*—The feed water was measured by means of two cylindrical tanks about 4 ft. in diameter and of 4,600 lb. capacity. These tanks, as shown in Fig. 3, had conical heads tapering to the size of a standard 6-inch nipple and were emptied through quick opening valves at the bottom. By this arrangement the error in filling was reduced to a minimum. They were calibrated at about 155° F., the calibration including all of the volume from the valve to the top of the nipple on the upper head. Making use of these data, a curve was calculated, taking

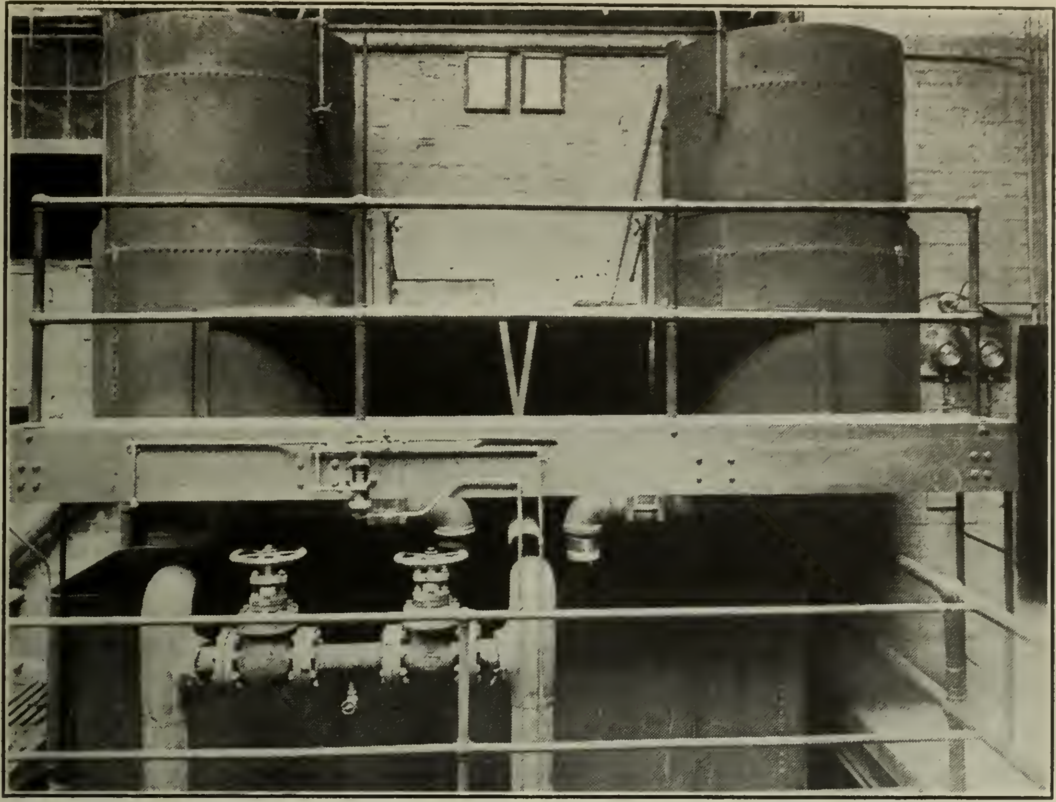

FIG. 3. FEED WATER TANKS.

into account the change in volume of the tank as well as the change in density of the water, from which the weight of water contained in the tank could be determined when its temperature was known. The temperature was then taken just before emptying a tank.

The measuring tanks emptied into a rectangular feed-tank 40 in. deep and of about 8,600 lb. capacity, from which the water gravitated to the feed pump supplying the boiler under test. This tank was also calibrated and a curve plotted giving the weight of water per inch of depth at any given temperature. The height of water in the tank was shown by a pointer which was attached to a float and moved over a scale having 0.25-in. divisions.

The boiler itself was calibrated and a family of curves plotted such that when the height of water in the gage glass and the pressure were known the total weight of water in the boiler could at once be determined.

Both boilers were fed by a single main, the two feed pumps discharging into it at points within about 3 feet of each other. Two valves were inserted in the main between the points where the pumps discharged and a bleeder was placed between the valves. Since the bleeder was always open during a test any leakage through either valve could be detected immediately. In this way the two parts of the main were completely isolated from each other and one pump could be used to deliver the water from the weighing tanks to the boiler under test, while the other was being used to feed the remaining boiler and to deliver water from the heater to the weighing tanks.

In the installation under discussion the water-back formed a circulating system entirely separate from the boiler, the water passing through it being measured by means of a calibrated meter and afterwards discharged into the feed water heater. It is not unusual for the water-back to be connected into the boiler in such a way as to form a closed system and to utilize the natural circulation taking place in the latter in which case it forms part of the heating surface the same as one of the tubes. If the water-back is disconnected from the boiler and a positive means of circulation adopted, the circulating water being taken from the feed water and returned to it subsequent to taking the weight and the temperature of the latter, the result is essentially the same as though the closed system had been used, and no account need be taken of the water thus circulated. In the present case, however, the temperature was taken close to the boiler and after the circulating water had gone into the feed water. The water-back has, therefore, been considered as part of the heating surface of the boiler and the heat absorbed by the circulating water has been reduced to terms of equivalent water evaporated and added to the boiler output as discussed under feed water corrections in the Appendix.

10. *Flue Gas.*—Flue gas samples were collected over half-hour periods. One sample was taken at the breeching about 2 ft. below the dampers and a second one at a point about 8 in. behind and below the end of the tile roof of the combustion chamber. This point was about 34 in. back of the bridge wall at the point where the gases turned to go up through the first pass. It was close enough to the furnace so that no air leakage chargeable to the boiler setting had as yet occurred, and far enough back so that combustion was complete, as may be seen from a comparison of the CO in the gas there and at the dampers.

The sampler used in the breeching is shown in Fig. 4 and the breeching itself in Fig. 5. In the latter it may be seen that the boiler had 3 nozzles opening from the last pass into the breeching with a damper in each one, the dampers being operated simultaneously from one control rod. A set of sample tubes cross connected as shown in Fig. 4 extended into each one of these passages. The two gate valves shown at A and B

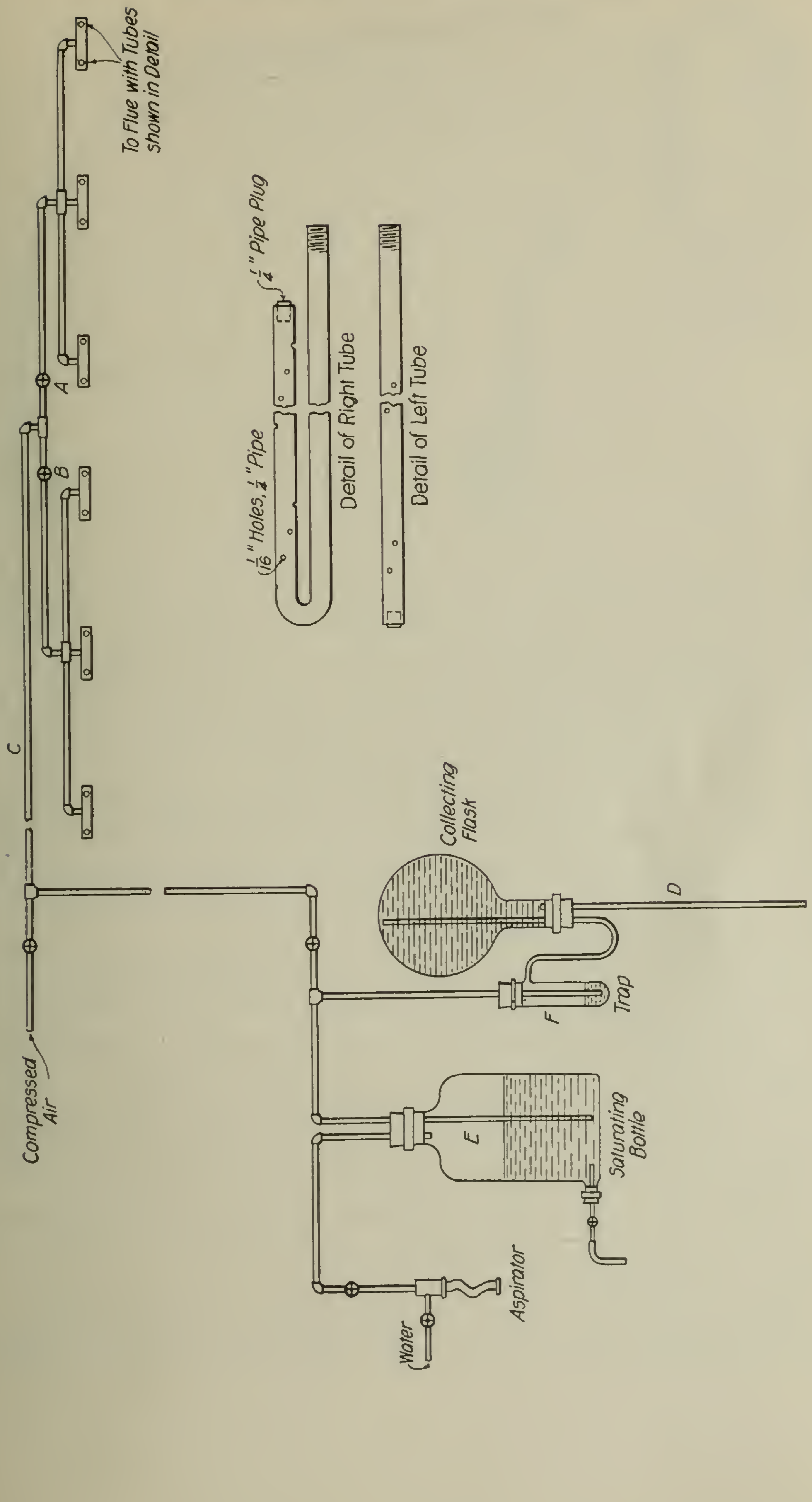

Fig. 4. Gas Sampling Apparatus for Flue.

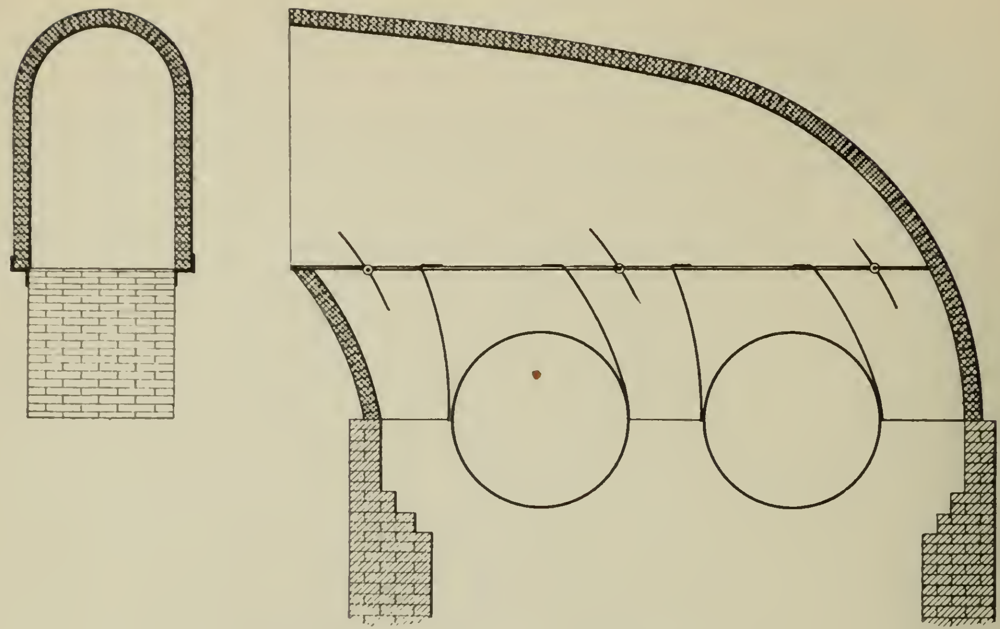

Fig. 5. Section Through Breeching.

made it possible to obtain a sample from either boiler. Before starting the tests the valve at B was closed and the whole apparatus tested for leaks by means of compressed air. The valve B then remained closed during the period over which tests were run. In order to keep the small holes open the tubes were blown occasionally with compressed air. The system was tested frequently to see that it did not leak. A continuous flow of gas was maintained through the pipe C by means of a small laboratory aspirator placed near the floor line. The sample bottle was attached at a tee near this point and any back flow of gas from the bottle was prevented by placing a small mercury trap (F) in the line between the latter and the tee.

The sampler used for the furnace gases is shown in Fig. 6. This consisted of a 2½-in. pipe extending in to the center of the furnace. Five small tubes were brazed into the pipe and water was kept flowing through the apparatus in order to prevent it becoming hot enough to react on the gas. It was assumed that the composition of the gas varied the same either side of the center toward the furnace walls and also that it was fairly well mixed by turning the corner at the point where the sample was taken. While this does not insure a strictly average sample, it is, however, believed to be within the limits of accuracy of the rest of the observation in boiler trials.

The gas samples were taken in one liter flasks as indicated in Fig. 4. Before being attached the latter were filled with water and inverted.

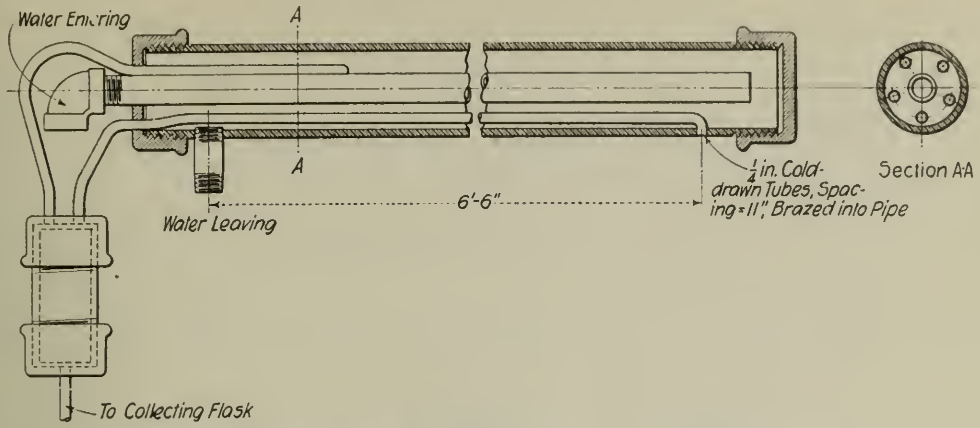
Water Entering
A
A
Water Leaving
6'-6"
$\frac{1}{4}$ in. Cold-drawn Tubes, Spacing = 11", Brazed into Pipe
Section A-A
To Collecting Flask

FIG. 6. GAS SAMPLING APPARATUS FOR FURNACE.

They were then allowed to empty under a head of about 3 ft., the rate of drawing the sample being regulated by means of a glass stop cock in tube D so that as nearly uniform flow as possible was maintained over the half-hour periods. A bottle of water was placed at E, between the sampler and aspirator, and all of the gas was drawn through it, thus keeping a supply of gas-saturated water for use in the flasks.

11. *Quality of Steam.*—The quality of steam was taken by means of a throttling calorimeter and the steam pressure was taken by a calibrated gage at this point.

12. *Temperature.*—Calibrated mercury thermometers were used for taking all temperatures, with the exception of that of the gas leaving the boiler. The latter was taken at points about 2 ft. below the dampers by means of 3 copper-constantan thermo-couples and a Siemens-Halske millivoltmeter. These were calibrated with the cold junctions at 100 degrees F. This temperature was read during each test and a correction applied in case it was not 100° F., the correction in no case being larger than about 5° F. The couples were calibrated before the tests and were then left in place for the rest of the series, a period of about 3 months. On being re-calibrated at the end of this time the new points fell on the curve of the previous calibration. It is, therefore, believed that the accuracy of the apparatus is within 5° F.

13. *Drafts.*—Drafts were taken at the points indicated in Fig. 1 by means of Ellison differential gages. The draft at (6) was taken at a point about 2 ft. below the center damper.

14. *Observation Periods, etc.*—All readings of pressure, temperature, drafts, etc., were taken at 15-minute intervals. The tests were of sufficient length to give a total coal consumption of 250 lb. per sq. ft.

of grate surface, with the exception of the weathered coal tests, which were necessarily limited by the amount of the samples. The heat balance shows a normal unaccounted-for loss for the latter, however, and it is believed that the accuracy has not suffered materially on this account.

## IV. Discussion of Results.

The general results of the tests are shown in Tables 1 to 16. The items in these tables are in accordance with the Boiler Test Code of the American Society of Mechanical Engineers, with the exception that some items have been added. These consist of three efficiencies, viz.: The efficiency of furnace and grate, the efficiency of the furnace, and the efficiency of the boiler exclusive of furnace and grate, and also some items referring to gas analysis at the furnace and to weights of air and

### Table 1.

### General Conditions.

| Test Number | Date | Duration, Hours | Thickness of Fuel Bed, Inches | Pressure, Lb. Per Sq. In. | | | Draft, Inches of Water | |
|---|---|---|---|---|---|---|---|---|
| | | | | Barometric | Steam (Gage) | Steam (Abs.) | Between Damper and Boiler | At End of 3rd Pass |
| Col. No. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| * a | 1 | 2 | 81 | | 11 | | 12 | |
| ‡ b | 2 | 3 | | 6 | 8 | 9 | 10 | 11 |
| 1 | 11–11–'13 | 13.0 | 6.0 | 14.4 | 128.2 | 142.6 | 0.091 | 0.134 |
| 2 | 11–18–'13 | 13.0 | 7.0 | 14.3 | 132.0 | 146.3 | 0.053 | 0.102 |
| 3 | 11–13–'13 | 13.0 | 8.0 | 14.4 | 131.4 | 145.8 | 0.077 | 0.118 |
| 4 | 11–20–'13 | 13.0 | 10.0 | 14.3 | 133.0 | 147.3 | 0.263 | 0.310 |
| 5 | 11–25–'13 | 9.0 | 6.0 | 14.4 | 129.4 | 143.8 | 0.379 | 0.433 |
| 6 | 11–29–'13 | 9.0 | 7.0 | 14.4 | 137.9 | 152.3 | 0.254 | 0.307 |
| 7 | 12– 2–'13 | 9.0 | 8.0 | 14.5 | 132.5 | 147.0 | 0.358 | 0.397 |
| 8 | 12– 9–'13 | 9.0 | 9.3 | 14.5 | 132.0 | 146.5 | 1.038 | 1.056 |
| 9 | 12– 4–'13 | 9.0 | 9.8 | 14.4 | 135.1 | 149.6 | 0.726 | 0.767 |
| 10 | 12– 5–'13 | 9.0 | 6.0 | 14.3 | 138.1 | 152.4 | 0.522 | 0.568 |
| 11 | 12–12–'13 | 9.0 | 6.0 | 14.5 | 134.2 | 148.7 | 0.984 | 1.011 |
| 12 | 12–13–'13 | 9.0 | 7.0 | 14.5 | 138.7 | 153.2 | 1.124 | 1.138 |
| 13 | 12– 6–'13 | 9.0 | 7.0 | 14.2 | 136.3 | 150.5 | 0.500 | 0.540 |
| 14 | 12– 8–'13 | 9.0 | 8.0 | 14.6 | 131.4 | 146.0 | 0.591 | 0.637 |
| 15 | 12–16–'13 | 9.0 | 9.4 | 14.4 | 131.4 | 145.8 | 1.126 | 1.127 |
| 16 | 12–19–'13 | 9.0 | 6.0 | 14.3 | 138.3 | 152.6 | 1.090 | 1.094 |
| 17 | 12–18–'13 | 9.0 | 7.0 | 14.4 | 133.9 | 148.3 | 1.147 | 1.153 |
| 18 | 12–15–'13 | 9.0 | 8.0 | 14.5 | 135.2 | 149.7 | 1.139 | 1.149 |
| 19 | 1–29–'14 | 7.0 | 7.0 | 14.4 | 141.0 | 155.4 | 1.124 | 1.160 |
| 20 | 12–30–'13 | 5.5 | 7.0 | 14.4 | 132.7 | 147.1 | 1.090 | 1.108 |
| 21 | 1– 2–'14 | 3.5 | 6.5 | 14.2 | 138.0 | 152.2 | 1.048 | 1.110 |
| 22 | 1–10–'14 | 4.5 | 7.0 | 14.4 | 134.0 | 148.4 | 1.250 | 1.240 |
| 23 | 1– 3–'14 | 6.0 | 7.0 | 14.3 | 134.7 | 149.0 | 1.055 | 1.159 |
| 24 | 1– 5–'14 | 6.0 | 7.0 | 14.5 | 133.0 | 147.5 | 1.225 | 1.206 |

* (a) Numbers correspond to the A. S. M. E. Code.
‡ (b) Numbers correspond to Calculating Code given in Appendix.

gas per lb. of coal. The heat balance has also been considerably extended and is based on 1 lb. of dry coal fired instead of on combustible.

The efficiencies given in the A. S. M. E. code are the over-all efficiencies for the boiler and furnace together and do not afford a means of determining whether a loss in efficiency of the unit is due to poor furnace construction or conditions or to dirty tubes or faults in the boiler itself. It was for this reason that the efficiencies indicated above were added and that the heat balance was extended and divided so as to separate from one another the losses chargeable to boiler and furnace alone. In order to do this it was necessary to obtain an analysis of gas at the furnace and to include some items on weights of air and gas. A discussion of all these items may be found in the Appendix.

### TABLE 2.

### GENERAL CONDITIONS (CONTINUED).

| Test Number | Draft, Inches of Water | | | | Temperature, Degrees Fahrenheit | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Between 1st & 2nd Pass | At End of Combustion Chamber | Over Fire in Furnace | In Ash Pit | Outside Air | Fire Room | Steam | Feed Water at Boiler | Flue Gases |
| Col. No. | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| a | | | 13 | 14 | 15 | 16 | 17 | 20 | 21 |
| b | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 21 |
| 1 | 0.045 | ..... | 0.088 | 0.002 | 36.0 | 74.0 | 354.5 | 137.3 | 451.0 |
| 2 | 0.020 | 0.079 | 0.072 | 0.001 | 64.8 | 78.1 | 356.5 | 123.2 | 445.7 |
| 3 | 0.048 | 0.117 | 0.108 | 0.004 | 58.8 | 73.9 | 356.1 | 127.3 | 435.3 |
| 4 | 0.168 | 0.217 | 0.184 | 0.009 | 69.6 | 83.5 | 357.0 | 114.0 | 460.7 |
| 5 | 0.208 | 0.219 | 0.171 | 0.032 | 48.8 | 78.6 | 355.2 | 128.8 | 551.0 |
| 6 | 0.141 | 0.179 | 0.149 | 0.015 | 59.9 | 79.8 | 359.7 | 125.2 | 515.0 |
| 7 | 0.247 | 0.274 | 0.232 | 0.008 | 53.8 | 70.7 | 356.9 | 128.8 | 532.3 |
| 8 | 0.721 | 0.654 | 0.549 | 0.047 | 34.9 | 83.1 | 356.6 | 146.5 | 573.7 |
| 9 | 0.499 | 0.449 | 0.389 | 0.029 | 51.2 | 74.5 | 358.2 | 125.9 | 555.0 |
| 10 | 0.320 | 0.289 | 0.200 | 0.042 | 52.3 | 73.1 | 359.7 | 126.4 | 596.6 |
| 11 | 0.631 | 0.524 | 0.330 | 0.051 | 47.7 | 84.1 | 357.8 | 136 5 | 638.3 |
| 12 | 0.748 | 0.656 | 0.421 | 0.053 | 47.9 | 79.1 | 360.1 | 138.8 | 644.4 |
| 13 | 0.320 | 0.290 | 0.227 | 0.023 | 54.2 | 77.8 | 358.8 | 126.0 | 544.8 |
| 14 | 0.404 | 0.391 | 0.327 | 0.015 | 26.5 | 71.4 | 356.3 | 130.4 | 557.5 |
| 15 | 0.818 | 0.748 | 0.540 | 0.020 | 40.4 | 78.7 | 356.2 | 142.5 | 563.5 |
| 16 | 0.712 | 0.594 | 0.320 | 0.078 | 37.3 | 76.2 | 359.8 | 154.0 | 612.5 |
| 17 | 0.779 | 0.667 | 0.364 | 0.043 | 37.2 | 78.2 | 357.6 | 147.7 | 620.3 |
| 18 | 0.778 | 0.666 | 0.416 | 0.041 | 39.6 | 84.2 | 358.3 | 140.5 | 616.4 |
| 19 | 0.746 | 0.546 | 0.375 | 0.017 | 45.7 | 70.1 | 361.2 | 147.0 | 632.8 |
| 20 | 0.624 | 0.537 | 0.416 | 0.108 | 32.6 | 77.0 | 357.0 | 154.3 | 602.0 |
| 21 | 0.664 | 0.590 | 0.467 | 0.085 | 34.7 | 72.6 | 359.6 | 144.4 | 613.8 |
| 22 | 0.733 | 0.587 | 0.482 | 0.070 | 23.4 | 77.0 | 357.6 | 166.9 | 618.3 |
| 23 | 0.705 | 0.649 | 0.540 | 0.099 | 34.3 | 63.1 | 357.9 | 142.1 | 597.0 |
| 24 | 0.716 | 0.572 | 0.441 | 0.089 | 30.3 | 77.5 | 357.1 | 149.4 | 621.0 |

15. *Fuel.*—The fuel used on tests 1 to 18 inclusive was Vermilion

County screenings from Mission Field Mine No. 3. The coal was of the same quality as that used for the normal operation of the plant and was all obtained from the same mine with the expectation that the ash and dust content would not vary much. So far as the ash was concerned this expectation was fairly well fulfilled but the dust varied considerably.

The fuel used on tests 20 to 24 inclusive was weathered coal. This had been exposed in bins since January, 1908, and consisted of one lot of each of the following: Sangamon County nut, Sangamon County screenings, Williamson County nut, Williamson County screenings, and Vermilion County nut. This coal had disintegrated considerably and when dry it presented a white appearance as though coated with clay. It was very easily broken and the surface thus exposed was black and rather oily.

## Table 3.

### Coal and Ash.

| Test Number | Kind of Fuel (District from which mined) | Condition of Fuel | Commercial Size | Amount Passing a ¼-in. Screen Per Cent |
|---|---|---|---|---|
| Col. No. | 19 | 20 | 21 | 22 |
| a | | 23 | 23 | |
| b | | | | |
| 1 | Mission Field | Fresh | Screenings | 35.0 |
| 2 | “ “ | “ | “ | 34.8 |
| 3 | “ “ | “ | “ | 40.0 |
| 4 | “ “ | “ | “ | 38.0 |
| 5 | “ “ | “ | “ | 37.8 |
| 6 | “ “ | “ | “ | 19.6 |
| 7 | “ “ | “ | “ | 26.3 |
| 8 | “ “ | “ | “ | 50.5 |
| 9 | “ “ | “ | “ | 35.7 |
| 10 | “ “ | “ | “ | 30.6 |
| 11 | “ “ | “ | “ | 33.2 |
| 12 | “ “ | “ | “ | 40.3 |
| 13 | “ “ | “ | “ | 26.3 |
| 14 | “ “ | “ | “ | 42.8 |
| 15 | “ “ | “ | “ | 22.3 |
| 16 | “ “ | “ | “ | 34.1 |
| 17 | “ “ | “ | “ | 24.7 |
| 18 | “ “ | “ | “ | 29.5 |
| 19 | “ “ | “ | Pea | 7.2 |
| 20 | Sangamon Co. | Weathered | Nut | 31.9 |
| 21 | “ “ | “ | Screenings | 45.1 |
| 22 | Williamson Co. | “ | Nut | 13.9 |
| 23 | “ “ | “ | Screenings | 42.9 |
| 24 | Vermilion Co. | “ | Nut | 35.0 |

16. *Relations Between Thickness of Fire, Capacity, and Efficiency.*—Curves are shown in Figs. 7 to 11 in which efficiency is plotted against thickness of fuel bed. For the sake of convenience in reference the points have been numbered to correspond to the tests from which they were taken. From the curves it becomes evident that the best efficiency under full load conditions was obtained with a fire of about 7 or 7½ inches in thickness. This seems to give a fuel bed resistance such that the normal amount of coal can be burned without excessive draft. If a thinner fire than this is used there is a greater probability that there will be holes or thin places in the fuel bed, forming paths of least resistance through which an excessive amount of air passes with a consequent decrease in efficiency. On the other hand if the thickness of the fire is materially increased it becomes necessary to use more draft in order to maintain the load. The increased draft causes an increase in the air leakage through the setting, etc., thus causing the efficiency

### Table 4.

### Coal and Ash (Continued).

| Test Number | Total Coal as Fired Lb. | Total Dry Coal Lb. | Combustible Consumed Lb. | Calorific Value by Calorimeter | | Total Ash and Refuse Lb. | Percentage of Ash and Refuse in Dry Coal Per Cent |
|---|---|---|---|---|---|---|---|
| | | | | Per Lb. Dry Coal B. t. u. | Per Lb. Combustible B. t. u. | | |
| Col. No. | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| a | 25 | 27 | 30 | 50 | 51 | 28 | 31 |
| b | 22 | 24 | 29 | 62 | 63 | 25 | 30 |
| 1 | 21954 | 18026 | 13026 | 10979 | 14177 | 4087 | 22.67 |
| 2 | 17935 | 14813 | 11061 | 11537 | 14287 | 3925 | 26.49 |
| 3 | 19135 | 15966 | 11661 | 11629 | 14318 | 3771 | 23.62 |
| 4 | 19840 | 16352 | 12742 | 11815 | 14276 | 3445 | 21.07 |
| 5 | 21805 | 17959 | 14081 | 11933 | 14381 | 3819 | 21.27 |
| 6 | 21035 | 17398 | 13418 | 11947 | 14437 | 4159 | 23.91 |
| 7 | 21512 | 18061 | 14328 | 12408 | 14478 | 3826 | 21.18 |
| 8 | 23325 | 19493 | 14982 | 11821 | 14409 | 3832 | 19 66 |
| 9 | 19257 | 16266 | 13164 | 12292 | 14439 | 2801 | 17.22 |
| 10 | 25397 | 21181 | 17421 | 12335 | 14452 | 4394 | 20.75 |
| 11 | 30173 | 25587 | 18255 | 10954 | 14377 | 5671 | 22.16 |
| 12 | 28490 | 23798 | 18369 | 11949 | 14320 | 4905 | 20.61 |
| 13 | 25678 | 21405 | 16549 | 11876 | 14230 | 4852 | 22.67 |
| 14 | 25390 | 21246 | 16474 | 12184 | 14462 | 4799 | 22.59 |
| 15 | 23416 | 19847 | 15320 | 11748 | 14350 | 4021 | 20.26 |
| 16 | 30955 | 26176 | 20872 | 12186 | 14381 | 5342 | 20.41 |
| 17 | 29235 | 24753 | 18924 | 11921 | 14324 | 5454 | 22.03 |
| 18 | 29309 | 24778 | 18183 | 11472 | 14260 | 5881 | 23.73 |
| 19 | 25087 | 21612 | 17617 | 12622 | 14465 | 5968 | 27.61 |
| 20 | 18226 | 14517 | 11743 | 11179 | 13469 | 2351 | 26.19 |
| 21 | 12658 | 9868 | 7618 | 10497 | 12707 | 1718 | 17.41 |
| 22 | 15660 | 13710 | 11009 | 12103 | 14067 | 2382 | 17.37 |
| 23 | 16917 | 14029 | 11002 | 12153 | 13911 | 2271 | 16.19 |
| 24 | 21383 | 17102 | 14193 | 11985 | 13787 | 2098 | 12.27 |

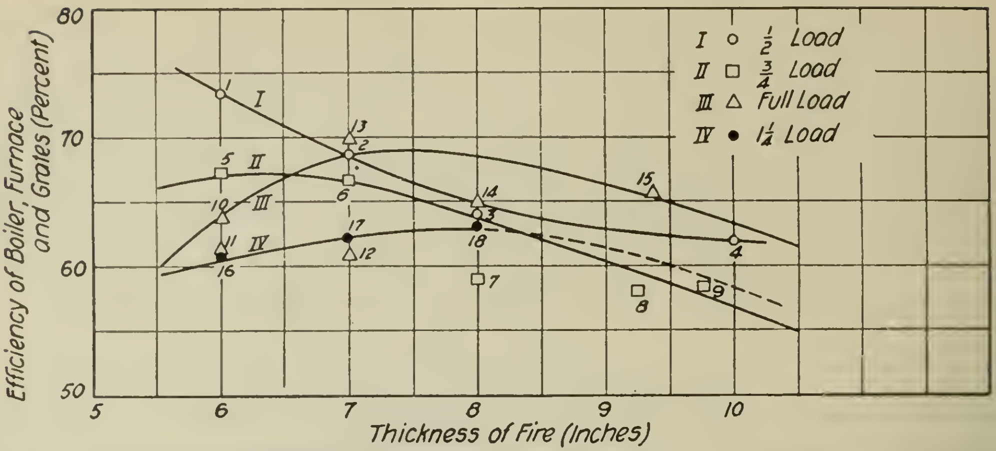

FIG. 7. RELATION BETWEEN THICKNESS OF FIRE AND EFFICIENCY OF BOILER, FURNACE, AND GRATES.

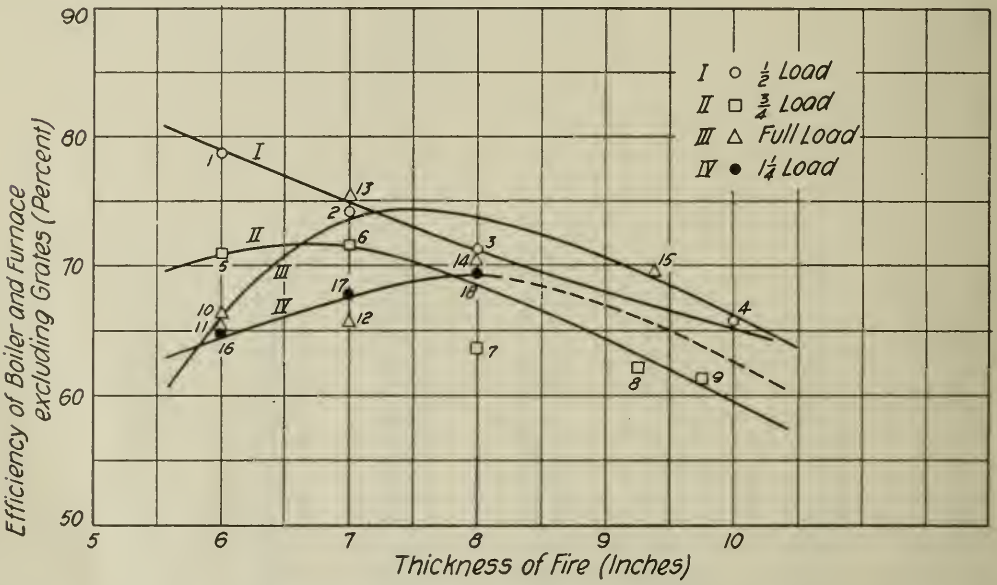

FIG. 8. RELATION BETWEEN THICKNESS OF FIRE AND EFFICIENCY OF BOILER AND FURNACE, EXCLUDING GRATES.

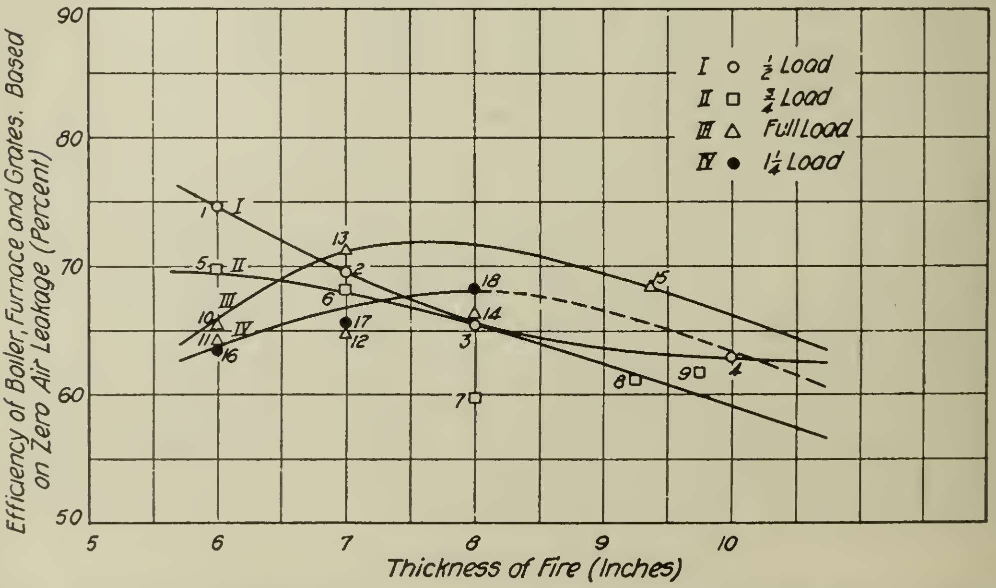

FIG. 9. RELATION BETWEEN THICKNESS OF FIRE AND EFFICIENCY OF BOILER, FURNACE, AND GRATES, BASED ON ZERO AIR LEAKAGE.

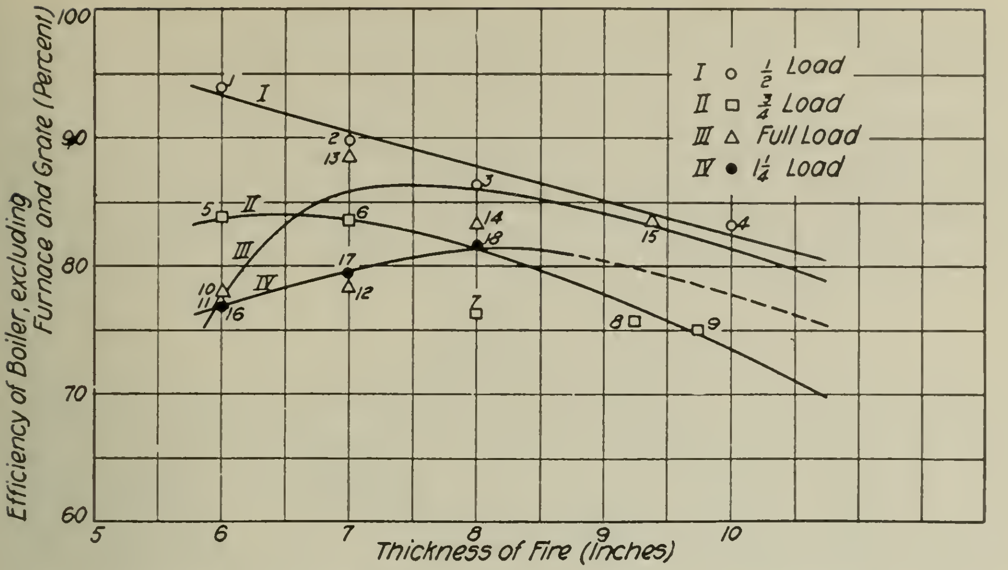

Fig. 10. Relation Between Thickness of Fire and Efficiency of Boiler, Excluding Furnace and Grate.

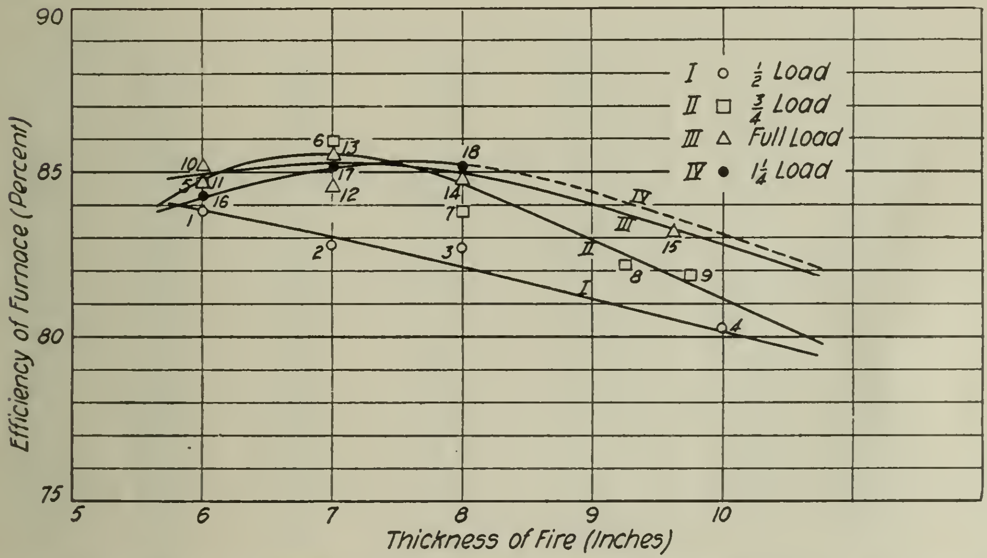

Fig. 11. Relation Between Thickness of Fire and Efficiency of Furnace.

curve to drop. The above discussion is based on the assumption that there is no more CO formed when using a thick fuel bed than with a thin one. An inspection of the gas analyses will show that the CO was never more than 0.24 per cent as a maximum and that this value was obtained on but two tests. For this reason it has been disregarded.

Another explanation of the decrease in efficiency when the thickness of fire is increased above 8 inches is offered by the fact that the arch is very low, it being only 10 inches above the grate at the front end. This causes an excessive draft loss toward the front and makes the combustion poor over that part of the grate thus reducing the effective grate surface. On this account it was found impossible to obtain loads as high as 1¼ capacity with a fire 9½ inches in thickness. That part of the 1¼ load curve to the right of the point representing the 8-in. fire has therefore been shown as a dotted line and has been made to parallel the full load curve.

## TABLE 5.

## STEAM AND WATER.

| Test Number | Quality of Steam | True Factor of Evaporation | Total Water to Weighing Tanks Lb. | Total Weight of Water in Boiler, Lb. | | Steam Pressure, Lb. per Sq. In. (Gage) | |
|---|---|---|---|---|---|---|---|
| | | | | Initial | Final | Initial | Final |
| Col. No. | 30 | 31 | 32 | 33 | 34 | 35 | 36 |
| a | | 60 | | | | | |
| b | 66 | 111 | 67 | 83 | 84 | 75 | 77 |
| 1 | 0.981 | 1.104 | 128849 | 34075 | 33000 | 128.0 | 135.0 |
| 2 | 0.958 | 1.098 | 101703 | 34700 | 34520 | 104.0 | 132.0 |
| 3 | 0.965 | 1.100 | 106005 | 32940 | 34300 | 140.0 | 111.0 |
| 4 | 0.959 | 1.109 | 106843 | 33800 | 32870 | 137.0 | 133.0 |
| 5 | 0.966 | 1.099 | 134113 | 33600 | 34970 | 140.0 | 138.0 |
| 6 | 0.969 | 1.107 | 121025 | 34420 | 33000 | 140.0 | 135.0 |
| 7 | 0.969 | 1.102 | 120655 | 33650 | 33765 | 135.0 | 126.0 |
| 8 | 0.962 | 1.078 | 124928 | 31650 | 32985 | 143.0 | 136.0 |
| 9 | 0.966 | 1.103 | 106858 | 33540 | 33300 | 131.0 | 138.0 |
| 10 | 0.972 | 1.108 | 153213 | 33860 | 33415 | 145.0 | 141.0 |
| 11 | 0.965 | 1.091 | 153191 | 34940 | 31750 | 126.0 | 135.0 |
| 12 | 0.965 | 1.090 | 162271 | 33215 | 34255 | 144.0 | 139.0 |
| 13 | 0.969 | 1.106 | 162830 | 33510 | 34960 | 147.0 | 139.0 |
| 14 | 0.966 | 1.098 | 158007 | 33070 | 34340 | 130.0 | 120.0 |
| 15 | 0.963 | 1.082 | 143285 | 33595 | 34450 | 140.0 | 123.0 |
| 16 | 0.985 | 1.091 | 180054 | 33440 | 34010 | 140.0 | 133 0 |
| 17 | 0.965 | 1.079 | 170912 | 33880 | 34380 | 143.0 | 129.0 |
| 18 | 0.968 | 1.090 | 166713 | 34265 | 34880 | 138.0 | 131.0 |
| 19 | 0.988 | 1.101 | 164818 | 33985 | 34805 | 148.0 | 138.0 |
| 20 | 0.983 | 1.088 | 101619 | 31500 | 33790 | 125.0 | 137.0 |
| 21 | 0.986 | 1.102 | 55580 | 36500 | 35170 | 123.0 | 137.0 |
| 22 | 0.985 | 1.077 | 105364 | 30755 | 33975 | 138.0 | 135.0 |
| 23 | 0.986 | 1.104 | 97300 | 34610 | 34595 | 125.0 | 140.0 |
| 24 | 0.982 | 1.093 | 124576 | 34375 | 34300 | 130.0 | 135.0 |

A study of the curves in Figs. 7 to 11 will show that for each load there was a well marked thickness of fire which gave the best efficiency and that as the load decreased this also became less. Thus for 1¼ load the maximum efficiency occurred with 8 to 8½-in. fires, for full load with 7 to 7½ inches and for ¾ load with about 6½ inches. It is probable that for ½ load the fire could be decreased to 5 inches before the efficiency would drop materially. This can only be inferred, however, from the action at other loads, since no data were taken using fires of less than 6 inches.

17. *Draft.*—When analyzed from the standpoint of draft the above condition is to be expected. For each load, or rate of combustion, a given amount of air is necessary. If the fuel bed is too thick, or dirty, it requires a heavy draft, with a consequent increase in air leakage through places other than the fire, thus causing a loss in efficiency. If it is too thin the draft may be reduced, but in this case the air leakage occurs through the fuel bed itself with the same result as before.

### Table 6.

### Steam and Water (Continued).

| Test Number | Total Weight of Water to Water-back Lb. | Temperature of Water to Water-back Deg. F. | | Feed-Water Corrections, Lb. | | | Total Water Fed to Boiler, Corrected, Lb. | Equivalent Evaporation from and at 212°. Lb. |
|---|---|---|---|---|---|---|---|---|
| | | Entering | Leaving | For Feed Tank Level | For Pressure and Gage Glass | For Water-back | | |
| Col. No. | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 |
| a | | | | | | | 57 | 61 |
| b | 90 | 91 | 92 | 72 | 102 | 105 | 110 | 112 |
| 1 | 88075 | 58.7 | 123.2 | + 318 | + 974 | + 5305 | 135446 | 149532 |
| 2 | 97760 | 60.3 | 109.1 | + 3086 | + 694 | + 4477 | 109960 | 120736 |
| 3 | 108095 | 59.5 | 96.0 | + 3083 | — 1594 | + 3696 | 111190 | 122309 |
| 4 | 99970 | 59.7 | 88.7 | + 1067 | + 648 | + 2695 | 111253 | 123380 |
| 5 | 70245 | 58.3 | 113.0 | — 2019 | — 1100 | + 3602 | 134596 | 147921 |
| 6 | 68310 | 59.3 | 117.8 | + 3512 | + 1020 | + 3721 | 129278 | 143111 |
| 7 | 68715 | 58.7 | 104.7 | + 319 | — 251 | + 2955 | 123678 | 136293 |
| 8 | 51930 | 58.5 | 110.8 | + 1796 | — 1174 | + 2597 | 128147 | 138142 |
| 9 | 65250 | 58.1 | 96.6 | — 319 | + 309 | + 2347 | 109195 | 120442 |
| 10 | 63810 | 58.3 | 112.8 | — 1382 | + 277 | + 3234 | 155342 | 172119 |
| 11 | 53640 | 59.0 | 136.3 | + 2332 | + 2679 | + 3917 | 162119 | 176872 |
| 12 | 52785 | 59.1 | 122.6 | — 1377 | — 906 | + 3169 | 163157 | 177841 |
| 13 | 55125 | 59.5 | 137.5 | — 213 | — 1262 | + 4008 | 165363 | 182891 |
| 14 | 53550 | 57.4 | 126.3 | — 3080 | — 1181 | + 3464 | 157210 | 172617 |
| 15 | 54630 | 58.3 | 106.4 | + 423 | — 826 | + 2502 | 145384 | 157306 |
| 16 | 58275 | 58.1 | 123.2 | + 316 | — 579 | + 3583 | 183374 | 200061 |
| 17 | 56925 | 58.2 | 123.3 | + 1319 | — 648 | + 3538 | 175121 | 188956 |
| 18 | 53370 | 58.8 | 127.7 | + 318 | — 614 | + 3477 | 169894 | 185184 |
| 19 | 50960 | 60.6 | 121.7 | + 1686 | — 822 | + 2913 | 168595 | 185623 |
| 20 | 44633 | 58.4 | 108.4 | — 2951 | — 1634 | + 2113 | 99147 | 107872 |
| 21 | 26863 | 59.7 | 113.2 | + 2801 | + 1331 | + 1344 | 61056 | 67284 |
| 22 | 35392 | 60.4 | 140.1 | — 575 | — 2669 | + 2699 | 104819 | 112890 |
| 23 | 48480 | 59.7 | 104.9 | — 4070 | + 280 | + 2046 | 95556 | 105494 |
| 24 | 46320 | 59.5 | 121.0 | — 3368 | + 151 | + 2687 | 124046 | 135582 |

The curves shown in Fig. 9 have been calculated on the assumption that no air leakage occurred. In this discussion air leakage has been defined as the air infiltration occurring in that portion of the setting from the bridge wall to the breeching and is chargeable against the boiler, and not against the furnace. It is believed that this efficiency forms a better basis for comparing the action of two different boilers and furnaces than the over-all efficiency. It was deemed advisable to calculate this because after the tests had been completed a place was found which permitted some air to leak into the second pass. On account of its inaccessible position this place had escaped detection from the time the boilers were installed. A close watch was kept and precautions were taken to prevent the development of leaks during the tests, and it was supposed that the setting was as tight as possible. With the possible exception of one or two tests, the difference between the percentage of $CO_2$ at the furnace and that at the breeching was not enough to arouse suspicion, since the setting was not a plastered one, and the

### Table 7.

### Fuel and Water per Hour.

| Test Number | Dry Coal, Lb. | | Combustible Consumed, Lb. | | Equivalent Evaporation, Lb. | |
|---|---|---|---|---|---|---|
| | Per Hour | Per Sq. Ft. of Grate Surface per Hour | Per Hour | Per Sq. Ft. of Water-heating Surface per Hour | Per Hour | Per Sq. Ft. of Water-heating Surface per Hour |
| Col. No. | 45 | 46 | 47 | 48 | 49 | 50 |
| a | 46 | 48 | 47 | 49 | 63 | 64 |
| b | 58 | 60 | 59 | 61 | 113 | 114 |
| 1 | 1387 | 15.41 | 1002 | 0.196 | 11502 | 2.25 |
| 2 | 1139 | 12.66 | 851 | 0.166 | 9287 | 1.81 |
| 3 | 1228 | 13.64 | 897 | 0.175 | 9408 | 1.84 |
| 4 | 1258 | 13.98 | 980 | 0.191 | 9491 | 1.85 |
| 5 | 1995 | 22.17 | 1565 | 0.306 | 16436 | 3.21 |
| 6 | 1933 | 21.48 | 1491 | 0.291 | 15901 | 3.11 |
| 7 | 2007 | 22.30 | 1592 | 0.311 | 15144 | 2.96 |
| 8 | 2166 | 24.07 | 1665 | 0.325 | 15349 | 3.00 |
| 9 | 1807 | 20.08 | 1462 | 0.286 | 13383 | 2.61 |
| 10 | 2353 | 26.14 | 1936 | 0.378 | 19124 | 3.74 |
| 11 | 2843 | 31.59 | 2028 | 0.396 | 19652 | 3.84 |
| 12 | 2644 | 29.38 | 2041 | 0.399 | 19760 | 3.86 |
| 13 | 2378 | 26.42 | 1839 | 0.359 | 20321 | 3.97 |
| 14 | 2361 | 26.23 | 1830 | 0.357 | 19180 | 3.75 |
| 15 | 2206 | 25.51 | 1702 | 0.332 | 17479 | 3.41 |
| 16 | 2908 | 32.31 | 2319 | 0.453 | 22229 | 4.34 |
| 17 | 2750 | 30.56 | 2103 | 0.411 | 20995 | 4.10 |
| 18 | 2753 | 30.59 | 2020 | 0.395 | 20576 | 4.02 |
| 19 | 3088 | 34.31 | 2517 | 0.492 | 26516 | 5.18 |
| 20 | 2639 | 29.32 | 2135 | 0.417 | 19613 | 3.83 |
| 21 | 2819 | 31.32 | 2177 | 0.425 | 19224 | 3.75 |
| 22 | 3047 | 33.86 | 2446 | 0.478 | 25087 | 4.90 |
| 23 | 2338 | 25.98 | 1834 | 0.358 | 17582 | 3.43 |
| 24 | 2850 | 31.67 | 2366 | 0.462 | 22597 | 4.41 |

ordinary brick setting is susceptible to some unavoidable air seepage. A comparison of this set of curves with the other efficiency curves indicates that the leakage was not sufficient to detract from any of the conclusions drawn from the data. A discussion of the method of calculation for the efficiency used in Fig. 9 may be found in the Appendix.

The curves in Figs. 12 to 15 inclusive show the relation existing between the various efficiencies and capacity or load in boiler horse power. They bring out about the same facts as those already discussed.

### TABLE 8.

### BOILER PERFORMANCE.

| Test Number | Horse Power Developed | Builder's Rating, Horsepower | Percentage of Builder's Rating Developed | Water Apparently Evaporated per Lb. of Coal as Fired, Lb. | Equivalent Evaporation from and at 212°. Lb. | | |
|---|---|---|---|---|---|---|---|
| | | | | | Per Lb. of Coal as Fired | Per Lb. of Dry Coal | Per Lb. of Combustible Consumed |
| Col. No. | 51 | 52 | 53 | 54 | 55 | 56 | 57 |
| a | 65 | 66 | 67 | 68 | 69 | 70 | 71 |
| b | 115 | 116 | 117 | 118 | 119 | 120 | 121 |
| 1 | 333.4 | 508 | 65.7 | 6.17 | 6.81 | 8.30 | 11.48 |
| 2 | 269.2 | " | 53.0 | 6.13 | 6.73 | 8.15 | 10.92 |
| 3 | 272.7 | " | 53.7 | 5.81 | 6.39 | 7.66 | 10.49 |
| 4 | 275.1 | " | 54.2 | 5.61 | 6.22 | 7.55 | 9.68 |
| 5 | 476.4 | " | 93.8 | 6.17 | 6.78 | 8.24 | 10.51 |
| 6 | 460.9 | " | 90.8 | 6.15 | 6.80 | 8.23 | 10.67 |
| 7 | 439.0 | " | 86.5 | 5.75 | 6.33 | 7.55 | 9.51 |
| 8 | 444.9 | " | 87.6 | 5.49 | 5.92 | 7.09 | 9.22 |
| 9 | 387.9 | " | 76.4 | 5.67 | 6.25 | 7.40 | 9.15 |
| 10 | 554.4 | " | 109.1 | 6.12 | 6.78 | 8.13 | 9.88 |
| 11 | 569.6 | " | 112.1 | 5.37 | 5.86 | 6.91 | 9.69 |
| 12 | 572.7 | " | 112.7 | 5.73 | 6.24 | 7.47 | 9.68 |
| 13 | 589.0 | " | 115.9 | 6.44 | 7.12 | 8.55 | 11.05 |
| 14 | 555.9 | " | 109.4 | 6.19 | 6.80 | 8.13 | 10.48 |
| 15 | 506.6 | " | 99.7 | 6.21 | 6.72 | 7.93 | 10.27 |
| 16 | 644.3 | " | 126.8 | 5.92 | 6.46 | 7.64 | 9.59 |
| 17 | 608.5 | " | 119.8 | 5.99 | 6.46 | 7.63 | 9.99 |
| 18 | 596.4 | " | 117.4 | 5.80 | 6.32 | 7.47 | 10.18 |
| 19 | 768.6 | " | 151.3 | 6.72 | 7.40 | 8.59 | 10.54 |
| 20 | 568.5 | " | 111.9 | 5.44 | 5.91 | 7.43 | 9.19 |
| 21 | 557.2 | " | 109.7 | 4.82 | 5.32 | 6.82 | 8.83 |
| 22 | 727.1 | " | 143.1 | 6.69 | 7.21 | 8.23 | 10.25 |
| 23 | 509.6 | " | 100.3 | 5.65 | 6.24 | 7.52 | 9.59 |
| 24 | 655.0 | " | 128.9 | 5.80 | 6.34 | 7.93 | 9.55 |

On the whole, the over-all efficiencies obtained are not surprisingly high. Under normal conditions of load and fire the over-all efficiency averaged about 65 per cent. As stated before, however, no attempt was made to obtain record tests, so that this represents the actual operating efficiency and as such is in accord with good practice. A study of column 60 in Table 9 will show that the efficiency of the furnace and grate was fairly constant for all loads and depths of fuel bed. Column 61 indicates that the loss in efficiency at higher loads was due to reduced heat absorption by the boiler. Also, a comparison of the values in this

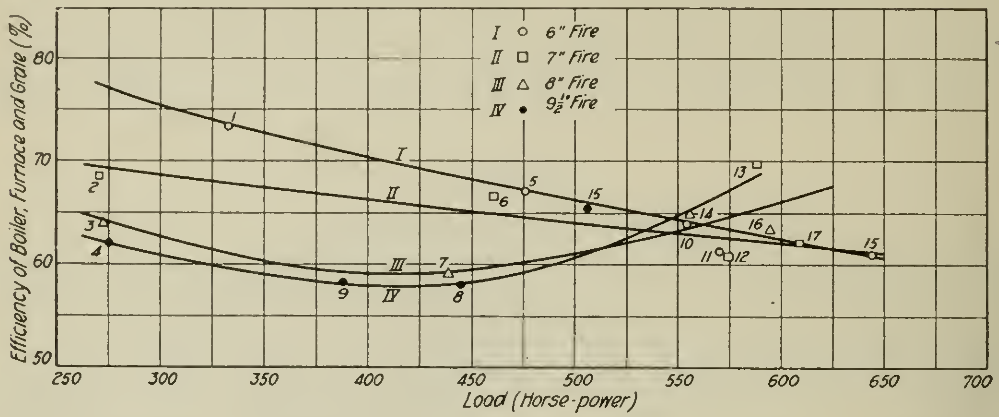

FIG. 12. RELATION BETWEEN LOAD AND EFFICIENCY OF BOILER, FURNACE, AND GRATE.

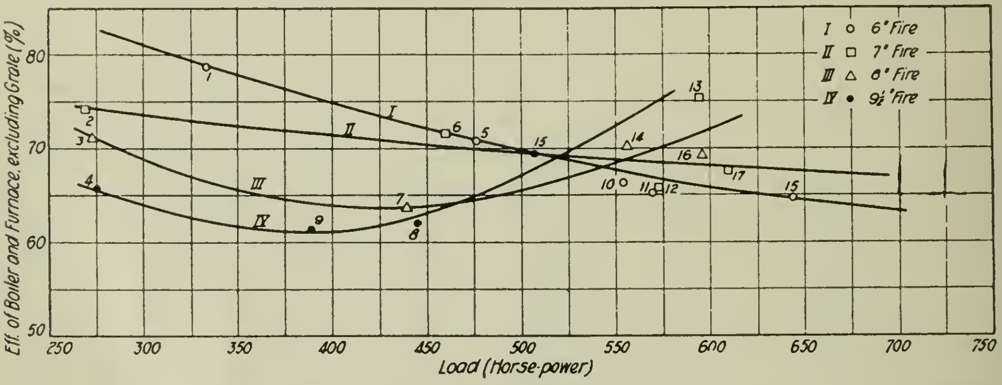

FIG. 13. RELATION BETWEEN LOAD AND EFFICIENCY OF BOILER AND FURNACE, EXCLUDING GRATE.

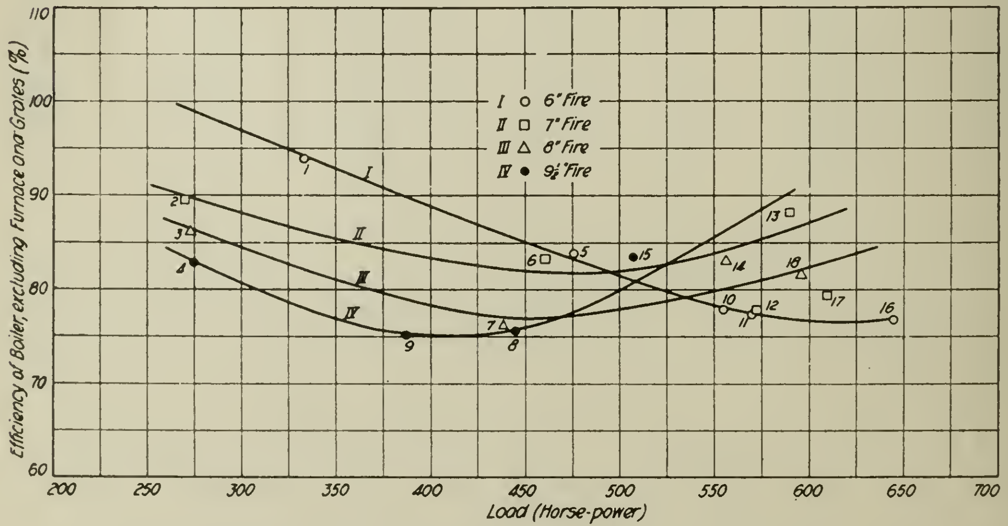

FIG. 14. RELATION BETWEEN LOAD AND EFFICIENCY OF BOILER, EXCLUDING FURNACE AND GRATES.

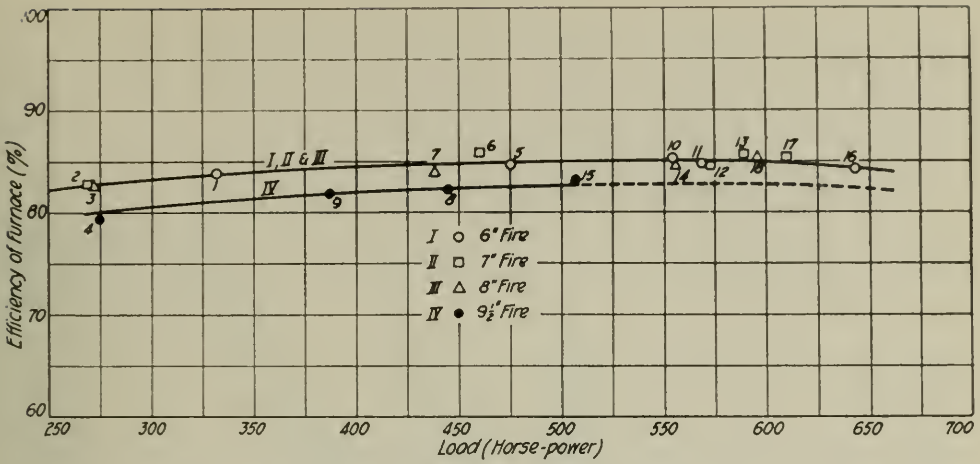

FIG. 15. RELATION BETWEEN LOAD AND EFFICIENCY OF FURNACE.

column with the temperature of the flue gas given in column 18, Table 2, indicates that in general the lower boiler efficiencies correspond with the higher temperatures.

The curves in Fig. 16 give the drafts throughout the boiler setting. The positions plotted as abscissae refer to the draft gage positions indicated in Fig. 1. It may be noticed that there is an unexpected drop in the curves at positions 4 and 6. At first it was supposed that this was due to error either in the gage itself or in the position of the tube. All the gages were then calibrated and found to be well within the limits of accuracy. The positions of the tubes were also changed, and they were moved backward and forward across the channels, with the same result each time. On comparing the cross-sectional areas of the various passes it was found that the area at point 4 was greater than that at point 3. Also that the wall forming part of the last pass was corbeled so that the area at point 6 was greater than that at point 5. This peculiarity in the curves, therefore, has been attributed to the dynamic effect of the flowing gases.

Referring to Fig. 17, it may be seen that the reading of a draft gage ($h'_1$), when connected to a chamber in which the pressure is less than atmospheric, indicates the difference between the barometric pressure (H) and the true pressure or static head ($h_1$) measured above absolute zero. If a Pitot tube is placed at the same point, and there is a flow of gas, the velocity head ($h_3$) will add to the static head ($h_1$), and the reading of the Pitot tube ($h'_2$) will be less than that of the draft gage ($h'_1$). According to Bernouilli's theorem, the total head at any point is the pressure head, plus the velocity head, plus the head lost up to that point. The atmospheric pressure (H) is the total head in this case, and $h_2$ is the sum of the pressure and velocity heads. Therefore $h'_2$ must represent the lost head or, in other words, the friction head. Proceeding from the front of the furnace to the breeching the quantity $h'_2$ increases more or less uniformly, and, since it is a loss, it would deduct from the value of $h_1$ and increase the gage reading ($h'_1$). In view of this fact, the natural expectation would be to find the gage readings increasing for the successive positions from the furnace to the breeching. Any decrease in velocity head ($h_3$), however, goes into an increase in

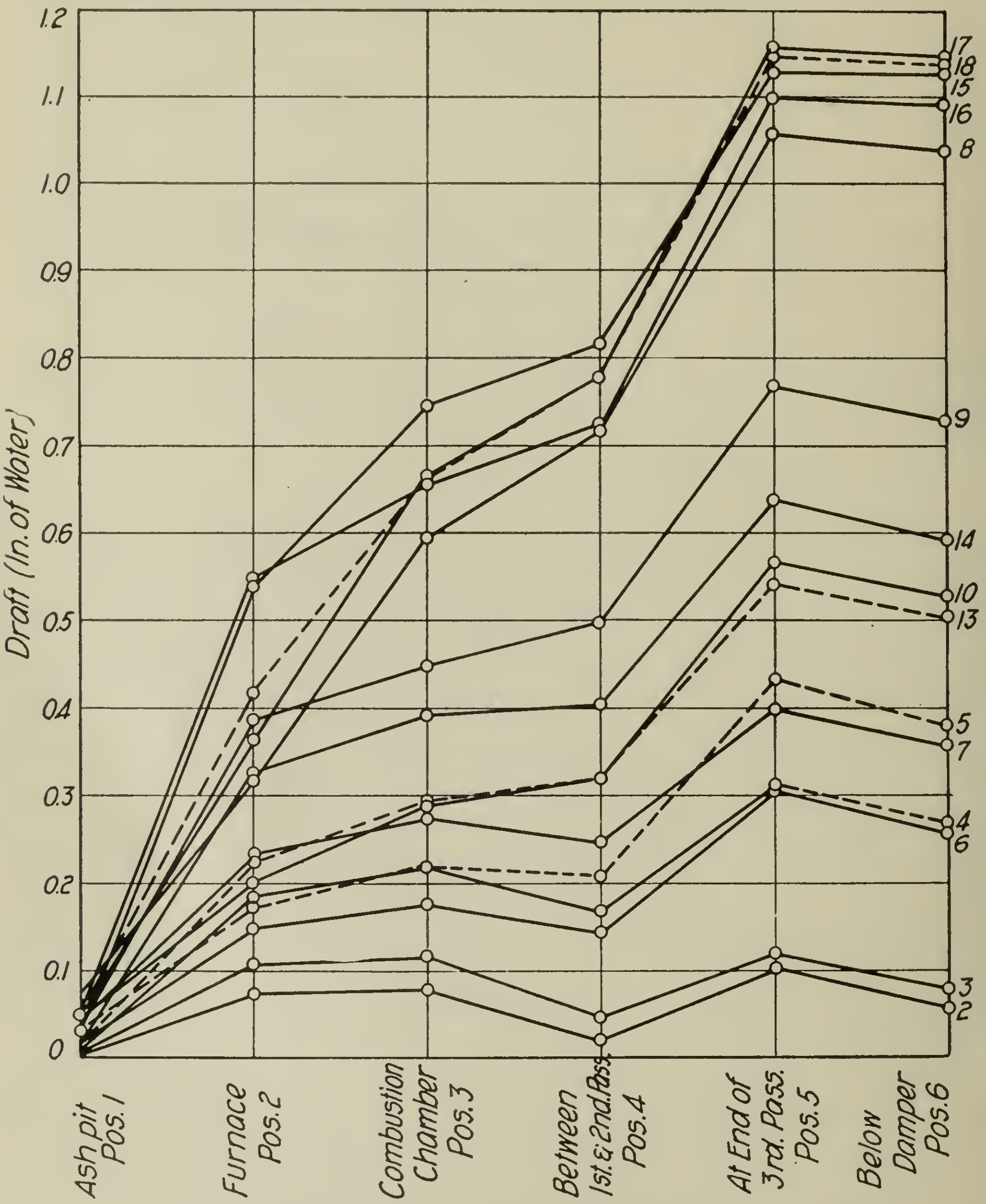

FIG. 16. DRAFTS THROUGHOUT THE BOILER SETTING.

pressure head ($h_1$) and therefore makes the gage reading ($h'_1$) less. Consequently, if a material increase in section, or reduction of gas volume occurs at any point, the decreased velocity head might diminish $h'_1$ enough to make it read less than the gage preceding it. This explains the dip in the curves at positions 4 and 6 in Fig. 16, since at position 4 there was an increase in section over that at position 3, and also at position 6 over that at position 5. It may, therefore, be seen that

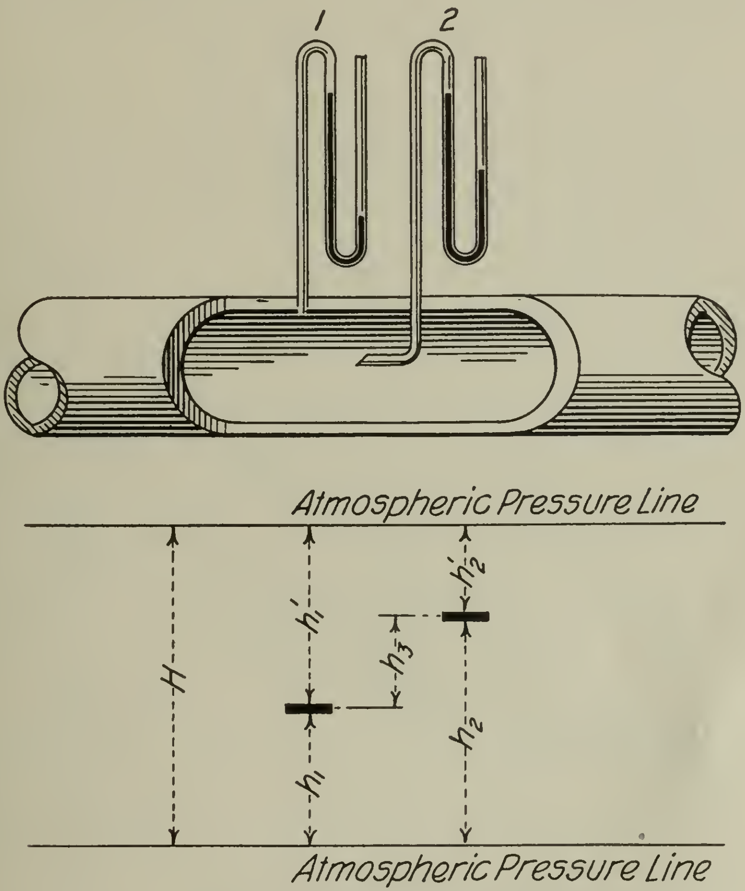

FIG. 17. DRAFT READINGS.

the practice of subtracting draft gage readings and calling the result the friction loss is incorrect unless the area of cross-section is the same at both points where the readings were taken. If this is not true a correction for velocity should be made.

Three factors determine the amount of draft necessary. They are the thickness of fire, the amount of dust in the coal, and the horsepower developed. Taking each load separately it may be seen from Fig. 16 that the 9½-in. fire invariably required the maximum draft for any given load. With the thinner fires sometimes one and sometimes another required more draft, depending largely upon the amount of dust, and slightly varying horsepower; conditions that seemed to affect the thinner fires to a larger extent than the thicker ones.

The curves in Fig. 18 show the relation existing between the draft loss from the ash-pit to the furnace, and the coal burned per square foot of grate surface per hour with varying thickness of fire. The amount of dust in the coal influences a number of these points to some extent. For instance, on test No. 12 the coal had a large percentage of dust. Points 17 and 19 were, therefore, given more weight in determining the end of the curve. From these curves it is evident that between the limits of 24 to 32 pounds per square foot of grate surface and with a normal

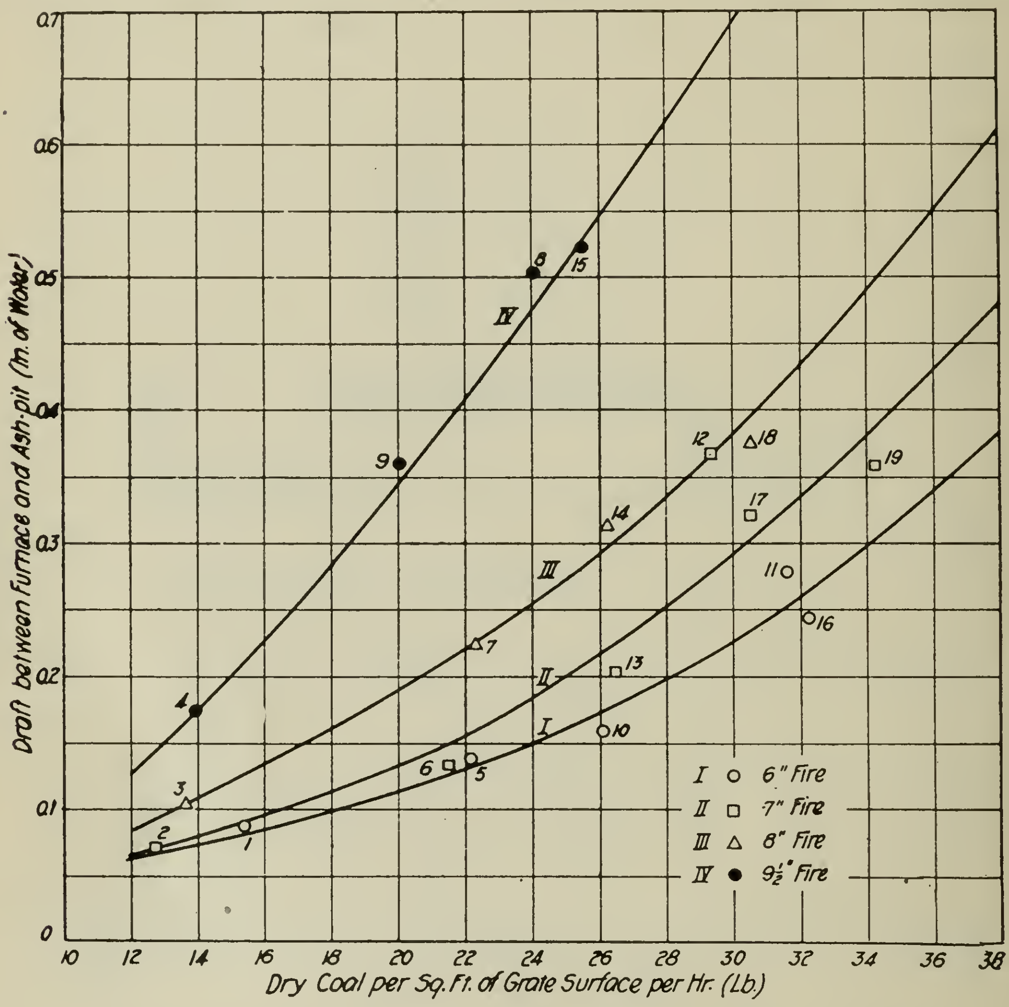

Fig. 18. Relation Between Draft Loss From Ash-Pit to Furnace, and Coal Burned per Sq. Ft. of Grate Surface per Hour With Varying Thickness of Fire.

thickness of fire of about 7½ inches, it requires a draft of approximately 0.01 in. of water per lb. of coal per square foot of grate surface per hour to burn Illinois screenings. This figure increases slightly at higher rates of combustion. With fires above 8 inches in thickness, the draft necessary for the coal to burn at a given rate becomes high enough to cause excessive air leakage, and it increases very rapidly with higher rates of combustion. The same draft required for the combustion of 38 lb. of coal per square foot per hour with a 7-inch fire is only sufficient for the combustion of 24 lb. per square foot per hour with a 9½-inch fire.

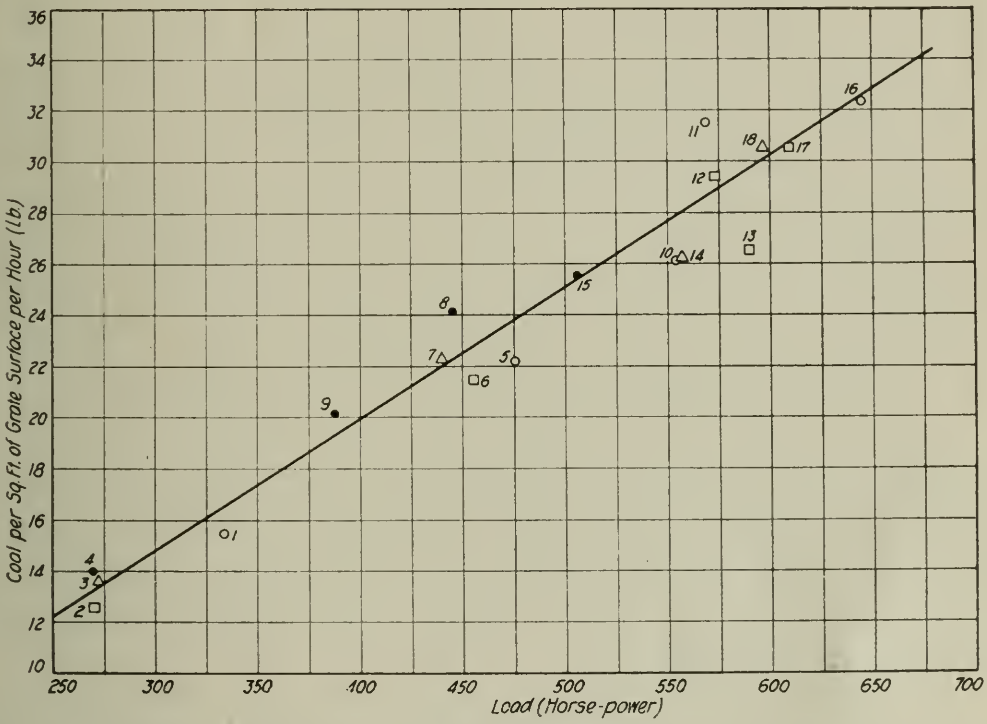

FIG. 19. RELATION BETWEEN LOAD AND COAL PER SQ. FT. OF GRATE SURFACE PER HOUR.

Fig. 19 shows the relation between rate of combustion and the horsepower developed. This relation may be represented by a straight line. The fact that the heating values of the coal varied somewhat, and also, that it was impossible to run all the tests with the tubes in exactly the same condition as regards soot deposits, tends to throw some of the points off of the curve.

Fig. 20 gives the draft requirements for a given horsepower using different thicknesses of fire. These curves necessarily assume the same form as those in Fig. 18, since the load is a straight line function of the rate of combustion. The points do not fall on these curves as they do

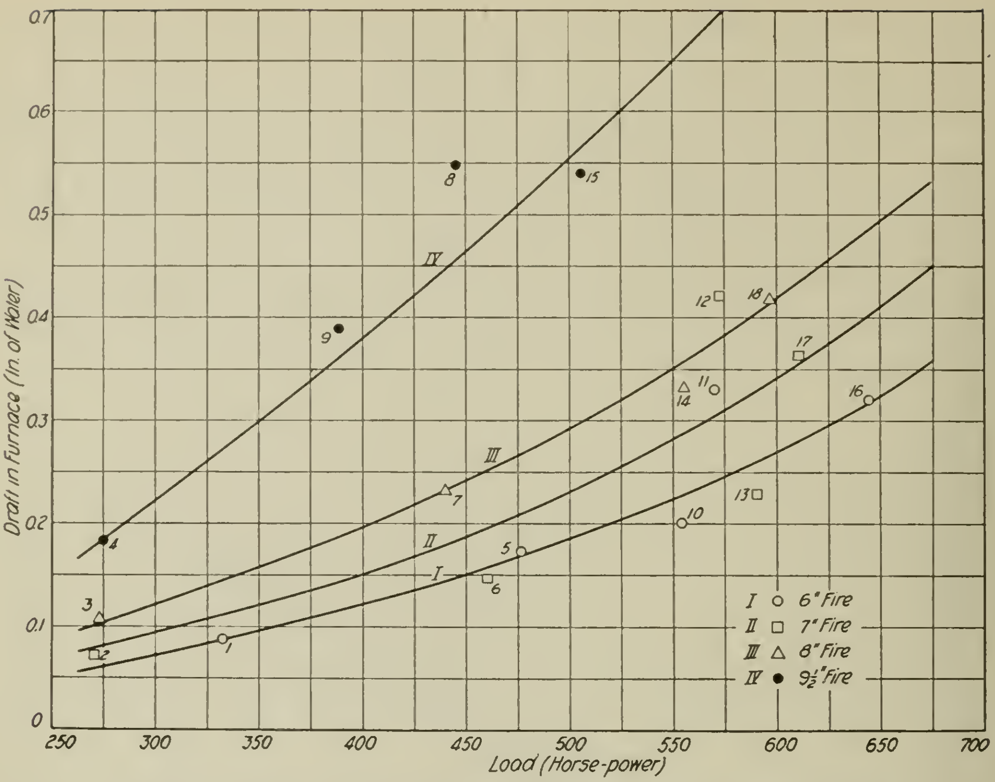

FIG. 20. RELATION BETWEEN LOAD AND DRAFT IN FURNACE.

on those of Fig. 18, however, because some factors such as soot on the tubes and the variable heating value of the coal influence the horsepower developed and do not influence the rate of combustion. Also they were plotted using draft over the fire instead of the difference between this and the draft in the ash pit.

18. *Excess Air.*—The curves in Fig. 21 give the relations between excess air, thickness of fire, and load. There are a number of other factors which affect the excess air, and the curves shown are at best only approximate. They are influenced by draft conditions, which in turn are somewhat dependent upon the dust content of the coal. For instance, it required practically the same draft to give ¾ load on test No. 9 as it did to give full load on test No. 15, due to the fact that on the former the dust content was higher than on the latter. This placed point 9 practically on the full load curve. In tests Nos. 11, 12, and 14 the dust was high, while in Nos. 10 and 13 it was normal. Hence more weight was given to the latter points in drawing the curves.

Taking the curve for each load separately it is evident that the percentage of excess air decreases to a minimum and then increases again

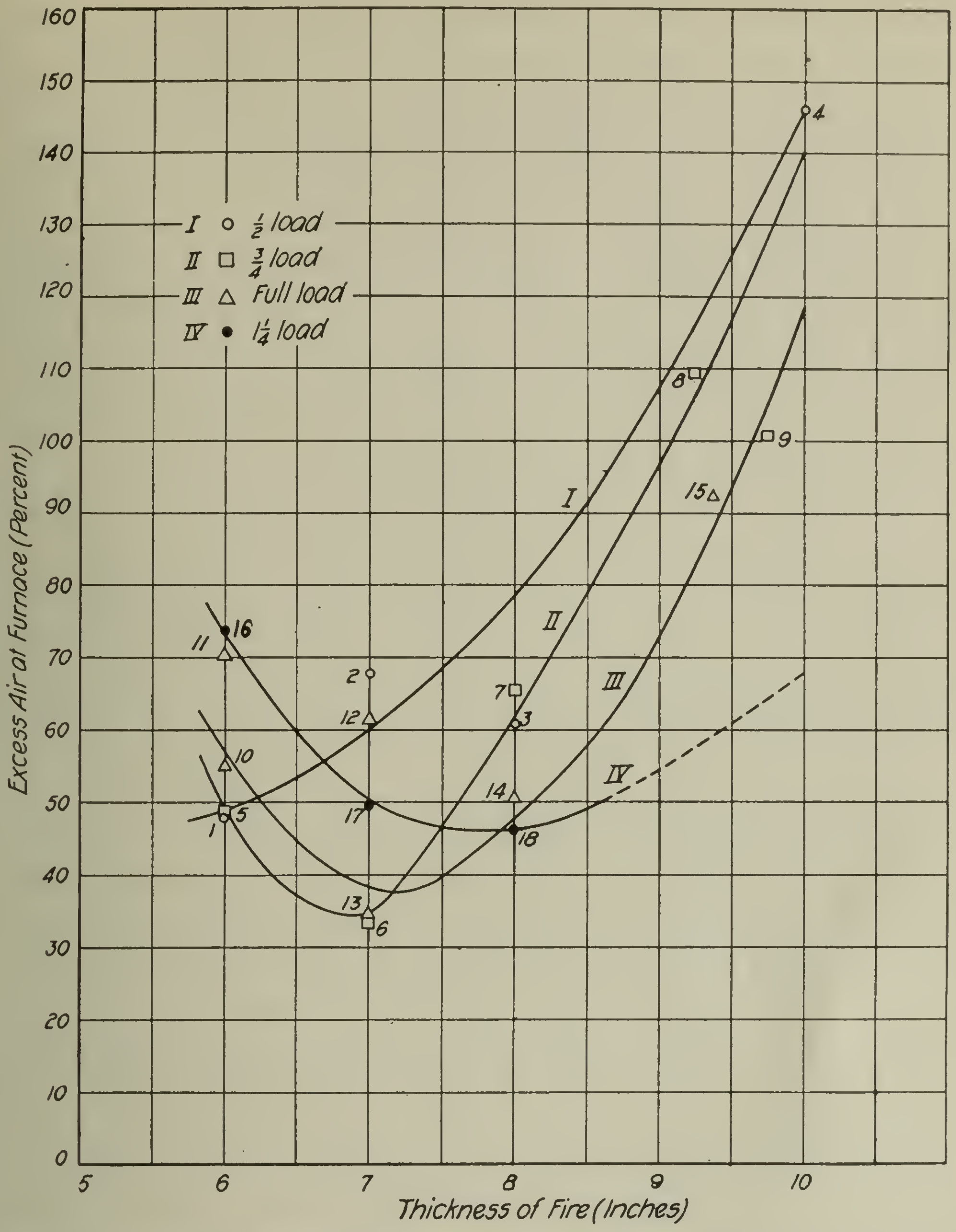

Fig. 21. Relation Between Thickness of Fire and Excess Air at Furnace.

as the thickness of fire increases. These minimum points correspond very closely to the maximum ones shown on the efficiency curves in Figs. 7 to 11. With thin fires there is a marked tendency toward the formation of holes in the fuel bed, through which an excessive amount of air can pass. Also the entire fuel bed is more open and porous, and the small channels formed by the voids are shorter. Hence the air is not brought into as intimate contact with the incandescent surface of the coal as it otherwise would be. When the thickness of the fire is increased the passage becomes more tortuous and thus a larger proportion of the volume of air passing through the fuel bed comes into contact with the surface of the coal. At the same time the total area of the incandescent surfaces increases. Hence the excess air decreases. As the depth of fuel bed increases, however, the ash, as it forms, interferes more with the processes of combustion. At a thickness of about 7½ inches it appears that the ash begins to protect the incandescent surface from the action of the air, thus reducing the total area exposed and serving to mitigate the effect gained by the increased turbulence of the air. Therefore, increasing the depth of fuel on the grate beyond this point results in an increase in the per cent of excess air.

Inspection of the curves in Fig. 21 will show that for fires less than 7½ inches thick, the excess air increases when the load or rate of combustion is increased if a constant depth of fuel bed is maintained, while for fires above 7½ inches thick the reverse is true. When the load is made greater a larger volume of air is drawn through the fuel bed. That part actually coming into contact with the fuel also increases, but, in the case of thin fires, not so rapidly as the total. Hence in this case the excess air will increase with load. As the fires become thicker the part coming into contact is a larger proportion of the whole and it is not necessary to increase the total volume as much in order to obtain a given rate of combustion. At about 8 inches the tendency for the ash to choke the fuel bed becomes manifest and with fires above this thickness, when the draft is made greater, the increased velocity of the air tends to sweep the surfaces clean and to permit better combustion. Hence the percentage of excess air is apt to be lower at the higher rates when using thick fires.

The trend of these curves is also influenced somewhat by air leakage along the sides and at the back of the stoker. This would have had a more marked effect when the heavier drafts were used. A study of the per cent $CO_2$ at the furnace, however, from which the excess air was calculated will prove that this was at no time excessive. On all but two of the tests the $CO_2$ ran from 8 to 13 per cent, representing an excess of from 100 to 25 per cent, and in most cases was about 50 per cent.

In Fig. 22 the percentage of $CO_2$ in the flue gas has been plotted against the excess air expressed in per cent. Most of the points for this curve were taken from the analysis of the gas at the furnace, a

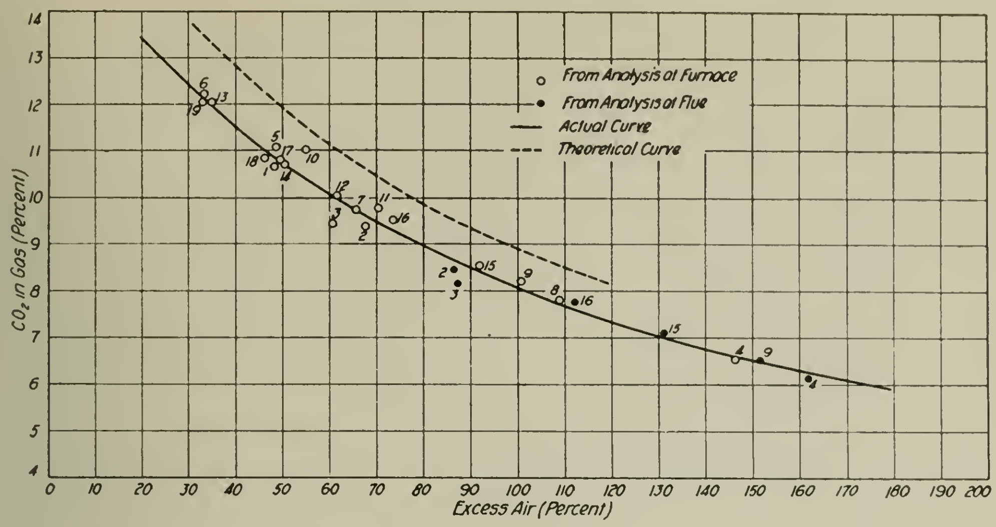

FIG. 22. RELATION BETWEEN EXCESS AIR AND $CO_2$ IN GAS.

few of the analyses at the flue being used, however, to establish some of the more remote points.

The theoretical curve shown in Fig. 22 was calculated from the following analysis, which was obtained by averaging the analyses for tests 1 to 18 inclusive:
C=65.77%, $H_2$=4.17%, $O_2$=6.27%, $N_2$=1.65%, S=4.52%, and Ash= 17.62%. On the assumption that complete combustion took place, a gas analysis was computed for the above coal using 100, 75, 50, and 25 per cent excess air, the results of which are shown as a dotted line in Fig. 22.

The fact that the curve plotted from the analyses so closely parallels the theoretical curve may be taken as an indication of the accuracy of the gas analyses made on the tests. The explanation for the theoretical curve falling above the other lies in the fact that some of the carbon in the coal actually used went through the grates, some went to form soot and a small amount to form CO. This made the carbon actually appearing in the gas per lb. of coal about 5 per cent lower than that used for the theoretical curve.

An inspection of the curve will show that all of the points in the group representing about 35 per cent, or the minimum excess air, were obtained from 7-inch fires. Of those in the 50 per cent excess air group, 4 were obtained from 6-inch, one from 7-inch, and one from 8-inch fires. This seems to verify the conclusion that a fire approximately 7 inches thick is the best to use for this kind of coal.

19. *$CO_2$ and CO Relations.*—In Fig. 23 the $O_2$ in the flue gas is plotted against per cent excess air. This gives the same results as Fig.

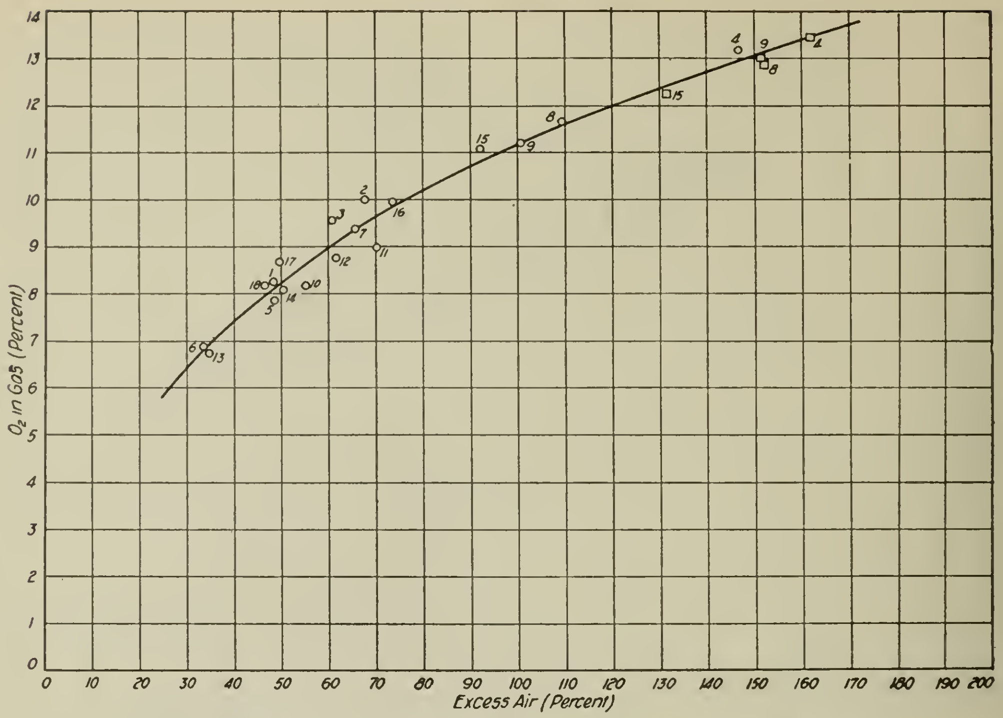

FIG. 23. RELATION BETWEEN EXCESS AIR AND $O_2$ IN GAS.

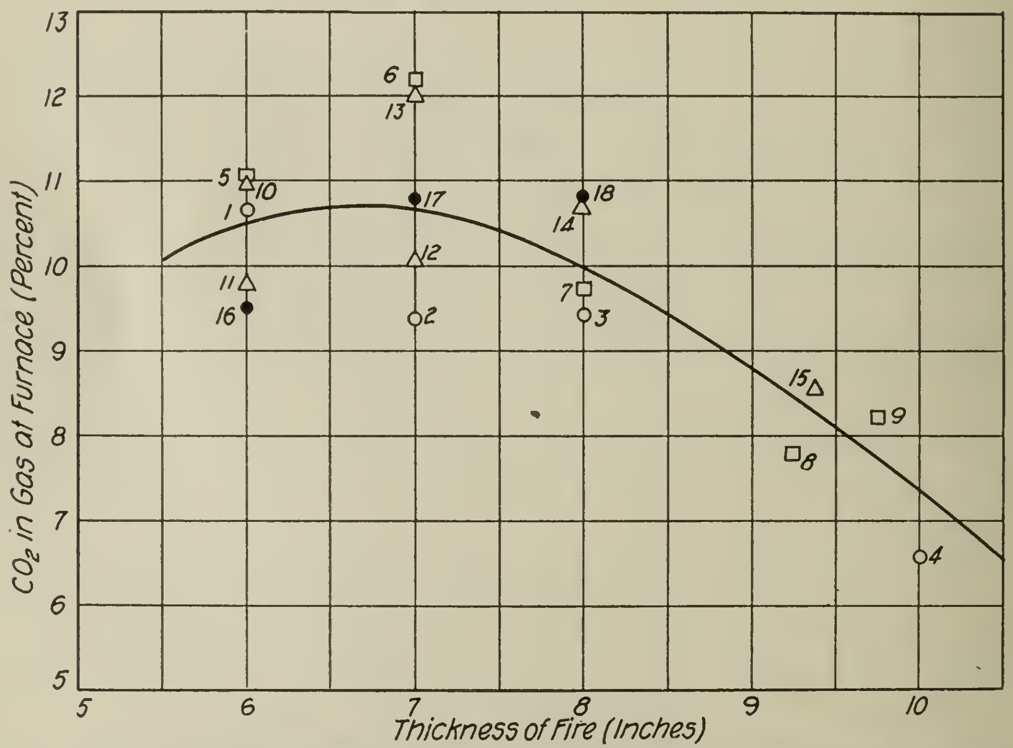

FIG. 24. RELATION BETWEEN THICKNESS OF FIRE AND $CO_2$ IN GAS AT FURNACE.

22, and of the two, the latter is the more valuable, since readings of $CO_2$ may be easily obtained in practice, while $O_2$ is not so readily determined. This curve, however, is given to serve as a check on the $CO_2$ curve.

Fig. 24 was plotted in an attempt to obtain a relation between the thickness of the fire and the percentage of $CO_2$. The latter seems to have a tendency to reach a maximum on the 7-inch fire, from which point it falls off rapidly as the thickness of the fire is increased.

In Figs. 25 and 26, the points fall with sufficient irregularity to require a slight stretch of the imagination in order to draw a curve. They seem to indicate straight line relations, however, the points being about equally distributed on both sides of a straight line.

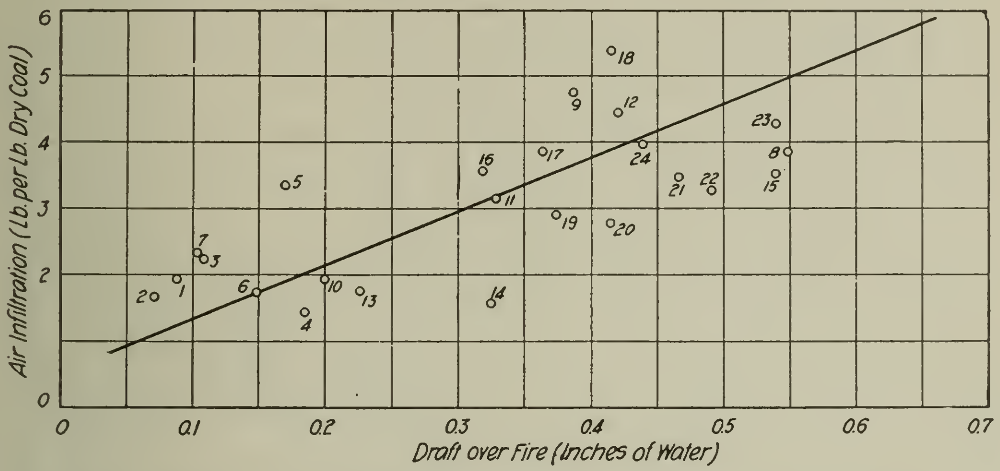

Fig. 25. Relation Between Draft Over Fire and Air Infiltration.

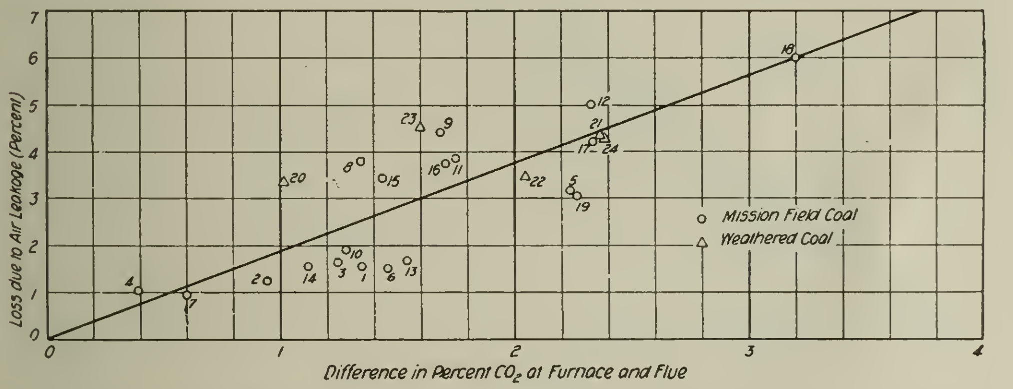

Fig. 26. Relation Between Loss Due to Air Leakage and Difference in Per Cent $CO_2$ at Furnace and Flue.

Fig. 25 indicates that the air leakage through the setting is roughly proportional to the draft over the fire. This curve is affected by the variation in the pounds of gas per pound of coal, which is in turn influenced by a number of factors previously discussed.

In Fig. 26 the loss due to air leakage is plotted against the difference in the $CO_2$ readings at the furnace and at the flue. This curve is also affected by the variation in the pounds of gas per pound of coal, but it shows the importance of stopping all the air leaks.

An attempt was made to find a relation between the loss due to excess air and the loss due to CO formed, but it was found impossible to deduce such a relation from the data obtained.

The curves in Fig. 27 and Fig. 28, however, have been calculated by making use of the coal analysis given under the discussion of Fig. 22. This analysis is fairly representative of the coals used. From these curves some idea may be gained of the relation between the losses caused by excess air and CO. Fig. 27 has been calculated for the different flue gas temperatures indicated, on the assumption that the gases enter the furnace at 75° F., and that the temperature of the flue gas would be the same for the different percentages of excess air.

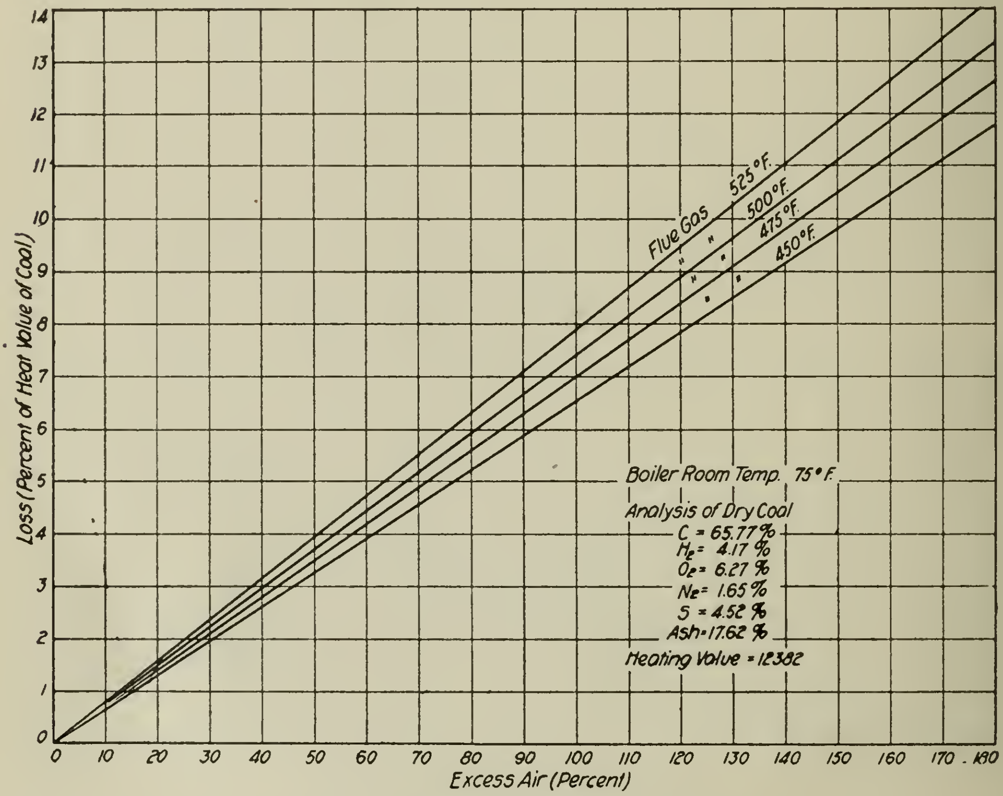

FIG. 27. LOSS CAUSED BY EXCESS AIR.

Assume that under certain operating conditions 11.5 per cent $CO_2$ is being obtained, and that there is no trace of CO in the gas. Reference to Fig. 22 will show that 40 per cent excess air is being used. Assume that the air supply is cut down, and when the per cent of $CO_2$ becomes 11.9, indicating 35 per cent excess air, a trace of CO begins to appear.

If the air supply is further reduced, so that the per cent of $CO_2$ becomes 12.9, indicating that only 25 per cent excess air is being used, reference to Fig. 27 will show that the loss has been cut down from 2.61 per cent when using 35 per cent excess air, with the flue gas at 500° F., to 1.88 per cent when using 25 per cent excess, with the flue gas at the same temperature, giving a difference of 0.73.

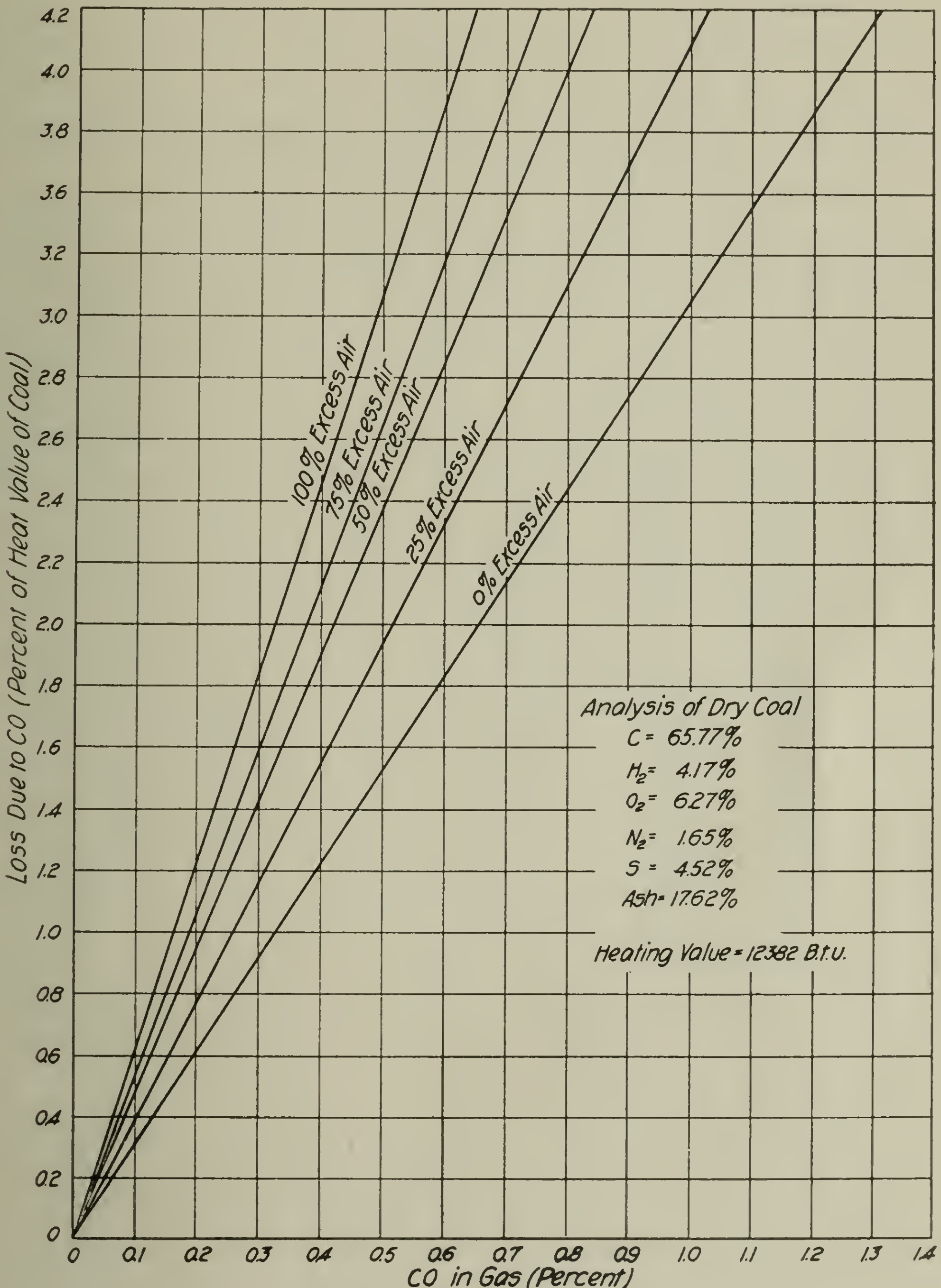

FIG. 28. LOSS CAUSED BY CO.

From Fig. 28 the percentage of CO which represents a loss equal to 0.73 is 0.19 when 25 per cent excess air is being used. Hence, in the case assumed, if 0.19 or more of CO appears in the gas when the excess air is reduced from 35 per cent to 25 per cent, the gain from reducing the air supply is more than offset by the loss due to the formation of CO.

For every furnace and for every coal, therefore, there will be well defined limits beyond which the air supply can not be reduced without causing a greater loss due to incomplete combustion. Just how much CO is permissible for any given coal and installation may be determined by plotting a set of curves similar to the above, and by finding by experiment at what value of $CO_2$ a trace of CO begins to appear.

Fig. 29 gives the relation between loss due to combustible in ash and loss due to excess air. This is influenced by so many factors, and the points fall so irregularly that it is impossible to draw any very exact conclusions from it. The general tendency shown by the points may be fairly well represented by the straight line. This shows that for the chain grate stoker the tendency is for the loss due to excess air to increase as the combustible in the ash is decreased, and that the sum of the losses remains fairly constant at from 12 to 14 per cent. This means

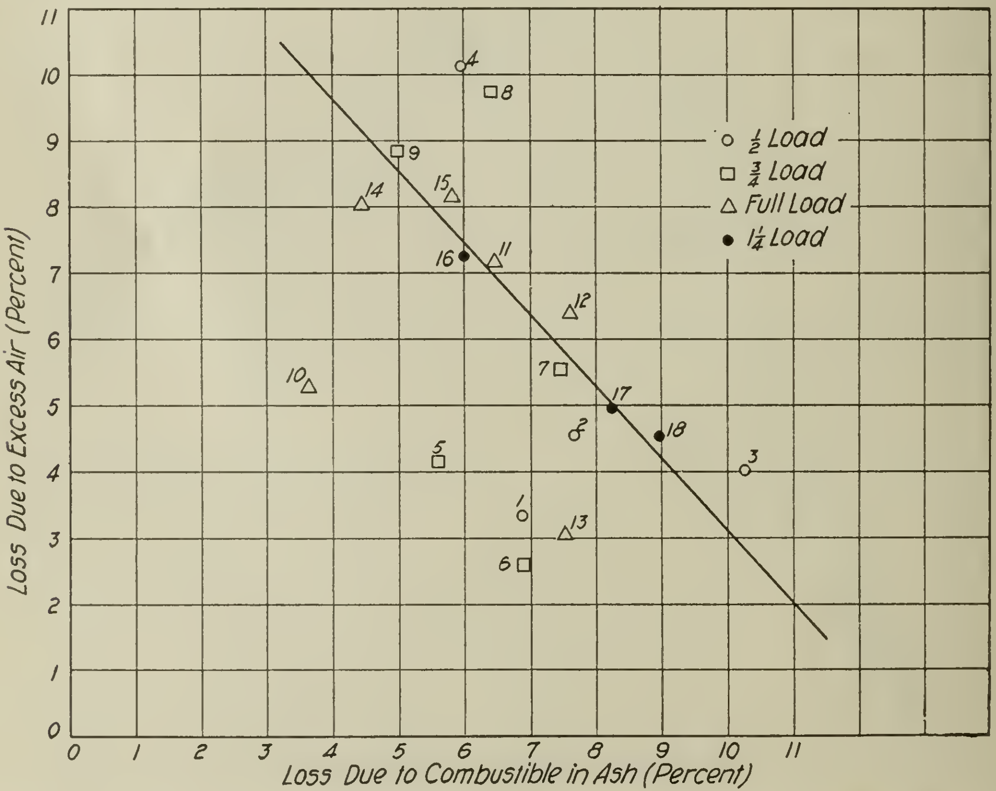

FIG. 29. RELATION BETWEEN LOSS DUE TO COMBUSTIBLE IN ASH AND LOSS DUE TO EXCESS AIR.

that in order to reduce the carbon appearing in the ash it is necessary to increase the air supply, which in turn entails a corresponding loss. It is probable that the combustible loss, being a visible one, will cause more concern than the invisible excess air loss, but the above, in some measure, indicates the futility of trying to cut down on the former at the expense of the latter. Since this conclusion is based entirely upon data obtained from a chain grate it is impossible to state whether or not it will hold for other types of stokers or methods of firing.

TABLE 9.

EFFICIENCY.

| Test Number | Efficiency, Per Cent | | | | | |
|---|---|---|---|---|---|---|
| | Of Boiler and Furnace Excluding Grates | Of Boiler, Furnace and Grates | Of Furnace and Grates | Of Boiler | Of Furnace | Over-all Based on Zero Air Leakage |
| Col. No. | 58 | 59 | 60 | 61 | 62 | 63 |
| a | 72 | 73 | | | | |
| b | 123 | 124 | 127 | 128 | 130 | 133 |
| 1 | 78.58 | 73.36 | 78.07 | 93.97 | 83.82 | 74.85 |
| 2 | 74.17 | 68.55 | 76.46 | 89.66 | 82.80 | 69.69 |
| 3 | 71.10 | 63.92 | 74.27 | 86.06 | 82.74 | 65.35 |
| 4 | 65.80 | 62.01 | 74.54 | 83.20 | 79.26 | 62.92 |
| 5 | 70.92 | 67.01 | 80.00 | 83.75 | 84.72 | 69.68 |
| 6 | 71.72 | 66.85 | 80.10 | 83.46 | 85.99 | 68.09 |
| 7 | 63.75 | 59.05 | 77.55 | 76.14 | 83.81 | 59.76 |
| 8 | 62.09 | 58.20 | 76.92 | 75.66 | 82.16 | 61.08 |
| 9 | 61.49 | 58.42 | 77.80 | 75.09 | 81.87 | 61.74 |
| 10 | 66.34 | 63.96 | 82.04 | 77.95 | 85.16 | 65.48 |
| 11 | 65.40 | 61.21 | 79.33 | 77.16 | 84.79 | 64.16 |
| 12 | 65.59 | 60.67 | 78.07 | 77.71 | 84.49 | 64.59 |
| 13 | 75.36 | 69.87 | 79.15 | 88.27 | 85.59 | 71.35 |
| 14 | 70.32 | 64.75 | 77.97 | 83.04 | 84.76 | 66.04 |
| 15 | 69.45 | 65.50 | 78.40 | 83.54 | 83.22 | 68.41 |
| 16 | 64.71 | 60.84 | 79.23 | 76.79 | 84.26 | 63.75 |
| 17 | 67.68 | 62.11 | 78.15 | 79.48 | 85.18 | 65.45 |
| 18 | 69.28 | 63.19 | 77.53 | 81.50 | 85.16 | 68.16 |
| 19 | 70.71 | 66.04 | 80.05 | 82.50 | 85.73 | 68.60 |
| 20 | 66.21 | 64.50 | 78.62 | 82.03 | 80.83 | 67.24 |
| 21 | 67.44 | 63.05 | 75.85 | 83.12 | 81.99 | 66.62 |
| 22 | 70.71 | 65.98 | 79.33 | 83 18 | 85.21 | 68.87 |
| 23 | 66.90 | 60.04 | 72.46 | 82.86 | 81.14 | 63.75 |
| 24 | 67.22 | 64.20 | 80.40 | 79.86 | 84.44 | 67.66 |

20. *Heat Balance.*—The heat balance as used on these tests has been based on one pound of dry coal and has been considerably extended over the one given in the A. S. M. E. code.

In considering the question of steam generation, there are two separate processes which require attention. First, the heat must be evolved from the coal, and second, it must be absorbed by the water. The first is the function of the furnace and the second is a function of the boiler. Each of these processes entails losses, some of which are mutually independent and some of which are more or less dependent one upon the other. Since the efficiency of the unit should be the product of the efficiency of the furnace and the efficiency of the boiler alone, it is evident that all losses should be tabulated under but two heads, viz., boiler losses and furnace losses.

TABLE 10.

WEIGHT OF AIR AND GASES PER LB. OF DRY COAL.

| Test Number | Weight of Gas, Lb. | | | Weight of Air, Lb. | | | | Weight of Excess Air, Lb. | | Per Cent Excess Air | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Theoretical for Complete Combustion | Actual, at Furnace | Actual, at Flue | Theoretical for Complete Combustion | Actual, at Furnace | Actual, at Flue | Leakage through Setting | At Furnace | At Flue | At Furnace | At Flue |
| Col. No. | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 |
| a | | | | | | | | | | | |
| b | 153 | 162 | 168 | 174 | 177 | 180 | 181 | 182 | 184 | 183 | 185 |
| 1 | 8.76 | 12.64 | 14.41 | 8.35 | 12.37 | 14.30 | 1.93 | 4.02 | 5.95 | 48.14 | 71.26 |
| 2 | 9.23 | 15.05 | 16.57 | 8.78 | 14.73 | 16.38 | 1.65 | 5.95 | 7.60 | 67.77 | 86.56 |
| 3 | 9.29 | 14.45 | 16.59 | 8.84 | 14.21 | 16.45 | 2.24 | 5.37 | 7.61 | 60.75 | 86.09 |
| 4 | 9.49 | 22.58 | 23.94 | 9.04 | 22.25 | 23.68 | 1.43 | 13.21 | 14.64 | 146.13 | 161.95 |
| 5 | 9.52 | 13.73 | 17.04 | 9.06 | 13.45 | 16.81 | 3.36 | 4.39 | 7.75 | 48.45 | 85.54 |
| 6 | 9.48 | 12.37 | 13.95 | 9.03 | 12.02 | 13.73 | 1.71 | 2.99 | 4.70 | 33.11 | 52.05 |
| 7 | 9.90 | 15.92 | 16.91 | 9.43 | 15.63 | 16.67 | 1.04 | 6.20 | 7.24 | 65.75 | 76 78 |
| 8 | 9.39 | 18.97 | 22.71 | 8.95 | 18.72 | 22.54 | 3.82 | 9.77 | 13.59 | 109.16 | 151.84 |
| 9 | 9.80 | 19.06 | 23.75 | 9.34 | 18.76 | 23.49 | 4.73 | 9.42 | 14.15 | 100.85 | 151.50 |
| 10 | 9.84 | 14.88 | 16.74 | 9.38 | 14.54 | 16.46 | 1.92 | 5.16 | 7.08 | 55.01 | 75.48 |
| 11 | 8.86 | 14.49 | 17.57 | 8.40 | 14.29 | 17.43 | 3.14 | 5.89 | 9.03 | 70.12 | 107.50 |
| 12 | 9.60 | 14.99 | 19.34 | 9.14 | 14.76 | 19.20 | 4.44 | 5.62 | 10.06 | 61.49 | 110.06 |
| 13 | 9.60 | 12.63 | 14.39 | 9.15 | 12.35 | 14.14 | 1.79 | 3.20 | 4.99 | 34.97 | 54.54 |
| 14 | 9.68 | 14.08 | 15.63 | 9.21 | 13.84 | 15.47 | 1.63 | 4.63 | 6.26 | 50.28 | 67.97 |
| 15 | 9.37 | 17.45 | 20.84 | 8.92 | 17.13 | 20.64 | 3.51 | 8.21 | 11.72 | 92.04 | 131.39 |
| 16 | 9.73 | 16.42 | 19.91 | 9.27 | 16.09 | 19.68 | 3.59 | 6.82 | 10.41 | 73.57 | 112.30 |
| 17 | 9.65 | 14.09 | 17.82 | 9.18 | 13.71 | 17.57 | 3.86 | 4.53 | 8.39 | 49.35 | 91.39 |
| 18 | 9.22 | 13.12 | 18.38 | 8.78 | 12.84 | 18.23 | 5.39 | 4.06 | 9.45 | 46.24 | 107.63 |
| 19 | 9.89 | 12.95 | 15.83 | 9.49 | 12.65 | 15.55 | 2.90 | 3.16 | 6.06 | 33.30 | 63.86 |
| 20 | 8.45 | 19.13 | 22.11 | 8.03 | 18.82 | 21.79 | 2.79 | 10.79 | 13.76 | 134.37 | 171.35 |
| 21 | 8.44 | 12.57 | 15.96 | 7.99 | 12.19 | 15.66 | 3.47 | 4.20 | 7.67 | 52.56 | 96.00 |
| 22 | 9.68 | 14.58 | 17.91 | 9.23 | 14.37 | 17.61 | 3.24 | 5.14 | 8.38 | 55.69 | 90.80 |
| 23 | 9.63 | 17.66 | 21.77 | 9.19 | 17.20 | 21.45 | 4.25 | 8.01 | 12.26 | 87.16 | 133.40 |
| 24 | 9.54 | 14.56 | 18.41 | 9.08 | 14.10 | 18.08 | 3.98 | 5.02 | 9.00 | 55.28 | 99.12 |

Since it requires a difference in temperature between the hot gases and the water for any heat to flow from the former into the latter, it is manifestly impossible to utilize any of the heat remaining in the gases after their temperature has fallen below the temperature of the steam, and hence of the water in the boiler. This introduces a set of unavoidable losses. In considering these unavoidable losses, and the loss due to excess air taken in at the furnace, it was not entirely self-evident under which head they logically belonged. Owing to reasons which will subsequently be discussed, the following division was finally decided upon as being the most convenient:

DEBIT.

1. Heat supplied by one pound of dry coal.

CREDIT.

2. Heat absorbed by boiler.

*Furnace Losses (unavoidable).*

3. Loss due to heat in theoretical weight of dry gas required for complete combustion, up to the temperature of the steam in the boiler.
4. Loss due to moisture in the coal, up to the temperature of the steam.
5. Loss due to moisture formed from $H_2$ in the coal, up to the temperature of the steam.
6. Loss due to moisture in the air, up to the temperature of the steam.

*Furnace Losses (controllable).*

7. Loss due to excess air up to the temperature of the steam in the boiler.
8. Loss due to CO formed.
9. Loss due to combustible in the ash.
10. Loss due to high temperature of the ash.

*Boiler Losses.*

11. Loss due to theoretical weight of dry gas for complete combustion, above the temperature of the steam in the boiler.
12. Loss due to moisture in coal, above the temperature of the steam.
13. Loss due to moisture formed from $H_2$ in coal, above the temperature of the steam.
14. Loss due to moisture in the air, above the temperature of the steam.
15. Loss due to excess air, above the temperature of the steam.
16. Total loss due to air leakage into the boiler setting.
17. Radiation, conduction, and unaccounted-for losses.

In making up this heat balance, excess air was defined as the air taken into the furnace in excess of that necessary for complete combustion, and was determined from the analysis of the gas at the furnace. Air leakage was defined as the air leaking into the gases after they had passed the bridge-wall. It was found by comparing the gas analyses at the flue and at the furnace. Therefore, all of it was charged against the boiler.

### TABLE 11.

### CHEMICAL ANALYSES.

| Test Number | Proximate Analysis of Coal as Fired | | | | | Ultimate Analysis of Dry Coal | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Fixed Carbon Per Cent | Volatile Per Cent | Moisture Per Cent | Ash Per Cent | Sulphur Separately Determined Per Cent | Carbon Per Cent | Hydrogen Per Cent | Oxygen Per Cent | Nitrogen Per Cent | Sulphur Per Cent | Ash Per Cent |
| Col. No. | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 |
| a | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 |
| b | 38 | 39 | 41 | 42 | 43 | 50 | 51 | 52 | 53 | 54 | 55 |
| 1 | 30.50 | 33.09 | 17.89 | 18.52 | 4.29 | 60.31 | 4.06 | 6.71 | 1.13 | 5.23 | 22.56 |
| 2 | 32.63 | 34.06 | 17.41 | 15.90 | 4.11 | 63.87 | 4.10 | 6.15 | 1.65 | 4.98 | 19.25 |
| 3 | 33.34 | 34.43 | 16.56 | 15.67 | 4.13 | 64.29 | 4.13 | 6.19 | 1.66 | 4.95 | 18.78 |
| 4 | 33.01 | 35.20 | 17.58 | 14.21 | 3.81 | 65.86 | 4.23 | 6.35 | 1.70 | 4.62 | 17.24 |
| 5 | 32.67 | 35.68 | 17.64 | 14.02 | 3.83 | 66.02 | 4.24 | 6.36 | 1.71 | 4.65 | 17.02 |
| 6 | 32.13 | 36.31 | 17.29 | 14.27 | 3.94 | 65.74 | 4.22 | 6.33 | 1.70 | 4.76 | 17.25 |
| 7 | 31.20 | 40.75 | 16.04 | 12.01 | 3.19 | 69.03 | 4.43 | 6.65 | 1.79 | 3.80 | 14.30 |
| 8 | 32.63 | 35.93 | 16.43 | 15.01 | 3.99 | 65.13 | 4.18 | 6.27 | 1.69 | 4.77 | 17.96 |
| 9 | 33.75 | 38.16 | 15.53 | 12.56 | 3.55 | 68.22 | 4.38 | 6.57 | 1.76 | 4.20 | 14.87 |
| 10 | 33.22 | 37 96 | 16.60 | 12.22 | 3.28 | 68.63 | 4.40 | 6.61 | 1.78 | 3.93 | 14.65 |
| 11 | 30.25 | 34.36 | 15.20 | 20.19 | 3.99 | 61.96 | 3.47 | 4.52 | 1.53 | 4.71 | 23.81 |
| 12 | 33.68 | 36.01 | 16.47 | 13.84 | 3.55 | 66.75 | 4.28 | 6.43 | 1.73 | 4.25 | 16.56 |
| 13 | 32.41 | 37.16 | 16.64 | 13.79 | 3.52 | 66.79 | 4.29 | 6.43 | 1.73 | 4.22 | 16.54 |
| 14 | 31.84 | 38.66 | 16.32 | 13.18 | 3.77 | 67.21 | 4.31 | 6.48 | 1.74 | 4.51 | 15.75 |
| 15 | 32.73 | 36.66 | 15.24 | 15.37 | 4.06 | 64.97 | 4.17 | 6.26 | 1.68 | 4.79 | 18.13 |
| 16 | 34.43 | 37.23 | 15.44 | 12.90 | 3.77 | 67.67 | 4.34 | 6.52 | 1.75 | 4.46 | 15.26 |
| 17 | 34.17 | 36.29 | 15.33 | 14.21 | 3.35 | 67.59 | 4.11 | 6.06 | 1.51 | 3.95 | 16.78 |
| 18 | 32.16 | 35.85 | 15.46 | 16.53 | 3.91 | 63.92 | 4.10 | 6.16 | 1.65 | 4.62 | 19.55 |
| 19 | 36.56 | 38.61 | 13.85 | 10.98 | 3.09 | 67.75 | 5.30 | 8.54 | 2.08 | 3.59 | 12.74 |
| 20 | 35.88 | 30.23 | 20.35 | 13.54 | 2.99 | 59.19 | 4.55 | 13.44 | 2.07 | 3.75 | 17.00 |
| 21 | 34.06 | 30.35 | 22.04 | 13.55 | 2.97 | 58.87 | 4.53 | 13.25 | 2.15 | 3.81 | 17.39 |
| 22 | 45.41 | 29.92 | 12.45 | 12.22 | 1.89 | 68.04 | 4.66 | 8.79 | 2.39 | 2.16 | 13.96 |
| 23 | 43.36 | 29.09 | 17.07 | 10.48 | 1.81 | 67.75 | 4.90 | 10.77 | 1.75 | 2.19 | 12.64 |
| 24 | 38.54 | 30.99 | 20.02 | 10.45 | 1.26 | 67.62 | 4.68 | 10.59 | 2.47 | 1.58 | 13.06 |

On the basis of a rigid analysis, regarding the furnace as being entirely separate from the boiler, the unavoidable loss might have been included under boiler losses. We can conceive of a unit so built that the water comes into it at room temperature, and is gradually passed through and heated up to steam temperature. In this kind of a unit the gas might also be made to leave at room temperature. In this hypothetical apparatus any heat in the coal which was not given to the gas would be chargeable to the furnace, while any that was not taken from the gas down to room temperature would be charged against the boiler. This conception leads to a difficulty with the excess air. The amount of excess air depends upon the design and operation of the furnace, and as a boiler furnace, the air should be charged against it. But under the above analysis so far as the furnace is concerned it merely takes this air in and heats it. All the heat that is put into it appears in the gas leaving the furnace, the only difference being that there is a larger volume at a lower temperature. A failure to utilize this heat would then be due to imperfections in the boiler.

### TABLE 12.

### CHEMICAL ANALYSES (CONTINUED).

| Test Nnmber | Analysis of Ash and Refuse | | Analysis of Dry Gases at Furnace Per Cent by Volume | | | | Analysis of Dry Gases at Flue Per Cent by Volume | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Carbon Per Cent | Earthy Matter Per Cent | $CO_2$ | $O_2$ | CO | $N_2$ By Difference | $CO_2$ | $O_2$ | CO | $N_2$ By Difference |
| Col. No. | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 |
| a | 44 | 45 | | | | | 84 | 85 | 86 | 88 |
| b | 56 | 57 | 139 | 140 | 141 | 142 | 143 | 144 | 145 | 146 |
| 1 | 22.84 | 77.16 | 10.69 | 8.23 | 0.23 | 80.85 | 9.34 | 8.98 | 0.18 | 81.50 |
| 2 | 22.93 | 77.07 | 9.39 | 9.94 | 0.18 | 80.49 | 8.45 | 10.49 | 0.20 | 80.86 |
| 3 | 34.67 | 65.33 | 9.43 | 9.52 | 0.24 | 80.81 | 8.18 | 10.58 | 0.20 | 81.05 |
| 4 | 22.95 | 77.05 | 6.54 | 13.18 | 0.12 | 80.16 | 6.15 | 13.47 | 0.12 | 80.26 |
| 5 | 21.49 | 78.51 | 11.06 | 7.84 | 0.16 | 80.94 | 8.82 | 10.15 | 0.14 | 80.89 |
| 6 | 23.55 | 76.45 | 12.20 | 6.87 | 0.06 | 80.87 | 10.74 | 7.88 | 0.05 | 81.33 |
| 7 | 30.07 | 69.93 | 9.74 | 9.31 | 0.08 | 80.87 | 9.14 | 9.82 | 0.08 | 80.96 |
| 8 | 26.36 | 73.64 | 7.80 | 11.61 | 0.02 | 80.57 | 6.45 | 12.86 | 0.05 | 80.64 |
| 9 | 24.37 | 75.63 | 8.20 | 11.16 | 0.13 | 80.51 | 6.51 | 12.97 | 0.13 | 80.39 |
| 10 | 14.96 | 85.04 | 11.00 | 8.14 | 0.04 | 80.82 | 9.72 | 9.23 | 0.04 | 81.01 |
| 11 | 21.86 | 78.14 | 9.78 | 8.99 | 0.05 | 81.18 | 8.03 | 10.90 | 0.02 | 81.05 |
| 12 | 30.34 | 69.66 | 10.07 | 8.75 | 0.01 | 81.17 | 7.74 | 11.21 | 0.00 | 81.05 |
| 13 | 27.12 | 72.88 | 12.03 | 6.72 | 0.05 | 81.20 | 10.49 | 8.34 | 0.05 | 81.12 |
| 14 | 29.72 | 70.28 | 10.70 | 8.06 | 0.05 | 81.19 | 9.58 | 9.03 | 0.06 | 81.33 |
| 15 | 23.10 | 76.90 | 8.54 | 11.01 | 0.04 | 80.41 | 7.10 | 12.21 | 0.04 | 80.65 |
| 16 | 24.52 | 75.48 | 9.50 | 9.91 | 0.01 | 80.58 | 7.79 | 11.46 | 0.00 | 80.75 |
| 17 | 30.72 | 69.28 | 10.78 | 8.67 | 0.04 | 80.51 | 8.44 | 10.75 | 0.03 | 80.78 |
| 18 | 29.78 | 70.22 | 10.81 | 8.18 | 0.05 | 80.96 | 7.61 | 11.39 | 0.04 | 80.96 |
| 19 | 20.81 | 79.19 | 12.02 | 6.84 | 0.03 | 81.11 | 9.75 | 9.38 | 0.02 | 80.85 |
| 20 | 13.00 | 87.00 | 7.37 | 12.34 | 0.01 | 80.28 | 6.35 | 13.50 | 0.01 | 80.14 |
| 21 | 31.08 | 68.92 | 10.58 | 9.17 | 0.07 | 80.18 | 8.22 | 11.36 | 0.09 | 80.33 |
| 22 | 33.02 | 66.98 | 10.68 | 7.95 | 0.01 | 81.36 | 8.63 | 10.75 | 0.01 | 80.61 |
| 23 | 55.20 | 44.80 | 8.23 | 11.92 | 0.04 | 79.81 | 6.63 | 13.17 | 0.03 | 80.17 |
| 24 | 32.15 | 67.85 | 10.96 | 8.84 | 0.01 | 80.19 | 8.58 | 10.93 | 0.01 | 80.48 |

The arrangement given was, therefore, considered the more practical, because the furnace in question is not a separate piece of apparatus for the purpose of heating gases, but is an integral part of the unit, and the heat in the gas below steam temperature is simply not available any more than that in the CO, which would ignite and burn if it were not cooled by the boiler tubes. On this basis any excess air taken by the furnace would increase the amount of heat unavailable, and this amount should be charged against the furnace; that part above steam temperature not absorbed by the boiler should be charged against the latter. The air leakage loss should be charged to the boiler, because this air went through the setting, and not through the furnace.

### TABLE 13.

### HEAT BALANCE, BASED ON 1 LB. OF DRY COAL.

| Test Number | Heating Value of 1 Lb. of Dry Coal B. t. u. | Heat Absorbed by Boiler B. t. u. | Furnace Losses, B. t. u. per Lb. of Dry Coal | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | Unavoidable Loss, up to Temperature of Steam | | | Controllable Loss | | |
| | | | Due to Theoretical Weight of Dry Gas | Due to Moisture in Coal | Due to $H_2$ in Coal | Due to Excess Air | Due to CO in Gas | Due to Combustible in Ash |
| Col. No. | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 |
| a | 50 | | | | | | | |
| b | 62 | 186 | 189 | 196 | 198 | 206 | 209 | 211 |
| 1 | 10979 | 8054 | 590 | 257 | 431 | 271 | 106 | 753 |
| 2 | 11537 | 7909 | 617 | 248 | 434 | 398 | 135 | 884 |
| 3 | 11629 | 7433 | 629 | 234 | 439 | 364 | 136 | 1190 |
| 4 | 11815 | 7327 | 623 | 250 | 446 | 867 | 118 | 704 |
| 5 | 11933 | 7996 | 632 | 252 | 449 | 292 | 97 | 664 |
| 6 | 11947 | 7986 | 637 | 246 | 447 | 201 | 28 | 819 |
| 7 | 12408 | 7327 | 680 | 226 | 472 | 426 | 55 | 926 |
| 8 | 11821 | 6880 | 616 | 230 | 441 | 641 | 47 | 753 |
| 9 | 12292 | 7181 | 667 | 217 | 466 | 641 | 127 | 611 |
| 10 | 12335 | 7889 | 676 | 236 | 469 | 355 | 28 | 451 |
| 11 | 10954 | 6705 | 582 | 210 | 366 | 387 | 14 | 705 |
| 12 | 11949 | 7249 | 647 | 232 | 454 | 379 | 0 | 909 |
| 13 | 11876 | 8297 | 647 | 235 | 455 | 216 | 29 | 894 |
| 14 | 12184 | 7889 | 662 | 231 | 459 | 317 | 39 | 976 |
| 15 | 11748 | 7695 | 624 | 211 | 441 | 547 | 34 | 680 |
| 16 | 12186 | 7414 | 662 | 216 | 461 | 464 | 0 | 728 |
| 17 | 11921 | 7404 | 647 | 213 | 435 | 304 | 22 | 984 |
| 18 | 11472 | 7249 | 607 | 214 | 432 | 267 | 30 | 1028 |
| 19 | 12622 | 8336 | 691 | 191 | 566 | 221 | 13 | 836 |
| 20 | 11179 | 7210 | 568 | 301 | 482 | 725 | 9 | 305 |
| 21 | 10497 | 6618 | 581 | 335 | 483 | 290 | 59 | 787 |
| 22 | 12103 | 7986 | 652 | 168 | 494 | 346 | 7 | 835 |
| 23 | 12153 | 7297 | 681 | 245 | 526 | 568 | 27 | 1300 |
| 24 | 11985 | 7695 | 640 | 295 | 496 | 337 | 7 | 574 |

This treatment might have been made slightly more rigid by including the unavoidable losses as a separate set of items charged up to the unit as a whole, but in this case the efficiency of the unit would not have been the product of the boiler efficiency and furnace efficiency. Any arrangement that is made, however, is merely a matter of distribution and in no way affects the calculation of the losses themselves.

Two items are given in the above heat balance which have not been calculated for this set of tests. They are the heat loss due to moisture carried in by the air, and the heat loss due to the temperature of the leaving ash. The moisture loss forms a very small percentage, and can not be determined with any very great degree of accuracy. It is true that the relative humidity may be found by means of a wet and dry bulb hygrometer, but this varies through a considerable range in different parts

### TABLE 14.

### HEAT BALANCE (CONTINUED).

| Test Number | Boiler Losses, B. t. u. per lb. of Dry Coal | | | | | |
|---|---|---|---|---|---|---|
| | Above Temperature of Steam | | | | Due to Air Leakage | Radiation, Conduction and Unaccounted-for |
| | Due to Theoretical Weight of Dry Gas | Due to Moisture in Coal | Due to $H_2$ in Coal | Due to Excess Air | | |
| Col. No. | 104 | 105 | 106 | 107 | 108 | 109 |
| a | | | | | | |
| b | 216 | 218 | 219 | 221 | 224 | 226 |
| 1 | 203 | 11 | 18 | 93 | 175 | + 17 |
| 2 | 198 | 9 | 16 | 127 | 146 | + 416 |
| 3 | 177 | 8 | 15 | 102 | 194 | + 708 |
| 4 | 236 | 11 | 20 | 329 | 129 | + 755 |
| 5 | 447 | 21 | 37 | 206 | 381 | + 459 |
| 6 | 353 | 16 | 29 | 112 | 179 | + 894 |
| 7 | 417 | 17 | 35 | 261 | 115 | + 1451 |
| 8 | 490 | 21 | 41 | 509 | 450 | + 702 |
| 9 | 463 | 18 | 39 | 445 | 545 | + 872 |
| 10 | 560 | 23 | 47 | 293 | 241 | + 1067 |
| 11 | 596 | 25 | 44 | 397 | 418 | + 505 |
| 12 | 655 | 28 | 55 | 383 | 603 | + 355 |
| 13 | 429 | 19 | 36 | 143 | 200 | + 276 |
| 14 | 467 | 20 | 39 | 224 | 190 | + 671 |
| 15 | 466 | 19 | 39 | 409 | 409 | + 174 |
| 16 | 590 | 23 | 49 | 414 | 462 | + 703 |
| 17 | 608 | 24 | 49 | 286 | 502 | + 443 |
| 18 | 571 | 24 | 48 | 251 | 688 | + 63 |
| 19 | 645 | 23 | 65 | 206 | 392 | + 437 |
| 20 | 495 | 31 | 50 | 635 | 374 | — 6 |
| 21 | 515 | 36 | 52 | 256 | 451 | + 34 |
| 22 | 606 | 19 | 55 | 322 | 421 | + 192 |
| 23 | 553 | 25 | 53 | 460 | 544 | — 126 |
| 24 | 604 | 33 | 56 | 318 | 519 | + 411 |

of the room, and is so affected by purely local conditions that it was not considered of much importance.

It was not possible to obtain a satisfactory temperature of the ash. As it leaves the grate the ash is probably in the neighborhood of 1800° F. Part of this heat, however, is given back to the setting and to the air entering the furnace. The ash leaving the pit was at a temperature of about 600° F., but the heat down to this point had not all been utilized because a large part of it was dissipated through the walls and door of the pit. The true temperature was probably somewhere between these two, but owing to the small size of the loss and the uncertainty regarding the temperature it was not considered worth while to calculate the loss.

### Table 15.

### Heat Balance (Continued.)

| Test Number | Heat Absorbed Per Cent | Furnace Losses, Per Cent of Heat in Dry Coal | | | | | |
|---|---|---|---|---|---|---|---|
| | | Unavoidable Loss, up to Temperature of Steam | | | Controllable Loss | | |
| | | Due to Theoretical Weight of Dry Gas | Due to Moisture in Coal | Due to $H_2$ in Coal | Due to Excess Air | Due to CO in Gas | Due to Combustible in Ash |
| Col. No. | 110 | 111 | 112 | 113 | 114 | 115 | 116 |
| a | | | | | | | |
| b | 227 | 228 | 229 | 230 | 232 | 233 | 234 |
| 1 | 73.36 | 5.37 | 2.34 | 3.93 | 2.47 | 0.97 | 6.86 |
| 2 | 68.55 | 5.35 | 2.15 | 3.76 | 3.45 | 1.17 | 7.66 |
| 3 | 63.92 | 5.41 | 2.01 | 3.78 | 3.13 | 1.17 | 10.23 |
| 4 | 62.01 | 5.27 | 2.12 | 3.77 | 7.34 | 1.00 | 5.96 |
| 5 | 67.01 | 5.30 | 2.11 | 3.76 | 2 45 | 0.81 | 5.56 |
| 6 | 66.85 | 5.33 | 2.06 | 3.74 | 1.68 | 0.23 | 6.86 |
| 7 | 59.05 | 5.48 | 1.82 | 3.80 | 3.43 | 0.44 | 7.46 |
| 8 | 58.20 | 5.21 | 1.95 | 3.73 | 5.42 | 0.40 | 6.37 |
| 9 | 58.42 | 5.43 | 1.77 | 3.79 | 5.21 | 1.03 | 4.97 |
| 10 | 63.96 | 5.48 | 1.91 | 3.80 | 2.88 | 0.23 | 3.66 |
| 11 | 61.21 | 5.31 | 1.92 | 3.34 | 3.53 | 0.13 | 6.44 |
| 12 | 60 67 | 5.41 | 1.94 | 3.80 | 3.17 | 0.00 | 7.61 |
| 13 | 69.87 | 5.45 | 1.98 | 3.83 | 1.82 | 0.24 | 7.52 |
| 14 | 64.75 | 5.43 | 1.90 | 3.77 | 2.60 | 0.32 | 8.01 |
| 15 | 65.50 | 5.31 | 1.80 | 3.75 | 4.66 | 0.29 | 5.79 |
| 16 | 60.84 | 5.43 | 1.77 | 3.78 | 3.81 | 0.00 | 5.97 |
| 17 | 62.11 | 5.43 | 1.79 | 3.65 | 2.55 | 0.19 | 8.25 |
| 18 | 63.19 | 5.29 | 1.87 | 3.77 | 2.33 | 0.26 | 8.96 |
| 19 | 66.04 | 5.47 | 1.51 | 4.48 | 1.75 | 0.10 | 6.62 |
| 20 | 64.50 | 5.08 | 2.69 | 4 31 | 6.49 | 0.08 | 2.73 |
| 21 | 63.05 | 5.53 | 3.19 | 4.60 | 2.76 | 0.56 | 7.50 |
| 22 | 65.98 | 5.39 | 1.39 | 4.08 | 2.86 | 0.06 | 6.90 |
| 23 | 60.04 | 5.60 | 2.02 | 4.33 | 4 67 | 0.22 | 10.70 |
| 24 | 64.20 | 5.34 | 2.46 | 4.14 | 2.81 | 0.06 | 4.79 |

Fig. 30 will give some idea as to the relative magnitude of the several losses. In this figure the heat balances for five typical tests have been plotted, from which it appears that the controllable losses due to the gases escaping at a temperature higher than that of the steam are in all cases less than the unavoidable losses up to the temperature of the steam. About the only method of diminishing these losses is by reducing the excess air and eliminating air leakage, since these are the only factors which are not at a minimum.

### TABLE 16.

### HEAT BALANCE (CONTINUED).

| Test Number | Boiler Losses, Per Cent of Heat in Dry Coal | | | | | |
|---|---|---|---|---|---|---|
| | Above Temperature of Steam | | | | Due to Air Leakage | Radiation, Conduction and Unaccounted-for |
| | Due to Theoretical Weight of Dry Gas | Due to Moisture in Coal | Due to $H_2$ in Coal | Due to Excess Air | | |
| Col. No. | 117 | 118 | 119 | 120 | 121 | 122 |
| a | | | | | | |
| b | 236 | 237 | 238 | 240 | 241 | 242 |
| 1 | 1.85 | 0.10 | 0.16 | 0.85 | 1.59 | 0.15 |
| 2 | 1.72 | 0.08 | 0.14 | 1.10 | 1.27 | 3.60 |
| 3 | 1.52 | 0.07 | 0.13 | 0.88 | 1.67 | 6.08 |
| 4 | 2.00 | 0.09 | 0.17 | 2.78 | 1.09 | 6.40 |
| 5 | 3.75 | 0.18 | 0.31 | 1.72 | 3.19 | 3.85 |
| 6 | 2.95 | 0.13 | 0.24 | 0.94 | 1.50 | 7.50 |
| 7 | 3.37 | 0.14 | 0.28 | 2.10 | 0.93 | 11.70 |
| 8 | 4.14 | 0.18 | 0.35 | 4.31 | 3.81 | 5.93 |
| 9 | 3.77 | 0.15 | 0.32 | 3.62 | 4.43 | 7.09 |
| 10 | 4.54 | 0.19 | 0.38 | 2.38 | 1.95 | 8.64 |
| 11 | 5.44 | 0.23 | 0.40 | 3.62 | 3.82 | 4.61 |
| 12 | 5.48 | 0.23 | 0.46 | 3.21 | 5.05 | 2.97 |
| 13 | 3.62 | 0.16 | 0.30 | 1.20 | 1.68 | 2.33 |
| 14 | 3.83 | 0.16 | 0.32 | 1.84 | 1.56 | 5.51 |
| 15 | 3.97 | 0.16 | 0.33 | 3.48 | 3.48 | 1.48 |
| 16 | 4.84 | 0.19 | 0.41 | 3.40 | 3.79 | 5.77 |
| 17 | 5.10 | 0.20 | 0.41 | 2.40 | 4.21 | 3.71 |
| 18 | 4.97 | 0.21 | 0.42 | 2.19 | 6.00 | 0.54 |
| 19 | 5.12 | 0.18 | 0.52 | 1.63 | 3.11 | 3.47 |
| 20 | 4.43 | 0.28 | 0.45 | 5.68 | 3.35 | |
| 21 | 4.91 | 0.34 | 0.50 | 2.44 | 4.30 | 0.32 |
| 22 | 5.01 | 0.16 | 0.45 | 2.66 | 3.48 | 1.58 |
| 23 | 4.55 | 0.21 | 0.44 | 3.79 | 4.48 | |
| 24 | 5.04 | 0.28 | 0.47 | 2.65 | 4.33 | 3.43 |

The largest loss seems to be due to the combustible in the ash. In a general way, however, where this is high, the excess air loss is low, tests No. 3 and No. 4 indicating this very well.

Since the unaccounted-for loss includes the effects of various experimental errors, as well as radiation and conduction, it has no definite relation to the other losses. It can be reduced, however, by adequate heat insulation for the setting.

A study of the boiler efficiency and furnace efficiency in Table 9 will show that these two are very nearly the same, thus indicating that the losses are about equally distributed between them. This should afford an easy means of checking them against each other, as any considerable loss in either one would immediately become apparent.

## V. Tests on Weathered Coal.

Five tests were run on weathered coal. As previously stated, this fuel had been placed in bins in January 1908, and formed part of the series reported in Engineering Experiment Station Bulletin No. 38, "The Weathering of Coal," by S. W. Parr and W. F. Wheeler. In Table 17 may be found a statement of the kind of coal, the conditions of storage, heating value, and dust content of this fuel. The data concerning original size and heating value have been taken from Bulletin No. 38.

From Table 17 it may be seen that from the physical standpoint, and in heating value, the coal had deteriorated considerably from its original condition, and that at the time of the tests it was in about the same class as the Mission Field screenings. A discussion of the physical and chemical properties, however, is outside of the scope of this work, it being concerned merely with the behavior of this fuel when used in a boiler furnace.

Table 17.

Deterioration Due to Weathering.

| Test No. | Coal | How Stored | Jan. 1908 |  | Jan. 1914 |  |
|---|---|---|---|---|---|---|
|  |  |  | Per Cent Passing a ¼″ Screen | B. t. u. per Lb. Dry Coal 7 Days After Mining | Per Cent Passing a ¼″ Screen | B. t. u. per Lb. Dry Coal at Time of Test |
| 20 | Sangamon Co. Nut | Open bin | 1.2 | 11800 | 31.9 | 11179 |
| 21 | Sangamon Co. Screenings | Covered bin | 26.3 | 11684 | 45.1 | 10497 |
| 22 | Williamson Co. Nut | Covered bin | 1.4 | 12341 | 13.9 | 12103 |
| 23 | Williamson Co. Screenings | Covered bin | 38.8 | 12287 | 42.9 | 12153 |
| 24 | Vermilion Co. Nut | Open bin | 13.6 | 12412 | 35.0 | 11985 |

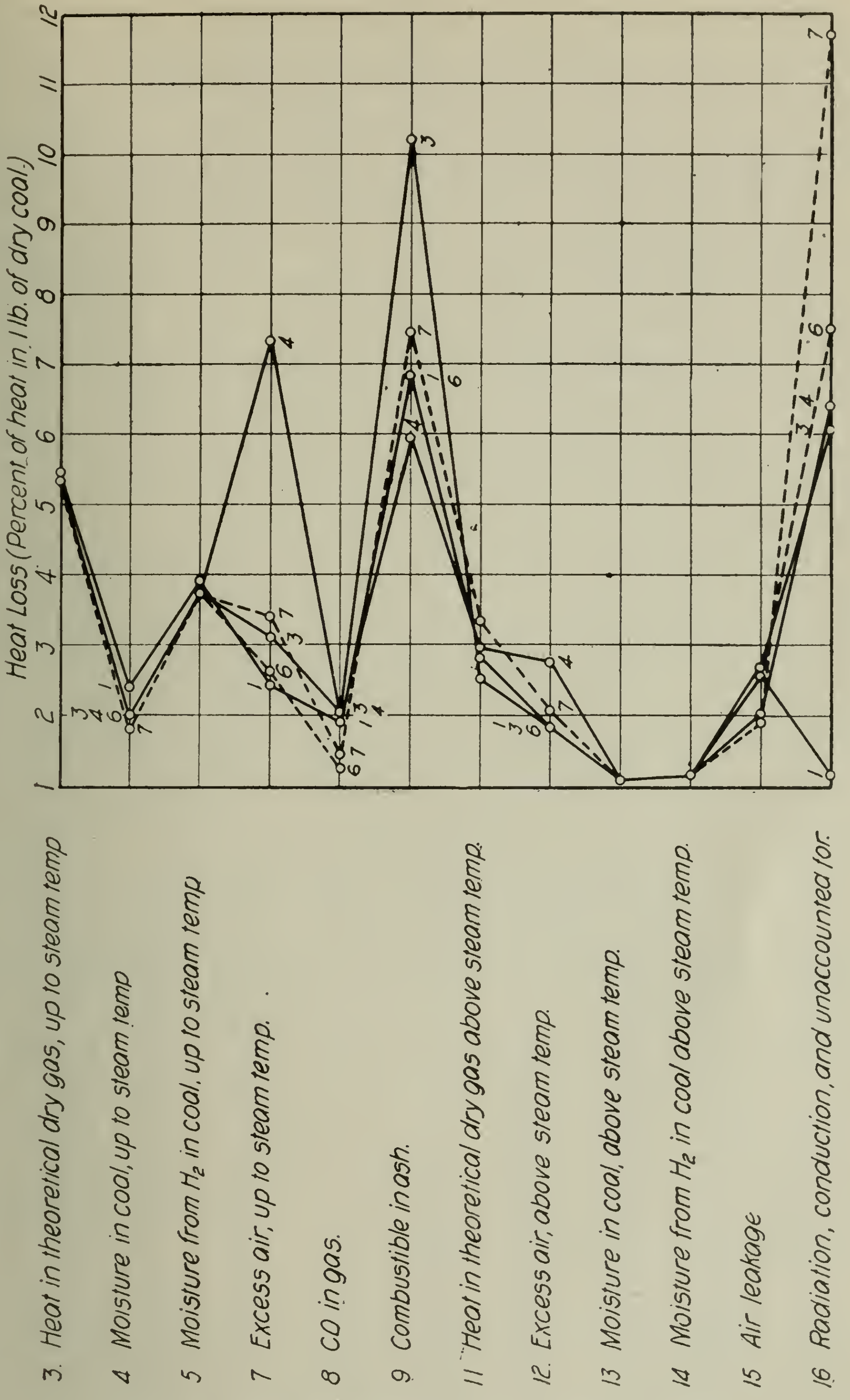

FIG. 30. RELATIVE MAGNITUDE OF VARIOUS LOSSES.

Since the samples were limited in quantity, these were necessarily short tests, but every effort was made to maintain uniform conditions, and particularly to have the fuel bed in the same condition at the start and close, thus reducing the chances of error as much as possible. An inspection of the heat balance will prove that no very serious error arose.

As the coal appeared to be covered with a coating of ash or clay, it was expected that considerable trouble would be experienced in burning it. This, however, did not prove to be the case. On the first test the coal banked slightly at the water-back, and the whole amount on the grate became clinkered. It immediately became evident that in order to run at all, the coal had to be kept away from the water-back. After the clinker had been removed, a fresh start was made and care was taken to keep the fuel bed from four to six inches away from the water-back. When this was done no further trouble was experienced.

All of the tests on the weathered coal were run with a 7-inch fire with the exception of test No. 21, which was run with a 6½-inch fire because the coal seemed to be broken up finer than the rest. In all cases but one the fire burned white and did not require much attention. The exception was test No. 23. On looking up the firing record, it was found that on this test the fire was rather cold and gave evidence of being too thick. It was found also that the fire required considerable attention, having to be barred and raked much oftener than the rest. This process, of course, let in air and lowered the $CO_2$. An inspection of the efficiencies in Table 9 will show that while the boiler efficiency compares favorably with the rest, the efficiency of the furnace and grate was low. Also, it appears from the heat balance that the loss due to combustible in the ash was high. A comparison of the dust in this coal with that in test No. 21 discloses the fact that they were very nearly the same. In that test a 6½-inch fire was used and the results were satisfactory. These facts all lead to the conclusion that a 7-inch fire was too thick in the case of test 23, and that the trouble encountered on this test should be charged against the lack of experience in handling the coal, rather than against the coal itself. The capacity obtained was also lower than in the other weathered coal tests.

A comparison of the draft required for a given rate of combustion for the weathered coal with that for the Mission Field screenings is made in Fig. 31. Referring to this figure it becomes evident that more draft is required to produce a given rate of combustion with the former than

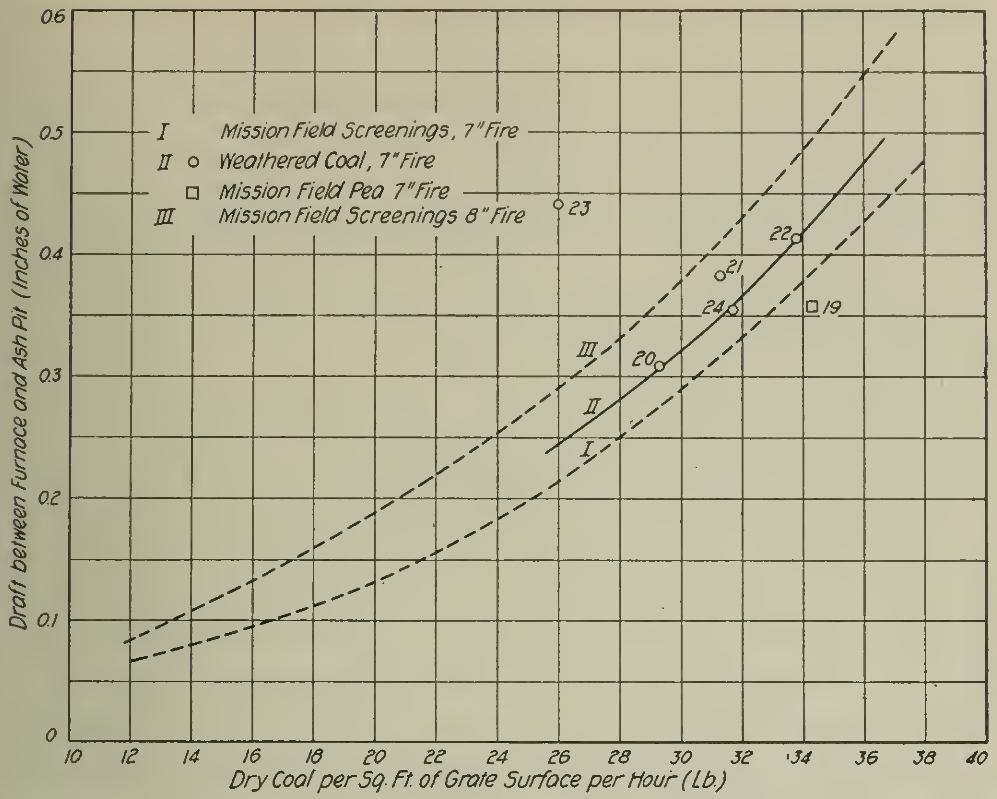

FIG. 31. DRAFT REQUIRED FOR A GIVEN RATE OF COMBUSTION FOR WEATHERED COAL AND MISSION FIELD SCREENINGS.

### TABLE 18.

### COMPARISON OF DUST CONTENT, MISSION FIELD SCREENINGS AND WEATHERED COAL.

| Mission Field Screenings | | Weathered Coal | |
|---|---|---|---|
| Test No. | Per Cent Dust | Test No. | Per Cent Dust |
| 2 | 34.8 | 20 | 31.9 |
| 6 | 19.6 | 21 | 45.1 |
| 12 | 40.3 | 22 | 13.9 |
| 13 | 26.3 | 23 | 42.9 |
| 17 | 24.7 | 24 | 35.0 |

with the latter. That this is not caused by dust in the coal may be shown by Table 18 and Fig. 18.

The dust content of the coals from which the two curves for the 7-inch fires were plotted is practically the same. In each case there were one or two tests in which the dust content was high, and the points representing these fell higher than the curve; i. e., point 12 was off of the curve in Fig. 18, while in Fig. 31 points 21 and 23 were not on the curve. In Fig. 18 the points of low dust content are below the curve for 7-inch fires, while in Fig. 31 all the points are above this curve, which is shown dotted. This seems to lead to the conclusion that the dust is not responsible for the high draft requirement. A possible explanation is that this coal is harder to ignite, owing to the partially oxidized condition indicated by the gray coating on the outside, and therefore it requires more draft than the Mission Field coal for a given rate of combustion.

The efficiencies obtained from the weathered coal compare very favorably with those obtained from the Mission Field coal. For the sake of convenience these have been tabulated for corresponding loads, and are shown in Table 19. The average for each set is about 64 per cent, the weathered coal showing an advantage at heavier loads. It is also worthy of note that greater capacity was obtained with the weathered coal than with the screenings, the heat value of both being about the same.

On comparing firing sheets, it was also found that on the whole, when weathered coal was being used, the fires required less attention, such as slicing and raking, than they did when fresh screenings were being used.

TABLE 19.

COMPARISON OF BOILER PERFORMANCES ON MISSION FIELD AND WEATHERED COAL.

| Mission Field Coal | | | Weathered Coal | | |
|---|---|---|---|---|---|
| Test No. | Boiler H. P. Developed | Efficiency of Boiler, Furnace and Grate | Test No. | Boiler H. P. Developed | Efficiency of Boiler, Furnace and Grate |
| 10 | 554.0 | 63.96 | 20 | 568.5 | 64.50 |
| 11 | 569.6 | 61.21 | 21 | 557.2 | 63.05 |
| 12 | 572.7 | 60.67 | 22 | 727.1 | 65.98 |
| 13 | 589.0 | 69.87 | 23 | 509.6 | 60.04 |
| 14 | 555.9 | 64.75 | 24 | 655.0 | 64.20 |
| 15 | 506.6 | 65.50 | | | |
| 16 | 644.0 | 60.84 | | | |

## VI. SUMMARY AND CONCLUSIONS.

*General Conclusions.*—1. The over-all efficiency of the unit tested, under the average conditions which prevailed during the tests, and when developing in the neighborhood of 550 boiler horsepower, is about 61 per cent.

2. The best thickness of fire for Mission Field screenings is about 7 or 7½ inches, depending upon the amount of fine material in the coal.

3. The maximum capacity developed was 768 boiler horsepower. This was obtained from pea coal with a 7-inch fuel bed and a draft over the fire of 0.375 inches of water.

4. The best rate of combustion is about 26 pounds of dry coal per sq. ft. of grate surface per hour. The drafts over the fire required to give this rate are as follows: 0.18 in. water for a 6-in. fire, 0.22 in. water for a 7-in fire, 0.29 in. water for an 8-in. fire, and 0.52 in. water for a 9½-in. fire.

5. The greatest losses were due to combustible in the ash, excess air, and air leakage.

6. The smallest loss was that due to the formation of CO.

7. With the exception of the air leakage loss, which can be practically eliminated without affecting any of the others, the rest of the losses are to some extent interrelated, and there are certain well defined limits within which the excess air must be maintained in order to reduce the others to a minimum. This is approximately 30 to 40 per cent.

8. The sum of the excess air loss and the loss due to combustible in the ash was nearly constant. It amounted to about 14 per cent, and a decrease in one source of loss could be accomplished only by an increase in the other. Hence the formation of CO was the factor having the greatest bearing on the extent to which excess air could be reduced. The latter could be decreased until CO appeared in excess.

9. The percentage of $CO_2$ depends to some extent on the attention required by the fire. The $CO_2$ was low on the tests during which the side door had to be frequently opened in order to allow barring and raking of the fire.

10. The form of baffling shown in Fig. 1 and used on these tests makes the maintenance of high furnace temperatures possible, and reduces the amount of smoke formed.

11. This form of baffling gives trouble due to the difficulty in removing the soot which collects in the corner where the first baffle joins the tile roof of the furnace.

21. *Conclusions Relative to Weathered Coal.*—12. Burning weathered coal is largely a question of correct handling and ignition. Under these circumstances it gives as good results as fresh screenings. It

must be borne in mind, however, that the conclusions drawn from the weathered coal tests are based solely on the burning of this fuel and do not deal with the deterioration in storage. In making comparisons it must also be noted that from the physical and chemical standpoint the original fuel had deteriorated and that at the time it was used it was on a basis of equality with the Mission Field screenings.

13. Weathered coal requires a little thinner fire and more draft than fresh screenings.

14. When using weathered coal the fuel bed should not approach any nearer to the water-back than from 4 to 6 inches, otherwise trouble with clinker is experienced.

15. Practically as high capacity was obtained with weathered coal as with the other coals used, and, if anything, the fuel bed required less attention.

# APPENDIX.

## FORMULAS AND FORMS FOR CALCULATING BOILER TRIALS.

Owing to the fact that the adding machine probably is the only type of calculating machine on which more than one mathematical process can be performed without resetting, it has been considered advantageous to arrange the code employed in calculating the results of these tests so that each item contains but one such process, except in the case of additions. A complicated formula may, therefore, be spread out over a number of items in the code. This form is convenient when a large number of tests are to be calculated, in that all like operations are grouped, so that in many cases a saving can be made in the number of resettings required on the machine. The code can also be arranged in columnar form, with the mathematical operation indicated at the head of each column, and the calculations may be made by a clerk having little knowledge of the technical significance of the processes involved. In such cases, however, considerable care must be exercised, and plus and minus signs rigorously observed.

Since, in the main, the methods of calculation recommended in the American Society of Mechanical Engineers' Boiler Code have been followed, it will not be necessary to discuss all of the items. In some cases, however, a departure has been made, and it has been thought advisable to take up these items and discuss the methods and formulas used in their calculation. In the following discussion the numbers refer to the calculating code given on pages 62 to 72.

### *Item 29. Weight of Combustible Consumed.*

It is usual to regard all of the coal as combustible, after deducting the ash and moisture. This being the case, the weight of combustible fired on the grates is $W\left(1-\frac{a}{100}\right)$, in which $W$ is the weight of dry coal and $a$ the percentage of ash taken from the analysis of the dry coal. Of this combustible, part goes through the grate in the form of unconsumed carbon. No essential error is made in regarding all of the combustible in the ash as carbon. Then the actual weight is $\frac{W_1c}{100}$, where $W_1$

is the actual weight of ash and refuse, and $c$ is the percentage of carbon taken from an analysis of the ash and refuse. Therefore:

Item 29 $= W\left(1 - \frac{a}{100}\right) - \frac{W_1c}{100}$. Item 29 is sometimes simply taken as the difference, $(W - W_1)$. This is in error, however, because all of the ash coming from the coal consumed may not appear in the weighed ash and refuse, for part of it is carried over the bridge wall in the form of fine dust. A larger source of error, however, is due to the fact that the fuel bed itself may contain more or less ash at the close than it did at the start of the test. Owing to this cause it is found that the result obtained by the simple subtraction is sometimes greater and at other times less than that obtained by making use of the analyses as outlined above.

*Items 31 to 55. Coal analyses.*

Where analyses are made direct on the samples of coal as fired, and both proximate and ultimate analyses reported by the chemist, the calculations included here are unnecessary. In the present case, however, the samples were partly dried before they were sent to the chemist, and the ultimate analyses were calculated from the individual proximate analyses and a composite ash, sulphur, and moisture free analysis.

In order to reduce the analyses as reported by the chemist to terms of coal as fired, each constituent had to be multiplied by the factor,

$$\left(1 - \frac{\text{per cent moisture lost before analysis}}{100}\right).$$

The separate ultimate analyses were then obtained by multiplying each constituent in the composite ash, sulphur, and moisture free analysis by the factor,

$\left(1 - \frac{\text{per cent ash in dry coal} + \text{per cent sulphur in dry coal}}{100}\right)$, which was obtained from the individual proximate analyses. The ash and sulphur in the dry coal were obtained by dividing the ash and sulphur in coal as fired by the factor,

$$\left(1 - \frac{\text{per cent moisture in coal as fired}}{100}\right).$$

*Item 110. Corrected total weight of water fed to boiler.*

The ultimate object of a boiler trial is to find the relation existing between the heat taken up by the boiler, and the heat supplied in the coal. It is, therefore, in reality a heat analysis. Since, however, items are not included in the standard code whereby these heat quantities may be separately tabulated, any heat with which the boiler is debited or credited that is not directly included in the weight of coal or water must be reduced to terms of one of these two, and added to or subtracted from the actual weighed amount. In order to put them into shape so that they may be conveniently handled in subsequent calculations, the usual method is to reduce all such quantities to terms of water. The following are some of the items that must be taken account of in this manner: (1) difference in water level at the start from that at the end of a test, (2) difference in pressure at the start from that at the end, (3) leakage from blow-off, etc., (4) heat going into the circulating water from the water-back in case it is not returned to the feed water at a point closer to the boiler than that at which the temperature of the latter is taken. Of these, No. 1 and No. 2 can be combined and treated under two heads.

Let $W$ = weight of water in the boiler.
$q$ = heat of the liquid.
$r$ = latent heat of steam.
$x$ = quality of steam.

Also let the subscript 1 refer to initial, the subscript 2 to the final, and the subscript 0 to average conditions in the boiler, and the subscript 3 to feed water conditions.

If the water level is higher at the close than at the start, the heat that has gone into the excess water, and into the change in the heat of the liquid due to the change in pressure, is the difference between the total heat in the water in the boiler at the close and that at the start. Or, taking all heat quantities above that of the liquid at feed water temperature it is:

$$W_2\,(q_2 - q_3) - W_1\,(q_1 - q_3).$$

The water that this amount of heat would have evaporated into steam as generated is; $\dfrac{W_2\,(q_2 - q_3) - W_1\,(q_1 - q_3)}{q_0 + x_0 r_0 - q_3}.$

This should be credited back to the boiler, after the difference between final and initial weights of water has been subtracted; or writing the total correction to automatically take care of signs, and calling the correction $W'$,

$$W' = (W_1 - W_2) + \frac{W_2\,(q_2 - q_3) - W_1\,(q_1 - q_3)}{q_0 + x_0 r_0 - q_3} \quad \text{I}$$

The case in which the water level is lower at the close than at the start is somewhat different from the above. In this event, all of the water in the boiler at the close of the test has been changed from a heat state represented by $q_1$, to one represented by $q_2$, while a weight corresponding to $(W_1 - W_2)$ has been evaporated. Since we do not know the exact conditions under which the latter took place, about the only assumption warranted is that it took place under the average conditions of the test. The total heat change to be taken into account then is:

$$W_2 (q_2 - q_1) + (W_1 - W_2)\ (q_0 + x_0 r_0 - q_1).$$

Calling the equivalent amount of water, or the correction, $W''$,

$$W'' = \frac{W_2 (q_2 - q_1) + (W_1 - W_2)\ (q_0 + x_0 r_0 - q_1)}{q_0 + x_0 r_0 - q_3} \ldots\ldots\ldots\ldots\ldots \text{II}$$

On expanding equations I and II, and collecting the terms, it becomes evident that they both reduce to the same expression, which is:

$$W''' = W_1\ \frac{(q_0 + x_0 r_0 - q_1) - W_2\ (q_0 + x_0 r_0 - q_2)}{q_0 + x_0 r_0 - q_3} \ldots\ldots\ldots\ldots\ldots \text{III}$$

Equation III has been made use of in the code for the combined corrections for changes in pressure and water levels. In case a blow-off leaks, or hot water is lost from any other point in the boiler, the heat loss may easily be seen to be $W_4\ (q_0 - q_3)$, in which $W_4$ = the weight of water leaking out. The correction then is:

$$W^{\text{IV}} = \left[\frac{W_4\ (q_0 - q_3)}{q_0 + x_0\ r_0 - q_3}\right] - W_4.$$

If the water back forms part of the circulating system of the boiler, or the water is returned at a point nearer to the boiler than that at which the temperature of the feed water is read, no account need be taken of it. If this is not the case, however, the water back should be regarded as part of the heating surface of the boiler, and the heat going to it credited to the latter.

Let $W_5$ = weight of water going through the water back, and $t_i$ its initial and $t_f$ its final temperature. Then the correction, $W^{\text{V}}$, is

$$\frac{W_5\ (t_f - t_i)}{q_0 + x_0 r_0 - q_3}$$

Designating by $W$ the total amount of water weighed to boiler, corrected for tank levels and items which do not include heat quantities,

$$\text{Item } 110 = W + W''' + W^{\text{IV}} + W^{\text{V}}.$$

*Item 111. Factor of evaporation.*

In this code the water evaporated has not been reduced to dry steam and the dry steam then reduced to equivalent evaporation, but the reduction from steam as generated to equivalent evaporation has been made in one step, making use of the "total" or "true" factor of evaporation:

$$F = \frac{q_0 + x_0 r_0 - q_3}{970.4},$$

in which $F =$ factor of evaporation, $q_0 =$ heat of the liquid at boiler pressure, $x_0 =$ quality of steam, $r_0 =$ latent heat at boiler pressure, and $q_3 =$ heat of the liquid at feed water temperature.

*Item 126. Heat available for use by the boiler.*

The heat available for use by the boiler has been regarded as all of the heat in the gas leaving the furnace, computed above the temperature of the steam in the boiler. This is equal to the heating value per pound of dry coal minus the furnace losses per pound of dry coal, up to the temperature of the steam in the boiler.

*Item 133. Efficiency of boiler, furnace, and grates, based on zero air leakage.*

This is an hypothetical quantity that permits of no rigid solution, and has been partly discussed on page 21. Of a number of possible assumptions the following seem to be the most tenable. All of the heat supplied to the boiler comes from the heat in the coal $(H)$. A certain portion of this $(H_1)$ is absorbed by the water in the boiler, and the rest goes to losses, of which the air leakage loss $(H_2)$ is one. If there had been no loss due to air leakage, the heat $(H_2)$ would have been available for absorption by the water. Since this process would not have taken place at 100 per cent efficiency, it seems reasonable to assume that it would have done so at the efficiency of the boiler alone. If we designate the latter by $(e)$, then the probable heat absorption would have been: $H_1 + eH_2$, and Item 133 $= \dfrac{H_1 + eH_2}{H}$.

*Item 153. Theoretical weight of dry gas per lb. of dry coal.*

Theoretically the gas given off per lb. of dry coal should consist of the sum of the products of combustion of the carbon and sulphur in the coal, plus the nitrogen carried in with the air necessary for the complete combustion of all the combustible constituents, including the hydrogen, plus the nitrogen in the coal. Since some of the oxygen for combustion came from the free $O_2$ in the coal, the nitrogen equivalent of this should be deducted from the above. Letting $C =$ per cent carbon in dry coal, $S =$ per cent sulphur, $H_2 =$ per cent hydrogen, $N_2 =$ per cent nitrogen, and $O_2 =$ per cent oxygen, the corresponding weights of each constituent in the dry coal will be 1/100 of each of the above.

From the stoichiometric relations, the sum of these products of combustion may now be deduced, or

$$\text{Item } 153 = 0.1261\ (C) + 0.0535\ (S) + 0.268\ (H_2) + 0.01\ (N_2) - 0.0335\ (O_2).$$

*Item 162. Actual weight of dry gas per lb. of dry coal.*

This calculation is based on the fact that all of the carbon in the coal which was actually burned, appears in the gas.

If $CO_2$, $CO$, and $O_2$ represent the parts by volume of the constituents in a given volume of flue gas, the parts by weight may be found by multiplying the above by their corresponding molecular weights. The sum of these relative weights in the flue gas is, $44\ (CO_2) + 28\ (CO + N_2) + 32\ (O_2)$. The relative weight of the $C$ in the same volume is $12\ (CO_2 + CO)$. Hence the weight of gas per lb. of carbon consumed is:

$$\frac{44\ (CO_2) + 28\ (CO + N_2) + 32\ (O_2)}{12\ (CO_2 + CO)} =$$

$$\frac{11\ (CO_2) + 7\ (CO + N_2) + 8\ (O_2)}{3\ (CO_2 + CO)}.$$

Since $N_2$ is always determined by difference: $N_2 = 100 - (CO_2 + CO + O_2)$, and the above reduces to: weight of gas per lb. carbon consumed $= \dfrac{4\ (CO_2) + O_2 + 700}{3\ (CO_2 + CO)}$.

Let $W_1 =$ weight of dry coal fired, $W_2 =$ weight of ash and refuse, $(a) =$ per cent carbon in dry coal and $(b)$ per cent carbon in ash. Then the weight of carbon actually consumed per lb. of dry coal $=$

$$\frac{0.01 W_1 a - 0.01 W_2 b}{W_1}, \text{ and Item } 162 =$$

$$\left[\frac{0.01\ (W_1 a - W_2 b)}{W_1}\right]\left[\frac{4\ (CO_2) + O_2 + 700}{3\ (CO_2 + CO)}\right].$$

This is slightly in error due to the fact that $SO_2$ is formed from the sulphur in the coal. Part of this is absorbed by the water in the sampling apparatus, while some of it probably goes into the $KOH$ pipette, and thus appears in the analysis as $CO_2$. It would be rather difficult to determine the exact distribution of these two. However, a simple calculation will show that the maximum error due to this source does not exceed 0.2 per cent.

*Item 174. Theoretical weight of air per lb. of dry coal.*

This can be calculated directly from the stoichiometric relations and the analysis of the coal.

If $C$, $H_2$, $S$, and $O_2$ represent the percentage by weight of the various constituents in the coal, then the weight of $O_2$ required is $0.0267\ (C) + 0.08\ (H_2) + 0.01\ (S) - 0.01\ (O_2)$, and,

$$\text{Item } 174 = \frac{0.0267\ (C) + 0.08\ (H_2) + 0.01\ (S) - 0.01\ (O_2)}{0.23} =$$

$0.116\ (C) + 0.348\ (H_2) + 0.0435\ (S - O_2)$.

*Item 177. Actual weight of air per lb. of dry coal.*

All of the $N_2$ appearing in the gas, with the exception of that already in the coal, came from the air taken into the furnace. By a line of reasoning similar to that discussed under Item 162, it may be shown that the weight of $N_2$ per lb. of carbon consumed $= \dfrac{7\ (N_2)}{3\ (CO_2 + CO)}$.

Then the weight of air per lb. of carbon consumed $= \dfrac{N_2}{0.33\ (CO_2 + CO)}$.

Taking the weight of carbon consumed per lb. of dry coal from the discussion under Item 162,

$$\text{Item } 177 = \left[\frac{0.01\ (W_1 a - W_2 b)}{W_1}\right]\left[\frac{N_2}{0.33\ (CO_2 + CO)}\right].$$

## FORMS FOR CALCULATING BOILER TRIALS.

(1) Test number.
(2) Date.
(3) Duration, hrs.
(4) Grate surface, sq. ft.
(5) Water heating surface, sq. ft.
(6) Barometer reading, inches of Hg.
(7) Atmospheric pressure, lb. per sq. in.; 0.491 (Item 6).
(8) Steam pressure, lb. per sq. in. gage.
(9) Steam pressure, lb. per sq. in. absolute; (Item 8) + (Item 7).
(10) Draft between damper and boiler, in. water.
(11) Draft at end of 3rd pass, in. water.
(12) Draft between 1st and 2nd pass, in. water.
(13) Draft at end of combustion chamber, in. water.
(14) Draft over fire in furnace, in. water.
(15) Draft in ash pit, in. water.
(16) Temperature of outside air, deg. F.
(17) Temperature of fire room, deg. F.
(18) Temperature of steam, deg. F.
(19) Temperature of feed water, at boiler, deg. F.
(20) Temperature of ash, deg. F.
(21) Temperature of gas escaping from boiler, deg. F.
(22) Weight of coal fired, lb.
(23) $\left(1 - \frac{\text{Item 41}}{100}\right)$.
(24) Weight of dry coal fired, lb.; (Item 22) (Item 23).
(25) Total ash and refuse, lb.
(26) $\left(1 - \frac{\text{Item 55}}{100}\right)$.
(27) Weight of combustible fired, lb.; (Item 24) (Item 26).
(28) Weight of carbon in ash, lb.; $\frac{(\text{Item 25})\ (\text{Item 56})}{100}$.
(29) Weight of combustible consumed, lb.; (Item 27) — (Item 28).

(30) Per cent ash and refuse in dry coal; $\frac{\text{(Item 25)}}{\text{(Item 24)}} \times 100.$

### PROXIMATE ANALYSIS (COAL AS RECEIVED BY CHEMIST).

(31) Fixed carbon, per cent.
(32) Volatile, per cent.
(33) Moisture, per cent.
(34) Ash, per cent.
(35) Sulphur (separately determined), per cent.
(36) Moisture loss previous to sending to chemist, per cent.
(37) $1 - \frac{\text{(Item 36)}}{100}.$

### PROXIMATE ANALYSIS (COAL AS FIRED).

(38) Fixed carbon, per cent; (Item 37) (Item 31).
(39) Volatile, per cent; (Item 37) (Item 32).
(40) Moisture (as received by chemist reduced to coal as fired), per cent; (Item 37) (Item 33).
(41) Total moisture in coal as fired, per cent; (Item 40) + (Item 36).
(42) Ash, per cent; (Item 37) (Item 34).
(43) Sulphur (separately determined), per cent; (Item 37) (Item 35).

### ULTIMATE ANALYSIS (ASH, MOISTURE, AND SULPHUR FREE COAL).

(44) Carbon, per cent.
(45) Hydrogen, per cent.
(46) Oxygen, per cent.
(47) Nitrogen, per cent.
(48) (Item 54) + (Item 55).
(49) $1 - \frac{\text{(Item 48)}}{100}.$

### ULTIMATE ANALYSIS (DRY COAL).

(50) Carbon (C), per cent; (Item 49) (Item 44).
(51) Hydrogen ($H_2$), per cent; (Item 49) (Item 45).
(52) Oxygen ($O_2$), per cent; (Item 49) (Item 46).
(53) Nitrogen ($N_2$), per cent; (Item 49) (Item 47).
(54) Sulphur (S), per cent; $\frac{\text{(Item 43)}}{\text{(Item 23)}}.$
(55) Ash, per cent; $\frac{\text{(Item 42)}}{\text{(Item 23)}}.$

ANALYSIS OF ASH (DRY).

(56) Carbon, per cent.
(57) Earthy matter, per cent.
(58) Dry coal fired per hour, lb.; $\frac{\text{(Item 24)}}{\text{(Item 3)}}$.
(59) Combustible consumed per hour, lb.; $\frac{\text{(Item 29)}}{\text{(Item 3)}}$.
(60) Dry coal per sq. ft. of grate surface per hour, lb.; $\frac{\text{(Item 58)}}{\text{(Item 4)}}$.
(61) Combustible per sq. ft. of water heating surface per hour, lb.; $\frac{\text{(Item 59)}}{\text{(Item 5)}}$.
(62) Calorific value per lb. of dry coal, by calorimeter, B. t. u.
(63) Calorific value per lb. of combustible, by calorimeter, B. t. u.; $\frac{\text{(Item 62)}}{\text{(Item 26)}}$.
(64) Temperature in atmospheric side of calorimeter, deg. F.
(65) Superheat in calorimeter, deg. F; (Item 64) — 211.5.
(66) Quality of steam ($x_0$); from Marks and Davis chart.
(67) Total weight of water, from weighing tanks, lb.
(68) Initial height in feed tank, in.
(69) Final height in feed tank, in.
(70) (Item 68) — (Item 69).
(71) Weight of water per inch of tank, lb.
(72) Feed tank correction; (Item 70) (Item 71).
(73) Initial height of water in gage glass, in.
(74) Final height of water in gage glass, in.
(75) Initial steam pressure, lb. per sq. in. gage.
(76) Initial steam pressure, lb. per sq. in. absolute; (Item 75) + (Item 7).
(77) Final steam pressure, lb. per sq. in. gage.
(78) Final steam pressure, lb. per sq. in. absolute; (Item 77) + (Item 7).
(79) Density of water at initial pressure, lb. per cu. ft.
(80) Density of water at final pressure, lb. per cu. ft.
(81) Initial volume of water in boiler, cu. ft.
(82) Final volume of water in boiler, cu. ft.

(83) Initial weight of water in boiler ($W_1$), lb.; (Item 79) (Item 81).

(84) Final weight of water in boiler ($W_2$), lb.; (Item 80) (Item 82).

(85) Heat of the liquid at initial steam pressure ($q_1$), B. t. u.

(86) Heat of the liquid at final steam pressure ($q_2$), B. t. u.

(87) Heat of the liquid at average steam pressure ($q_0$), B. t. u.

(88) Heat of the liquid at feed water temperature ($q_3$), B. t. u.

(89) Latent heat at average steam pressure ($r_0$), B. t. u.

(90) Weight of water to water-back, lb.

(91) Temperature of water entering water-back, deg. F.

(92) Temperature of water leaving water-back, deg. F.

(93) Total weight of hot water leakage from blow-offs, etc., lb.

(94) $x_0 r_0$; (Item 66) (Item 89).

(95) $q_0 + x_0 r_0$; (Item 94) + (Item 87).

(96) $q_0 + x_0 r_0 - q_1$; (Item 95) − (Item 85).

(97) $W_1 (q_0 + x_0 r_0 - q_1)$; (Item 83) (Item 96).

(98) $q_0 + x_0 r_0 - q_2$; (Item 95) − (Item 86).

(99) $W_2 (q_0 + x_0 r_0 - q_2)$; (Item 84) (Item 98).

(100) $W_1 (q_0 + x_0 r_0 - q_1) - W_2 (q_0 + x_0 r_0 - q_2)$; (Item 97) − (Item 99).

(101) $q_0 + x_0 r_0 - q_3$; (Item 95) − (Item 88).

(102) Correction for pressure and boiler level, lb.; $\frac{\text{(Item 100)}}{\text{(Item 101)}}$.

(103) (Item 92) − (Item 91).

(104) (Item 90) (Item 103).

(105) Water-back correction; $\frac{\text{(Item 104)}}{\text{(Item 101)}}$.

(106) (Item 87) − (Item 88).

(107) (Item 106) (Item 93).

(108) Steam equivalent of blow-off leak; $\frac{\text{(Item 107)}}{\text{(Item 101)}}$.

(109) Correction for leakage from blow-off, etc.; (Item 93) − (Item 108).

(110) Total weight of water to boiler, corrected; (Item 67) + (Item 72) + (Item 102) + (Item 105) + (Item 109).

(111) Factor of evaporation; $\frac{\text{(Item 101)}}{970.4}$.

(112) Equivalent evaporation from and at 212° F., lb.; (Item 110) (Item 111).

(113) Equivalent evaporation per hr., lb.; $\frac{(\text{Item } 112)}{(\text{Item } 3)}$.

(114) Equivalent evaporation per sq. ft. of water heating surface per hr., lb.; $\frac{(\text{Item } 113)}{(\text{Item } 5)}$.

(115) Horsepower developed; $\frac{(\text{Item } 113)}{34.5}$.

(116) Builder's rated horsepower.

(117) Percentage of builder's rating developed, per cent; $\frac{(\text{Item } 115)}{(\text{Item } 116)}\,100$.

(118) Water apparently evaporated per lb. of coal fired, lb.; $\frac{(\text{Item } 110)}{(\text{Item } 22)}$.

(119) Equivalent evaporation per lb. of coal fired, lb.; $\frac{(\text{Item } 112)}{(\text{Item } 22)}$.

(120) Equivalent evaporation per lb. of dry coal, lb.; $\frac{(\text{Item } 112)}{(\text{Item } 24)}$.

(121) Equivalent evaporation per lb. of combustible consumed, lb.; $\frac{(\text{Item } 112)}{(\text{Item } 29)}$.

(122) Heat absorbed by boiler per lb. of combustible consumed, B. t. u.; 970.4 (Item 121).

(123) Efficiency of boiler and furnace excluding grate, per cent; $100\,\frac{(\text{Item } 122)}{(\text{Item } 63)}$.

(124) Efficiency of boiler, furnace, and grate, per cent; $100\,\frac{(\text{Item } 186)}{(\text{Item } 62)}$.

(125) (Item 189) + (Item 196) + (Item 198) + (Item 204) + (Item 206) + (Item 209) + (Item 211) + (Item 214).

(126) Heat available for use by boiler, B. t. u.; (Item 62) − (Item 125).

(127) Efficiency of furnace and grate, per cent; $100\,\frac{(\text{Item } 126)}{(\text{Item } 62)}$.

(128) Efficiency of boiler, per cent; $100\ \frac{(\text{Item } 186)}{(\text{Item } 126)}$.

(129) (Item 62) − (Item 211).

(130) Efficiency of furnace, per cent; $100\ \frac{(\text{Item } 126)}{(\text{Item } 129)}$.

(131) $\frac{(\text{Item } 224)\ (\text{Item } 128)}{100}$.

(132) (Item 186) + (Item 131).

(133) Over-all efficiency based on zero air leakage, per cent; $100\ \frac{(\text{Item } 132)}{(\text{Item } 62)}$.

(134) Cost of coal per ton of 2,000 lb., dollars.

(135) Tons of coal per 1,000 lb. of water apparently evaporated; $\frac{0.5}{(\text{Item } 118)}$.

(136) Cost of coal to evaporate 1,000 lb. of water under observed conditions, dollars; (Item 134) (Item 135).

(137) Tons of coal per 1,000 lb. of water evaporated from and at 212°F.; $\frac{0.5}{(\text{Item } 119)}$.

(138) Cost of coal to evaporate 1,000 lb. of water from and at 212°F., dollars; (Item 134) (Item 137).

### ANALYSIS OF DRY GAS AT FURNACE.

(139) $CO_2$, per cent.
(140) $O_2$, per cent.
(141) CO, per cent.
(142) $N_2$ (by difference), per cent.

### ANALYSIS OF DRY GAS AT FLUE.

(143) $CO_2$, per cent.
(144) $O_2$, per cent.
(145) CO, per cent.
(146) $N_2$ (by difference), per cent.
(147) 0.1261 (Item 50).
(148) 0.0535 (Item 54).
(149) 0.268 (Item 51).

(150) 0.01 (Item 53).

(151) 0.0335 (Item 52).

(152) (Item 147) + (Item 148) + (Item 149) + (Item 150).

(153) Theoretical dry gas for complete combustion per lb. of dry coal, lb.; (Item 152) — (Item 151).

(154) 4 (Item 139).

(155) (Item 154) + (Item 140) + 700.

(156) (Item 139) + (Item 141).

(157) 3 (Item 156).

(158) Actual weight of dry gas per lb. of carbon consumed, at furnace, lb.; $\frac{\text{(Item 155)}}{\text{(Item 157)}}$.

(159) Total weight of carbon fired, lb.; $\frac{\text{(Item 24) (Item 50)}}{100}$.

(160) Weight of carbon consumed, lb.; (Item 159) — (Item 28).

(161) Weight of carbon consumed per lb. of dry coal fired, lb.; $\frac{\text{(Item 160)}}{\text{(Item 24)}}$.

(162) Actual weight of gas per lb. of dry coal, at furnace, lb.; (Item 161) (Item 158).

(163) 4 (Item 143).

(164) (Item 163) + (Item 144) + 700.

(165) (Item 143) + (Item 145).

(166) 3 (Item 165).

(167) Actual weight of gas per lb. of carbon consumed, at flue, lb.; $\frac{\text{(Item 164)}}{\text{(Item 166)}}$.

(168) Actual weight of dry gas per lb. of dry coal, at flue, lb.; (Item 161) (Item 167).

(169) 0.116 (Item 50).

(170) 0.348 (Item 51).

(171) (Item 52) — (Item 54).

(172) 0.0435 (Item 171).

(173) (Item 169) + (Item 170).

(174) Theoretical weight of air per lb. of dry coal, for complete combustion, lb.; (Item 173) — (Item 172).

(175) 0.33 (Item 156).

(176) Actual weight of air per lb. of carbon consumed, at furnace, lb.; $\frac{\text{(Item 142)}}{\text{(Item 175)}}$.

(177) Actual weight of air per lb. of dry coal, at furnace, lb.; (Item 161) (Item 176).

(178) 0.33 (Item 165).

(179) Actual weight of air per lb. of carbon consumed, at flue, lb.; $\frac{\text{(Item 146)}}{\text{(Item 178)}}$.

(180) Actual weight of air per lb. of dry coal, at flue, lb.; (Item 161) × (Item 179).

(181) Weight of air leakage, per lb. of dry coal, lb.; (Item 180) — (Item 177).

(182) Weight of excess air per lb. of dry coal, at furnace, lb.; (Item 177) — (Item 174).

(183) Per cent excess air, at furnace; $100 \frac{\text{(Item 182)}}{\text{(Item 174)}}$.

(184) Weight of excess air per lb. of dry coal, at flue, lb.; (Item 180) — (Item 174).

(185) Per cent excess air, at flue; $100 \frac{\text{(Item 184)}}{\text{(Item 174)}}$.

### HEAT BALANCE.

(186) Heat absorbed by boiler per lb. of dry coal, B. t. u.; 970.4 (Item 120).

(187) (Item 18) — (Item 17).

(188) 0.24 (Item 153).

(189) Loss due to theoretical gas up to temperature of steam, B. t. u.; (Item 188) (Item 187).

(190) 212 — (Item 17).

(191) (Item 18) — 212.

(192) 0.5 (Item 191).

(193) 970.4 + (Item 190) + (Item 192).

(194) 0.01 (Item 22) (Item 41).

(195) $\frac{\text{(Item 194)}}{\text{(Item 24)}}$.

(196) Loss due to moisture in coal, up to steam temperature, B. t. u.; (Item 195) (Item 193).

(197) 0.09 (Item 51).

(198) Loss due to $H_2$ in coal, up to steam temperature, B. t. u.; (Item 197) (Item 193).

(199) Wet bulb reading, deg. F.

(200) Dry bulb reading, deg. F.

(201) Grains of moisture per lb. of air (from Carrier's chart).

(202) Lb. moisture per lb. air; $\frac{(\text{Item } 201)}{7{,}000}$.

(203) Moisture from air per lb. dry coal, lb.; (Item 177) (Item 202).

(204) Loss due to moisture in air, up to steam temperature, B. t. u.; (Item 203) (Item 193).

(205) 0.24 (Item 182).

(206) Loss due to excess air, up to steam temperature, B. t. u.; (Item 205) (Item 187).

(207) $\frac{(\text{Item } 145)}{(\text{Item } 165)}$.

(208) 10,140 (Item 207).

(209) Loss due to CO formed, B. t. u.; (Item 208) (Item 161).

(210) $\frac{(\text{Item } 28)}{(\text{Item } 24)}$.

(211) Loss due to combustible in ash, B. t. u.; 14,540 (Item 210).

(212) (Item 20) — (Item 17).

(213) 0.28 (Item 212).

(214) Loss due to temperature of leaving ash and refuse, B. t. u.; 0.01 (Item 30) (Item 213).

(215) (Item 21) — (Item 18).

(216) Loss due to theoretical dry gas, above steam temperature, B. t. u.; (Item 215) (Item 188).

(217) 0.5 (Item 215).

(218) Loss due to moisture in coal, above steam temperature, B. t. u.; (Item 195) (Item 217).

(219) Loss due to $H_2$ in coal, above steam temperature, B. t. u.; (Item 197) (Item 217).

(220) Loss due to moisture in air, above steam temperature, B. t. u.; (Item 203) (Item 217).

(221) Loss due to excess air, above steam temperature, B. t. u.; (Item 215) (Item 205).

(222) (Item 21) − (Item 17).

(223) 0.24 (Item 222).

(224) Loss due to air leakage, total, B. t. u.; (Item 181) (Item 223).

(225) (Item 186) + (Item 216) + (Item 218) + (Item 219) + (Item 220) + (Item 221) + (Item 224).

(226) Radiation, conduction, and unaccounted-for loss, B. t. u.; (Item 126) − (Item 225).

(227) Heat absorbed by boiler, per cent; $\frac{100 \text{ (Item 186)}}{\text{(Item 62)}}$.

(228) Loss due to theoretical gas, up to steam temperature, per cent; $\frac{100 \text{ (Item 189)}}{\text{(Item 62)}}$.

(229) Loss due to moisture in coal, up to steam temperature, per cent; $\frac{100 \text{ (Item 196)}}{\text{(Item 62)}}$.

(230) Loss due to $H_2$ in coal, up to steam temperature, per cent; $\frac{100 \text{ (Item 198)}}{\text{(Item 62)}}$.

(231) Loss due to moisture in air, up to steam temperature, per cent; $\frac{100 \text{ (Item 204)}}{\text{(Item 62)}}$.

(232) Loss due to excess air, up to steam temperature, per cent; $\frac{100 \text{ (Item 206)}}{\text{(Item 62)}}$.

(233) Loss due to CO formed, per cent; $\frac{100 \text{ (Item 209)}}{\text{(Item 62)}}$.

(234) Loss due to combustible in ash, per cent; $\frac{100 \text{ (Item 211)}}{\text{(Item 62)}}$.

(235) Loss due to temperature of ash and refuse, per cent; $\frac{100 \text{ (Item 214)}}{\text{(Item 62)}}$.

(236) Loss due to theoretical gas, above steam temperature, per cent; $\frac{100 \text{ (Item 216)}}{\text{(Item 62)}}$.

(237) Loss due to moisture in coal, above steam temperature, per cent; $\frac{100 \text{ (Item 218)}}{\text{(Item 62)}}$.

(238) Loss due to $H_2$ in coal, above steam temperature, per cent; $\frac{100\ (\text{Item } 219)}{(\text{Item } 62)}$.

(239) Loss due to moisture in air, above steam temperature, per cent; $\frac{100\ (\text{Item } 220)}{(\text{Item } 62)}$.

(240) Loss due to excess air, above steam temperature, per cent; $\frac{100\ (\text{Item } 221)}{(\text{Item } 62)}$.

(241) Loss due to air leakage, total, per cent; $\frac{100\ (\text{Item } 224)}{(\text{Item } 62)}$.

(242) Radiation, conduction, and unaccounted-for loss, per cent; $\frac{100\ (\text{Item } 226)}{(\text{Item } 62)}$.

## PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS

ENGINEERING EXPERIMENT STATION

BULLETIN NO. 79 MAY, 1915

# THE COKING OF COAL AT LOW TEMPERATURES WITH SPECIAL REFERENCE TO THE PROPERTIES AND COMPOSITION OF THE PRODUCTS

BY S. W. PARR, PROFESSOR OF APPLIED CHEMISTRY, AND H. L. OLIN, INSTRUCTOR IN CHEMISTRY

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# THE COKING OF COAL AT LOW TEMPERATURES WITH SPECIAL REFERENCE TO THE PROPERTIES AND COMPOSITION OF THE PRODUCTS

## I. Introduction.

1. *Preliminary.*—This report covers a series of studies made during the period from 1911 to 1913 on the coking properties of Illinois coal. It is a continuation of the work described in Bulletin No. 60 of the University of Illinois Engineering Experiment Station.* Its distinctive feature has been the use of an apparatus which would yield the main products of coke, gas, and tar in quantities sufficient for a detailed study of these products, and, to a certain extent, quantities sufficient for a determination of their values by practical tests on a commercial scale.

2. *Resumé of Previous Work.*—In the experiments described in Bulletin No. 60 the apparatus used had a capacity of 6 to 8 pounds of coal at a charge. Notwithstanding this limited capacity, certain fundamental facts were developed as follows:

(a) The formation of coke depends upon the presence of certain constituents having a melting point which is lower than the temperature at which decomposition or carbonization takes place.

(b) Oxidation of these compounds may easily take place and the greatest coking effect is obtained where the opportunity for the minimum amount of oxidation has occurred. The condition prescribed, therefore, is that there shall be the least possible exposure to oxidation either before or during the process of carbonization.

(c) Coals containing an excessive quantity of the coking substance produce a light porous coke. The texture of the product may be modified by use of pressure and by close packing of the charge and especially by mixing with material which has already passed through the coking process. Such a mixture provides the physical conditions whereby the gases formed may readily pass out of the mass without carrying along the cementing substances.

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*The Coking of Coal at Low Temperatures, by S. W. Parr and H. L. Olin.

(d) By use of temperatures between 400° and 500°C. all of the resulting products are of a type distinctly different from those obtained by the usual high temperature procedure.

3. *Outline of Present Investigation.*—An apparatus was designed to utilize about 100 pounds of coal. Experience in the use of the apparatus indicated also the main principles which should be embodied in a commercial equipment. The coking process was studied, and the mixture for producing the best product determined. It was found that a smokeless fuel may be produced especially well adapted to domestic purposes, including its use in open grates. Its freedom from tar or condensable hydrocarbons makes it easily adapted to generating producer gas, thus affording a good substitute for anthracite coal in suction gas producer practice.

In the study of the composition and properties of the tar, this material was found to have a very low content of free carbon, a relatively high percentage of light boiling distillate, and an unusually high content of tar acids or phenols. The latter fact is of special interest to the wood preserving industry.

## II. Experimental Work.

4. *Description of Coking Oven.*—The apparatus used in the experiments is an elaboration of that employed in 1910, described on page 5 of Bulletin 60, and is capable of producing material in greater quantities than was possible with the older type.

The device was manufactured by Burr and Company of Champaign, Illinois, and is illustrated in Fig. 1. As shown in the detailed diagram Fig. 2, it consists of a boiler plate shell A, lined with asbestos to prevent excessive radiation of heat; within this, forming the coking chamber, is a shell B of the same material containing a cone of light sheet iron C, perforated with 3/16 in. holes, designed to confine the coal charge and to allow a free circulation of gases. To obtain the non-oxidizing atmosphere such as was used with the old apparatus, steam was admitted from the high pressure main at E, passed through the coil F where it was superheated by the hot currents ascending from the gas burner and then conducted into the coking chamber. The heat was supplied by a blast ring burner D, connected with the gas and air

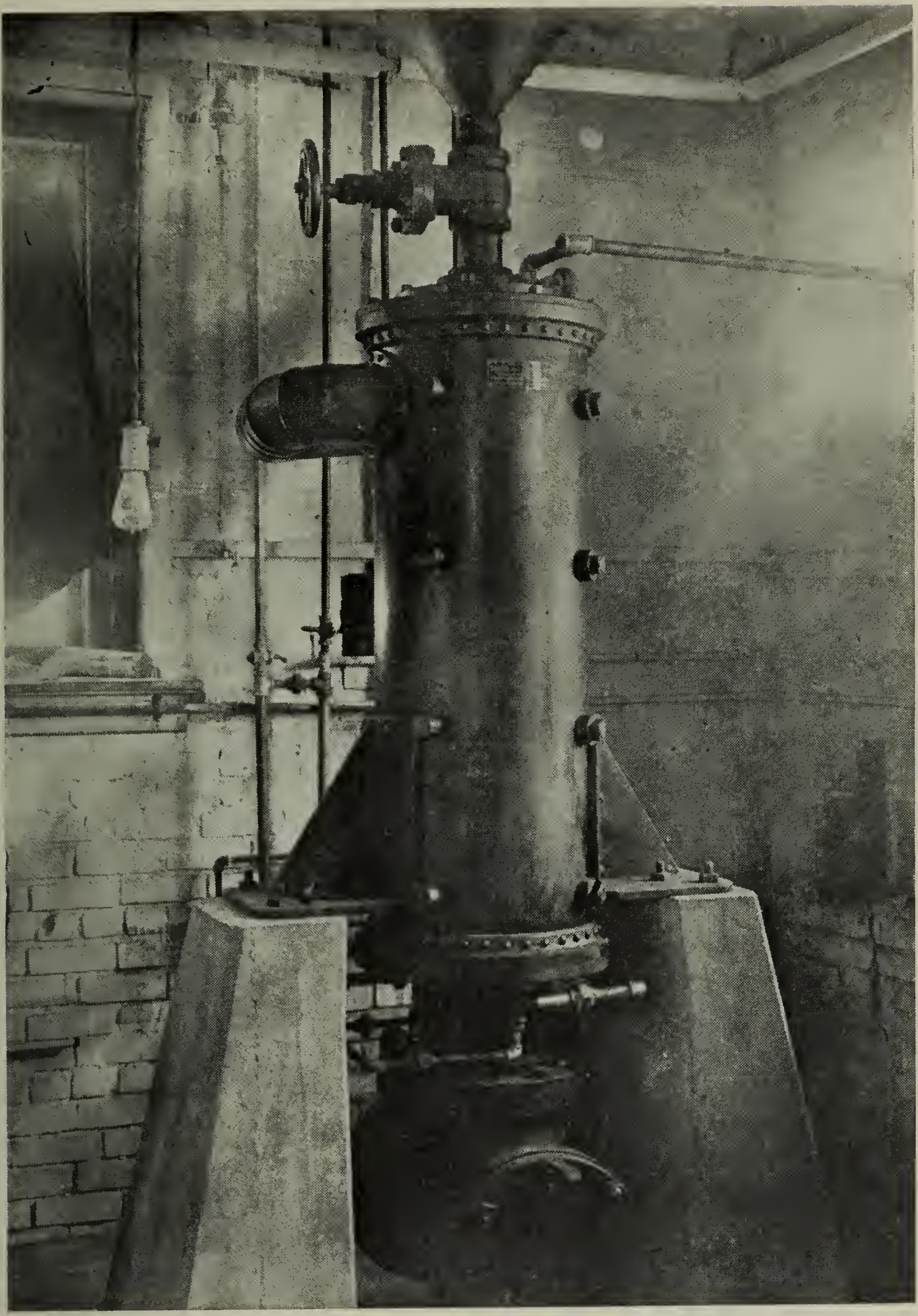

Fig. 1. Apparatus for Low Temperature Distillation of Coal

mains, and no difficulty was experienced in producing the desired temperatures. The charge of crushed coal was fed into the hopper G and admitted to the retort through a large gate valve. The coked residue was removed, after cooling the apparatus, through the bottom at H. Gases of combustion escaped through the opening J, which was connected with a flue, while the distillates were conducted through an outlet pipe to a condenser consisting of several four-foot lengths of inch pipe connected by return elbows. Cold water was allowed to drip over this gridiron-like contrivance. The tars were passed through a water sealed exit at the bottom, while the gases, fairly clean, were collected in a gasometer.

5. *Coking Tests.*—To study further the coking qualities of Illinois coals at temperatures ranging from 400°C. to 500°C. and to obtain a quantity of the coke residue sufficient in amount for practical tests in order to determine its commercial value, a series of runs was made using the apparatus described. Numerous coals from the different fields of the State were included in this set of experiments. In this report, however, products from representative types only are illustrated and described since the results of tests of different coals of a given field showed little variation. Of particular interest during the process of distillation was the behavior of the coals from the northern districts, especially those from Vermilion County. As the heating progressed, a black pitchy substance dripped from the joints of the containing ovens but it hardened immediately on cooling, forming a brittle mass much resembling asphalt. This was undoubtedly the cementing principle which is instrumental in forming coke. According to Lewes it consists of substances derived from the resins of the original coal sources, which melt at about 300°C. and decompose at slightly higher temperatures yielding on the one hand liquid products which distil out as tar vapors and hydrocarbon gases, and on the other, a pitch residuum, which at 500°C. forms a mass of coke. His general theory that these resinic substances are readily oxidizable and in their oxidized condition have much to do with coke formation is borne out in these experiments by the fact that no weathered coal of any type produced the gummy exudation mentioned, although there was no apparent diminution in the amount of gases given off. Compared with the bituminous coals of the Eastern States, those of Illinois are exceedingly rich in this resinous binding material. Even those from the southern districts of the State while not as ''fat'' as those from Sangamon and Vermilion Counties, nevertheless, much surpass in this respect coals like the Pocahontas, and those from Ohio and Pennsylvania. Strangely enough the abun-

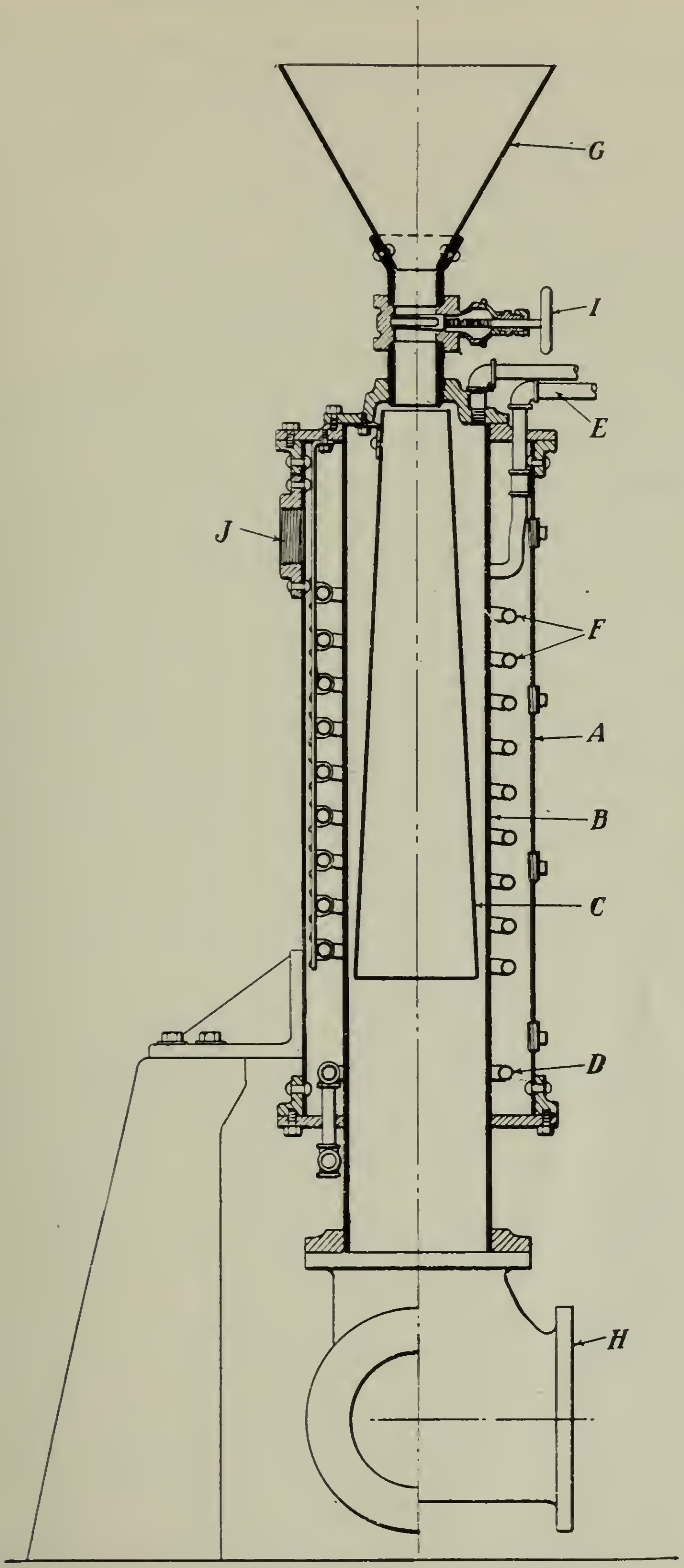

FIG. 2. CROSS SECTION OF APPARATUS FOR LOW TEMPERATURE DISTILLATION OF COAL.

dance of the coking material which the western types possess is the cause of their inferiority in the matter of making dense, hard coke; for with the decomposition of the resinic bodies and the deposition of cementing carbon there occurs at the same time an evolution of large quantities of gases which inflate the pasty mass and make the resulting coke more or less light and spongy. Indeed, certain Vermilion County coals, after being heated under conditions which allow free expansion, present the appearance of hardened froth, because of the excessive development of cell structure in the coke.

As has been shown in the previous work the porosity of the product may be appreciably reduced by subjecting the contents of the oven during the heating period to a considerable pressure. Another means to the same end which was recognized in the first experiments and which has been applied in the latter series is the addition of inert coke dust to the raw coal as fed into the retort. This material acting in the capacity of a "blotter" reduces the plasticity of the softened mass and allows the gases to escape freely without producing a blowing effect.

The diluting medium in the case of the specimens shown here was a mixture of various semi-cokes which had accumulated in the course of the work. The material was crushed and ground to a fineness of 40 or 50 mesh and thoroughly mixed with the rest of the charge in ball mills.

An analysis of a composite sample of this semi-coke breeze gave the following results:

TABLE 1.

COMPOSITION OF INERT COKE MIXTURE.

| | Actual | Dry |
|---|---|---|
| Moisture | 1.85 | |
| Ash | 11.90 | 12.15 |
| Volatile Matter | 19.85 | 20.22 |
| Sulphur | 2.62 | 2.67 |
| Fixed Carbon | 66.40 | 67.65 |
| Calorific Value (B. t. u.) | 11243 | 11454 |

The effect of the addition of this foreign substance is exceedingly interesting. Certain types of coal, as for instance those from Vermilion County, produce hard firm coke when mixed with as much as 100 per cent of their own weight of the coke dust; those from the southern districts with higher fixed carbon do not need so great a dilution. Not only is the texture made firmer by this treatment but the density is increased. Attention will be called to the differences in the properties of the cokes in discussing the individual samples.

Coke was made first from Vermilion County screenings such as were furnished the power plant of the University of Illinois. In order to remove the dust which may have been more or less weathered, the coal was passed over a quarter inch screen.

The resulting coke, shown in Fig. 3, is extremely light and porous, having a specific gravity of only .650. Its texture is firm, notwithstanding the low temperature at which it was formed and it has proved its ability to stand considerable handling without excessive dusting. Moreover, its porosity probably accounts in a large measure for the success obtained in using it in the gas producer test described later, since the carbon is easily accessible to the blasts of hot air and steam.

The coke shown in Fig. 4 was made from Saline County coal and was obtained from a run in which the time and temperature conditions were the same as in that of B-10. It illustrates the superiority of a coal from the southern field for coking purposes. Compared with the Vermilion County sample its texture is hard and firm and it has a density more than 6 per cent higher than the latter. It may be seen by referring to the photographs that the cellular structure of the Saline County sample is relatively close and solid, and bears little resemblance to the other.

One of the best products obtained in this series of experiments was from a coal from Williamson County. This sample, B-16, Fig. 5, shows a fine even grain and has a density of .750, more than 15 per cent higher than B-10. It has remarkable strength, both tensile and compressive, and stands rough handling without appreciable dusting.

Other coals from Williamson County have shown the good coking qualities which characterize the southern types. The particular sample illustrated in Fig. 6 shows an uncarbonized center but the outer portion is consistently dense and hard and has considerable strength.

In the course of the work tests were made of many coals from other localities as well as from different beds in the same locality. Included in the list are samples from Perry, Franklin, Jefferson, and Jackson Counties, but the cokes made from them are so similar in quality to those already described that they are not separately discussed.

TABLE 2.

COMPOSITION OF COKE FROM RAW COALS.

| Sample | Moisture | Ash | Volatile Matter | Fixed Carbon | S | Heat Value B. t. u. |
|---|---|---|---|---|---|---|
| B-10 | 1.85 | 13.8 | 13.20 | 71.10 | 3.0 | 11891 |
| B-12 | 1.20 | 11.9 | 10.85 | 75.95 | 1.6 | 12520 |
| B-16 | 3.25 | 12.4 | 11.50 | 72.85 | 1.5 | 12600 |
| B-19 | 2.25 | 11.75 | 12.30 | 73.70 | 1.55 | 12415 |

The effect of the addition of a fine inert diluting medium to different types of raw coal is seen in samples B-18 and B-21, Fig. 7 and Fig. 8. The former is the product from the heating of a mixture of equal parts of Williamson County coal and the coke dust described on page 8. It is loose in texture and crumbles easily showing a deficiency in bonding material. On the other hand, B-21, made from equal parts of Vermilion County coal and the coke in question, is quite as hard and firm as the raw coal product and considerably denser. B-22, Fig. 9, is from Vermilion County coal diluted with one-half its weight of inert material and while it resembles B-21 it has a lower specific gravity.

Williamson County coal although unable to cement itself firmly when mixed with an equal weight of non-coking material nevertheless works successfully in a mixture of two parts coal to one part inert material. B-25, Fig. 10, is both coherent and dense proving that the dilution limit has at least not been exceeded in applying this mixing ratio.

Saline County coal likewise produces better coke with the addition of foreign material. B-13, Fig. 11, raw Saline County coal two parts and coke dust one part, is superior in every way to the coke from the raw coal alone.

The composition of this series of samples is given in the following table.

TABLE 3.

COMPOSITION OF COKES FROM MIXTURES OF COAL AND COKE DUST.

| Sample | Moisture | Ash | Volatile Matter | Fixed Carbon | S | Heat Value B. t u. |
|---|---|---|---|---|---|---|
| B-21 | 1.35 | 14.95 | 13.7 | 70.0 | 3.15 | 11750 |
| B-18 | 1.30 | 12.9 | 13.90 | 71.90 | 2.50 | 12295 |
| B-22 | 1.60 | 14.50 | 12.70 | 71.20 | 2.9 | 11825 |
| B-13 | 1.40 | 14.2 | 15.22 | 69.18 | 2.8 | 12150 |
| B-25 | 1.45 | 15.4 | 10.50 | 72.65 | 3.0 | 11920 |

The rise in the density of the coke with the addition of the inert substance is shown by the results of the following specific gravity measurements.

### TABLE 4.

### DENSITIES OF COKES.

| Description | | Density |
|---|---|---|
| Saline County Coal | B-12 (unmixed) | .687 |
| | B-24 (raw ¾, dust ¼) | .775 |
| | B-13 (raw ⅔, dust ⅓) | .868 |
| Williamson County Coal | B-16 (unmixed) | .750 |
| | B-25 (raw ⅔, dust ⅓) | .969 |
| Vermilion County Coal | B-10 (unmixed) | .650 |
| | B-22 (raw ⅔, dust ⅓) | .848 |
| | B-21 (raw ½, dust ½) | 852 |

For purposes of comparison it is interesting to note that B-29, Fig. 12, a 16-hour Solvay coke from an Illinois coal has a density of .830; B-30, Fig. 13, a 48-hour compressed charge, .986; while a representative sample of bee-hive Connellsville has a density of about 1.12.

While the methods described in the foregoing may not be applied directly on a commercial scale to operations for making metallurgical coke, still it is hoped that the results obtained may at least help to establish the characteristics of the coals of the State with regard to their adaptability to this use and to suggest methods and principles on which to work in solving the problem of making cokes of good quality from such material. Unquestionably they possess coking properties to a marked degree but need to be treated differently from the eastern coals with their higher percentages of fixed carbon.

6. *Gas Producer Tests.*—Early in the course of this work, the study of the products resulting from the distillation of bituminous coals at low temperatures suggested the possibility of using them as fuels for the gas producer whereby it was hoped to obtain gases sufficiently free from tar to be suitable for use in the internal combustion engine. Previous experiments had shown that the moderate temperatures of the preliminary heating period were effective in expelling the major part of the tar-producing substances and that the residue, distilled a second time at high temperatures, yielded gases remarkably free from heavy condensation products.

In order to give the matter a practical test, arrangements were made with Professor C. R. Richards of the Department of Mechanical Engineering of the University of Illinois for the use of the necessary apparatus of his department and with Mr. A. P. Kratz of the Engineering Experiment Station for his services in conducting the trial run. By means of the device already described a quantity of coke was prepared and it was fired in the usual manner.

The fuel used in this test was the semi-coke product obtained in subjecting Vermilion County screenings from the University power plant to low temperature distillation (400°C–550°C). This test material, four or five hundred pounds in weight, consisted of pieces varying in size from three-fourths of an inch to two inches in diameter, but the charge as fired contained some dust. It was light and porous and lay on the fuel bed without showing much tendency to pack. The following tables show its composition.

### TABLE 5.

### PROXIMATE ANALYSIS OF PRODUCER TEST FUEL.

| | |
|---|---|
| Moisture | 2.28 |
| Ash | 15.82 |
| Sulphur | 3.13 |
| Volatile Matter | 18.00 |
| Fixed Carbon | 63.90 |
| Calorific Value (B. t. u.) | 11601 |

### TABLE 6.

### ULTIMATE ANALYSIS OF PRODUCER TEST FUEL.

| | |
|---|---|
| Carbon | 69.86 |
| Hydrogen | 2.76 |
| Oxygen | 5.15 |
| Nitrogen (estimated) | 1.00 |
| Sulphur | 3.13 |
| Ash | 15.82 |
| Water | 2.28 |

The producer used was a Number 3 Otto, designed to operate on anthracite pea coal, with a wet scrubber attached. The latter was merely a shell filled with coke through which the gas, admitted at the bottom, passed counter to a stream of water flowing from the top. Using anthracite coal the normal capacity of the producer was 4500 cu. ft. of gas per hour.

In place of the gas engine which under normal working conditions draws the gas from the scrubber, a Schütte-Köerting steam ejector was used. This delivered the mixture of steam and gas to a condenser and thence to a Westinghouse meter of the wet type which had been calibrated just before the test was made.

Inasmuch as the supply of fuel was limited the usual method of starting and stopping was varied somewhat. Before beginning the experimental part of the work, the producer was fired with anthracite and thoroughly warmed up. This fire was then drawn and a new one made with a weighed amount of wood and the fuel to be tested, and the operation continued until gas of a good quality was given off, when the test was declared begun. During this preliminary period, as in the rest of the run, the measured gas was sampled by means of the continuous sampler which is a part of the installation.

At the close of the test, the fuel bed was burned as low as was deemed practicable, the ash pit cleaned, and the ash weighed and sampled. The residue on the grates was then drawn out, quenched, weighed, and sampled.

After the test was started all the coal put into the producer was weighed. The total fuel charge used in the test then included the coal equivalent of the wood and coal used to start, plus the coal fired during the test, minus the coal equivalent of the carbon in the gas given off before the formal start, minus the coal equivalent of the residue in the fuel bed at the close. Putting this into a formula:

$$W = W_1 + W_2 - \frac{(W_3 + W_5)\ 14560 + W_4 \times 62000}{H}$$

$W$ = Total weight of equivalent coal fired during test.
$W_1$ = Total weight of equivalent coal in producer at start.
$W_2$ = Total weight of coal fired during test.
$W_3$ = Total weight of carbon appearing in gas before starting.
$W_4$ = Total weight of hydrogen appearing in gas before starting.
$W_5$ = Total weight of carbon within fuel bed at close of test.
$H$ = Heating value of the coal, B. t. u.

Since the first sample contained practically no hydrogen the last term in the above formula becomes zero.

Gas samples were drawn from a point beyond the ejector where the gas was under pressure, and were taken over mercury. Coal and ash samples were taken in the usual manner.

Water fed into the vaporizer was weighed in a tank on scales. The weight of the overflow from the vaporizer was obtained in the same manner, and the weight of the water going into the fuel bed from this source was the difference between these two.

All temperatures were taken with mercury thermometers with the exception of that of the gas at the producer outlet which was obtained by means of a Hoskins pyrometer.

An attempt was made to get tar samples just as the gas left the producer but it was found that the amount of tar formed was not appreciable and the sampling was discontinued.

The Junker calorimeter was used to obtain the heating value of the gas, about one sample an hour being taken.

The form for recording the results of this test is abridged from the one given in Bulletin* No. 50 of the University of Illinois Engineering Experiment Station, where details of methods of computation may also be found.

The firing sheet for this test showed that the producer ran successfully and gave little trouble. Because of the small diameter of the fuel bed, resulting in considerable friction, the lightness of the material used, and its slight tendency to coke and arch, the fuel above the bed proper did not feed down as rapidly as it was burned. Hence it was necessary about once an hour to poke it down with a slice bar and in this respect it probably required a little more attention than a charge of anthracite.

It may be noted, however, that it was necessary to poke and clean the grates from the bottom but once during the six hours of the trial. With so small a producer this indicates a remarkable freedom from ash and clinker trouble. On cleaning the fire small pieces of clinker were found in the ash, but there was none at all sticking to the sides, where it usually collects. The high grate efficiency, 98.2 per cent, also shows that the fires required little attention, since poking and cleaning invariably force much unburned carbon out into the ash.

The fact that the fire required so little attention has an added significance in connection with a producer having a small fuel bed, such as the one used in this test. Usually, the inrush of air, when the doors are opened for cleaning, dilutes the gases sufficiently to make them too lean to burn, but with the fuel in question less trouble was experienced in this respect than is usually the case with anthracite, for at no time did the pilot flame go out and there was no great variation in the calorimeter readings.

A study of the temperature of the gas leaving the producer throws considerable light on the condition of the fuel bed. If the bed becomes clinkered, or if holes form in the fire, air goes through without coming into intimate contact with the carbon. This chimney effect causes the gas to burn at the surface of the bed and the temperature to rise. Sometimes it is very difficult tc keep the temperature down and cleaning and poking have little effect. In the case under discussion, however, the temperature never rose above 420°C except during the last twenty minutes of the test, when the fire was so low that it was essentially a furnace and not a producer.

---

*Tests of a Suction Gas Producer. Garland and Kratz. 1912.

No trouble was experienced with tar. There was little evidence of its presence in the fuel bed, and an attempt to get a sample of it from the gas leaving the producer showed that it was present in negligible quantities only. On cleaning the ejector at the close of the run a small amount of tar was noticed but most of the deposit was of the nature of scale. It is to be remembered that before going to the ejector the gas had first passed through the scrubber.

The efficiencies (hot gas 74.4 per cent and cold gas 64.9 per cent) compare very favorably with those obtained in using anthracite coal of the same ash content.

Data and results are given in the following table.

### TABLE 7.

### RESULTS OF GAS PRODUCER TRIAL.

Made by A. P. Kratz and H. L. Olin.
Mechanical Engineering Laboratory, University of Illinois.
Make of Producer, Otto.
Kind of Fuel—Semi-coked Vermilion County Coal.
Type of Producer—Suction for Anthracite.

| | |
|---|---|
| Duration of Trial—hours | 6.33 |

DIMENSIONS AND PROPORTIONS

| | |
|---|---|
| Great area, sq. ft. | 1.666 |
| Mean diameter of fuel bed, ft. | 1.545 |
| Depth of fuel bed, ft. | 2.21 |
| Area of fuel bed, sq. ft. | 1.877 |
| Approximate width of air spaces in grate, inches | 0.5 |
| Area of air space, sq. ft. | 0.722 |
| Ratio of air space to whole grate | 0.433 |
| Area of discharge pipe, sq. ft. | 0.165 |
| Water heating surface in vaporizer, sq. ft. | 2.7 |

AVERAGE PRESSURES

| | |
|---|---|
| Average corrected barometer reading, inches mercury | 29.52 |
| Draft in ash pit, inches water | 0.45 |
| Suction at producer outlet, inches water | 0.88 |
| Pressure at meters, inches water | 3.69 |
| Absolute pressure at meters, inches mercury | 29.69 |
| Vapor pressure at meters, inches mercury | 1.9 |
| Dry gas pressure at meters, inches water | 27.79 |

## TABLE 7. (CONTINUED)

### AVERAGE TEMPERATURES (Degrees Centigrade)

| | |
|---|---|
| At barometer, | 28.3 |
| Of fire room, | 33.3 |
| Of feed water entering vaporizer, | 23.4 |
| Overflow from vaporizer, | 84.8 |
| Of water entering scrubber, | 14.8 |
| Of water leaving scrubber, | 45.7 |
| Of gases leaving producer | 378.0 |
| Of gases leaving first scrubber, | 34.5 |
| Of gases entering meters, | 38.2 |
| Of gases entering meters, (absolute) | 311.2 |

### FUEL

| | |
|---|---|
| Weight of wood used in starting, lb. | 9 |
| Volume of gas given off before start of test, cu. ft. | 2238 |
| Volume of standard gas given off before start of test, cu. ft. | 2010 |
| Weight of equivalent coal as fired, lb. | 381 |
| Percentage of moisture in coal | 2.28 |
| Total weight of dry coal fired, lb. | 372 |
| Total ash and refuse, lb. | 37 |
| Total weight of residual, lb. | 49 |
| Total weight of combustible, lb. | 280.6 |

### COMPOSITION OF PRODUCER FUEL

| | |
|---|---|
| Fixed carbon, per cent | 70.77 |
| Volatile matter, per cent | 18.00 |
| Moisture, per cent | 2.28 |
| Ash, per cent | 15.82 |
| Sulphur, separately determined, per cent | 3.13 |

### ULTIMATE COMPOSITION OF PRODUCER FUEL

| | |
|---|---|
| Carbon, per cent | 71.50 |
| Hydrogen, per cent | 2.82 |
| Oxygen, per cent | 5.26 |
| Nitrogen, per cent | 1.02 |
| Sulphur, per cent | 3.20 |
| Ash, per cent | 16.20 |
| Moisture in sample coal as received, per cent | 2.28 |

### COMPOSITION OF DRY ASH AND REFUSE

| | |
|---|---|
| Carbon, per cent | 14.48 |
| Earthy matter, per cent | 85.52 |

### COMPOSITION OF RESIDUAL FUEL ON GRATE

| | |
|---|---|
| Carbon | 36.46 |
| Volatile matter | 7.10 |
| Moisture | 1.18 |
| Ash | 55.30 |

### FUEL CONSUMPTION PER HOUR

| | |
|---|---|
| Dry coal per hour, lb. | 58.8 |
| Dry coal per sq. ft. of grate area per hr., lb. | 35.3 |
| Dry coal per sq. ft. of fuel bed per hr., lb. | 31.3 |

### Table 7. (Continued)

#### CALORIFIC VALUE OF FUEL

| | |
|---|---|
| Calorific value by oxygen calorimeter, per lb. dry coal, B. t. u. | 11601 |

#### WATER

| | |
|---|---|
| Total weight fed to vaporizer, lb. | 440.0 |
| Total weight of overflow, lb. | 256.0 |
| Water actually evaporated in vaporizer, lb. | 184.0 |
| Weight of water fed to producer, lb. | |
| (a) From vaporizer | 184.0 |
| (b) In air (estimated) | 6.0 |
| (c) In coal | 8.7 |
| Total | 199 |
| Total weight of water decomposed as used in calculations, lb. | 91. |
| Total weight of moisture in gas leaving producer, lb. | 108. |
| Weight of water decomposed per lb. of gas generated, lb. | 0.0588 |
| Weight of water decomposed per lb. of dry coal fired, lb. | 2.42 |
| Total weight of scrubber water, lb. | 5880 |

#### GAS

| | |
|---|---|
| Calorific value per cu. ft. of standard gas from calorimeter, B. t. u. (high value) | 121 |
| Specific weight of standard gas, lb. per cu. ft. | 0.0670 |
| Specific heat of dry gas leaving producer (estimated) | 0.260 |
| Total volume of gas from meters, cu. ft. | 26927 |
| Total volume of standard gas at 16.5°C. and 30 in., cu. ft | 23150 |
| Volume of standard gas per hr. cu. ft. | 3660 |
| Volume of standard gas per lb. of dry coal cu. ft | 62.3 |
| Total weight of standard gas, lb. | 1552. |
| Weight of standard gas per lb. of dry coal, lb. | 4.17 |

#### COMPOSITION OF GAS BY VOLUME

| | |
|---|---|
| Carbon dioxide, $CO_2$ | 4.15 |
| Carbon monoxide, CO | 21.10 |
| Oxygen, $O_2$ | 0.30 |
| Hydrogen, $H_2$ (estimated from calorific value) | 11.80 |
| Marsh gas, $CH_4$ | 1.50 |
| Nitrogen, $N_2$ by difference | 61.00 |

#### COMPOSITION OF GAS BY WEIGHT

| | |
|---|---|
| Carbon dioxide, $CO_2$ | 7.19 |
| Carbon monoxide, CO | 23.17 |
| Oxygen, $O_2$ | 0.38 |
| Hydrogen, $H_2$ | 0.93 |
| Marsh gas, $CH_4$ | 0.96 |
| Nitrogen, $N_2$ by difference | 67.37 |

#### EFFICIENCY

| | |
|---|---|
| Grate efficiency, per cent | 98.2 |
| Hot gas efficiency, based on high heating value, per cent | 74.4 |
| Cold gas efficiency, based on low heating value, per cent | 64.9 |

7. *Adaptation of the Coke to Domestic Appliances.*—A limited amount of the coke was available for testing its adaptability to house-heating purposes. The ordinary household appliance is especially well adapted for distilling off the hydrocarbons of coal and sending them into the air unburned or partly burned and accompanied by large volumes of smoke.

In the coke product here described the heavy hydrocarbons have been removed. The coke itself, therefore, is clean both in handling and in burning. However, there remains approximately 20 per cent of volatile matter which enters into the process of combustion after the coke has attained a temperature at or beyond the point to which it had been subjected in the coking process. These conditions result in a free burning substance yet one whose combustible constituents may not produce smoke in burning.

An open grate was selected as furnishing the best opportunity for observing the behavior of the material. The results are summed up as follows: the coke ignites readily, it retains its shape through the process of combustion, a bed of glowing coals quickly results, the very indifferent provision for draft as found in an open grate is sufficient for keeping the combustion lively, there is no smoke produced, and fire is retained over a long period of time, because the interior of the larger pieces holds the fire and continues the combustion until all of the carbonaceous matter is consumed. While the temperature commonly attained by a grate fire would not furnish positive evidence as to the formation of clinker, the indications so far as they went, were altogether favorable. A small amount of coke was tested by burning in a hot air furnace. Not enough material was at hand for a complete test, but so far as observations could be made, it was as favorable as the test in the open grate.

8. *The Sources and Uses of Coal Tar.*—The installation of the first American by-product recovery coke plant in 1893 marks the beginning of a notable rise in the production and use of coal tar in this country.

Previous to that time the isolated illuminating-gas plant was the only source of supply and, indeed, in the smaller places at least, the tar was looked upon as a troublesome waste product rather than as a thing of value.

For making metallurgical coke, types of beehive ovens of varying degrees of efficiency were employed and these, of course, allowed the volatile constituents of the coal, both gaseous and liquid, to be wasted.

In 1912, not twenty years after the introduction of the new type of oven, 165,000,000 gallons of tar were produced in the United States, and two thirds of this quantity, according to Perry,* came from the by-product coke plant. With the steady increase in by-product oven construction the tar refining industry will assume a proportionate importance.

Of the various fractions obtained in distilling the crude tar the most important are;—benzol and its homologues, used in the color industry and as paint and fat solvents; the carbolic oils, much in demand for disinfectants of various kinds; and the creosote and anthracene oils, of great importance in the preserving of wood. The pitch residue is used in roofing, paving, and road-building, but in these fields it meets strong competition with the mineral bitumens and consequently does not find so ready a market as the other constituents named. But pitch makes up the greatest part of the crude tar aggregate, being in most cases considerably more than half. The bulk of the raw material therefore, is in the form of high temperature condensation products and free carbon of relatively small value, while the supply of the lighter fractions is insufficient to meet the demand. This is especially true of those compounds of the tar which have bactericidal properties. With the rapid depletion of the forests and the cutting off of lumber supplies, wood preserving treatment has become imperative in many industries—particularly that of the railroads—and the lack of available material for this process is causing much concern. American supplies are inadequate for home consumption and nearly 45,000,000 gallons of creosote oils were imported from Europe in 1911. Any modification of the coking process, therefore, that will increase the yield of the light tars by preventing their condensation to compounds of high molecular weight will effect a considerable economy. Considerations of this kind have lent interest to the study of the tars obtained in the course of these experiments, in which temperatures were kept much below those of the gas retort and the commercial coke oven.

9. *Methods for Testing and Analyzing Tars.*—Of the methods proposed and outlined for testing tar, perhaps the best are those published by S. R. Church in a paper on "Methods for Testing Coal Tar and Refined Tars, Oils and Pitches Derived Therefrom."† These tests, as

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*Eighth Int. Cong. of App. Chem., 10, 233.

†The Gas Age, 32, 103. Jour. of Ind. and Eng. Chem., 3, 227.

he says in the introduction, were not put forward as methods for the scientific examination of, or research into, the products of coal tar, but rather as an attempt, in cooperation with other chemists of the company with which he was connected, to revise and standardize the everyday tests applied to the raw materials and products of the American tar distiller.

In this article he takes up the determination of water, free carbon, fixed carbon, and ash, and the measuring of the specific gravity and viscosity of the raw tar, distilled tars, and pitch. He then outlines the chemical investigation of the light oils, carbolic oils, benzols and creosotes, including the estimation of napthalene. In a later paper[1] he gives some supplementary methods.

Prevost Hubbard's "Methods for the Examination of Bituminous Road Materials"[2] approaches the matter solely from the standpoint of the road-builder, and takes up the physical tests necessary to determine the fitness of the material for this special purpose. In its field it is, unquestionably, authoritative.

Among the publications devoted particularly to methods for testing wood preservatives may be mentioned those of the American Railway Engineering Association,[3] the National Electric Light Association,[4] and the Forest Service of the Department of Agriculture.[5]

In addition to the foregoing, the following works take up to a greater or less extent the subject of coal tar analysis:

Sadtler, Industrial Organic Chemistry, 3rd. ed. J. B. Lippincott Co.

Rogers and Aubert, Industrial Chemistry, 1913, Van Nostrand Co.

Lunge, Coal Tar and Ammonia, 4th ed. 1909. Van Nostrand Co.

Allen, Commercial Organic Analysis. Vol. II, part 2, 3rd. ed. 1909. Blakiston's Son and Co., Philadelphia.

Mulliken, Identification of Pure Organic Compounds. 1904. Wiley and Sons.

10. *Properties of Tar Products From Low Temperature Coal Distillation.*—The tar used in this investigation was the product obtained in a separate series of runs made with Vermilion County coal.

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[1] Jour. of Ind. and Eng. Chem., 5, 195.

[2] U. S. Dept. of Agriculture, Office of Public Roads, Bulletin 38, 1911.

[3] Bulletin 65, American Railway Eng. Assoc.

[4] Report of Committee on Preservative Treatment of Poles and Crossarms, 1911.

[5] Dean and Bateman, Circular 112, "Analysis and Grading of Creosotes."

To prevent oxidation and evaporation the tar was protected as soon as formed with a water seal, and kept in air-tight cans until used for analysis.

The tar is fluid at temperatures considerably below room conditions. In appearance it is black to rich brown with varying thickness of layer, and it possesses an exceedingly disagreeable odor even when judged by coal tar standards.

The specific gravity measured with a pyknometer at 20° was 1.069. Its low density makes its separation from water by gravity somewhat difficult and there is a tendency, moreover, for high fractions to float on the surface of the water. Its viscosity is low and in this respect it resembles a mineral oil rather than a high temperature coal tar.

The percentage of free carbon, or more accurately, of substances insoluble in toluene and benzene, was found by treating a comparatively large sample of the dry tar, (about 36 grams) with toluene, and heating the mixture on the steam bath. The solution was decanted through two S and S filter cones (33 mm. and 26 mm.) one within the other. After three treatments the whole mass was transferred to the cones and extracted with benzene in a Soxhlet apparatus until the filtrate was colorless. Check results showed a free carbon content of 1.35 per cent.

Fixed or combined carbon may be determined by a method based on the report of the Committee on Coal Analysis of the American Chemical Society.* The cracking of an oil or tar is, however, so closely dependent upon time, temperature, and pressure conditions that a simple laboratory test of the kind outlined by the Society would have little significance.

11. *Distillation of Tar.*—In order to dry the tar a preliminary distillation 125° was made and the light oil runnings after being separated from the water were returned to the retort. The apparatus used was an ordinary Jena distilling flask attached to a Liebig condenser.

In the earlier attempts to make this separation much trouble was caused by the tendency of the water in the crude material to produce a succession of ''bumpings.'' These explosions were often so sudden and violent as to throw a considerable part of the charge out of the retort. This annoying feature was prevented, however, by running a slow stream of air through a tube extending nearly to the bottom of the vessel, and later, in order to prevent oxidation which seemed to take place at the higher temperatures under these conditions, carbon dioxide, which had been previously washed and dried, was substituted. This method was used in all subsequent distillations of the crude, wet tar and served the double purpose of providing an inert atmosphere in which to carry on the heating of the substances making up the material, many of which show remarkable chemical activity, and of making the boiling proceed smoothly and quietly. It is to be noted, however, that the current of gas sent through the apparatus carries over with it a considerable quantity of material whose boiling point is at a temperature above that maintained in the retort. In redistilling the dry tar to fractionate the sample this precautionary measure was unnecessary, and therefore was not taken.

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*Jour. Am. Chem. Soc., 21, 1116.

The escape of the first light oil runnings was prevented by surrounding the receiving vessel with a freezing mixture. To condense the heavier part of the second fraction, only the inner tube of the condenser was used so that heat might be applied to facilitate the flow of the viscous fluid.

To insure greater accuracy a subsequent redistillation of the light oil fraction was made with the use of a Lebel-Henninger tube, with the thermometer so adjusted that the top of the bulb was level with the side neck. The Liebig condenser was used as before. With this apparatus it was possible to secure fractions with sharply defined boiling points, an important prerequisite to identifying individual compounds.

TABLE 8.

RESULTS OF PRELIMINARY DISTILLATION OF TAR.

| Fraction | Temperature Range | Percentage |
|---|---|---|
| Light oil | below 210° C | 17.2 |
| Heavy oil | 210°–325° | 52.7 |
| Pitch | above 325° | 30.1 |

12. *Examination of Light Oil.*—The low boiling fraction purified as indicated above is a clear amber colored liquid with a disagreeable odor. It is very susceptible to the action of light and air and finally turns dark red even when sealed in an atmosphere of carbon dioxide.

The separation and identification of all the individual constituents of a substance like coal tar is a great task. Lunge, in his work "Coal Tar and Ammonia" has described more than two hundred distinct compounds which occur in tars of different qualities and his list is probably not complete. We are concerned in this work merely with a

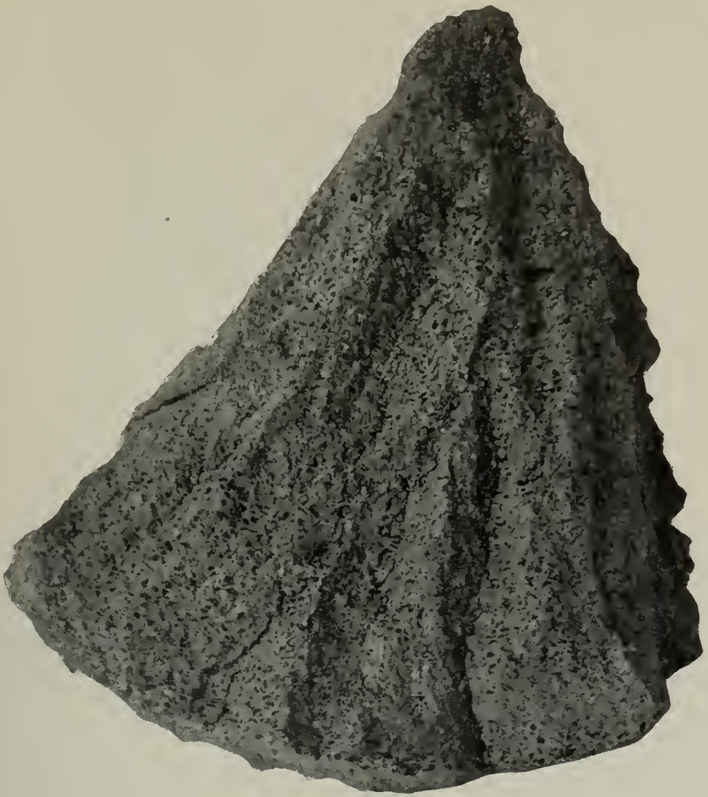

FIG. 3. B-10. VERMILION COUNTY COKE

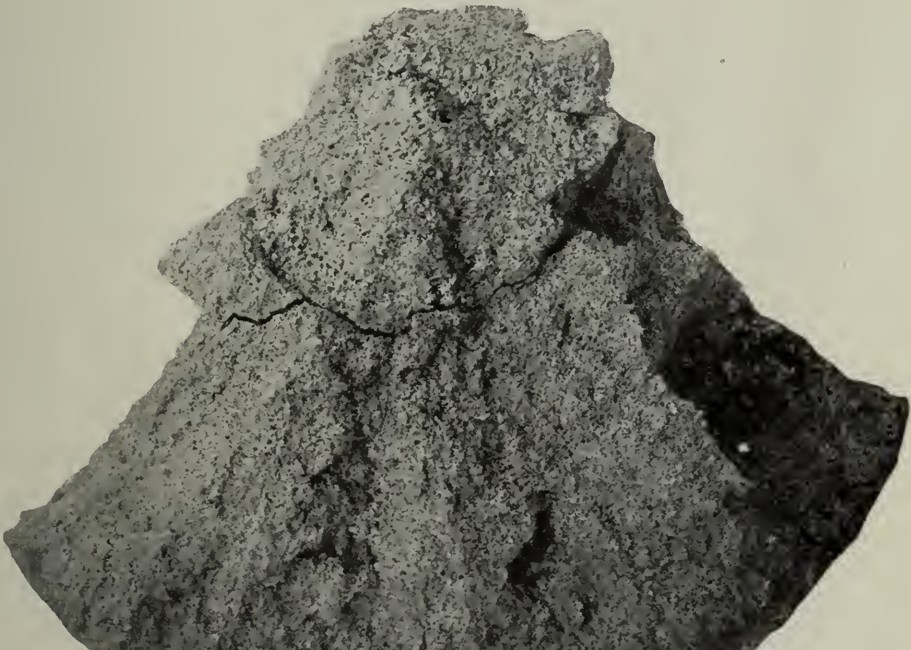

FIG. 4. B-12. SALINE COUNTY COKE

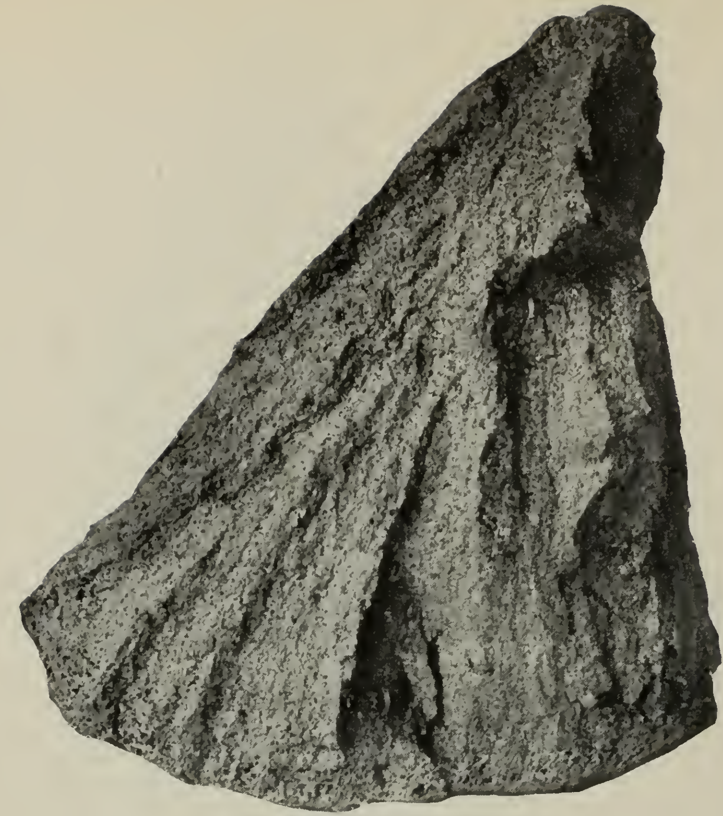

FIG. 5. B-16. WILLIAMSON COUNTY COKE

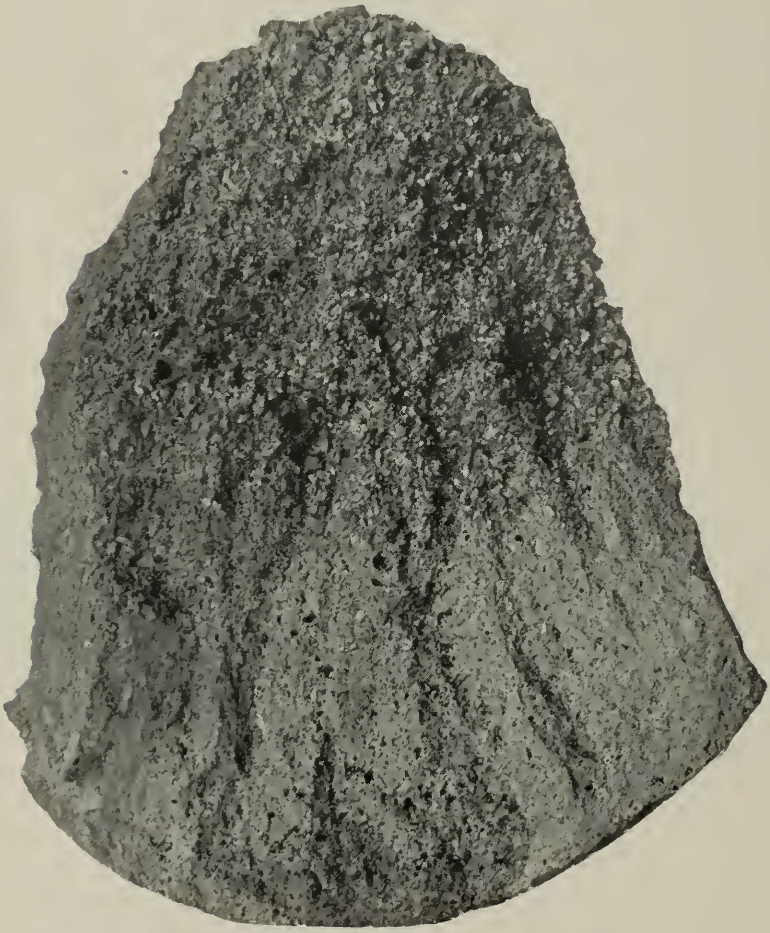

FIG. 6. B-19. WILLIAMSON COUNTY WASHED NUT COKE

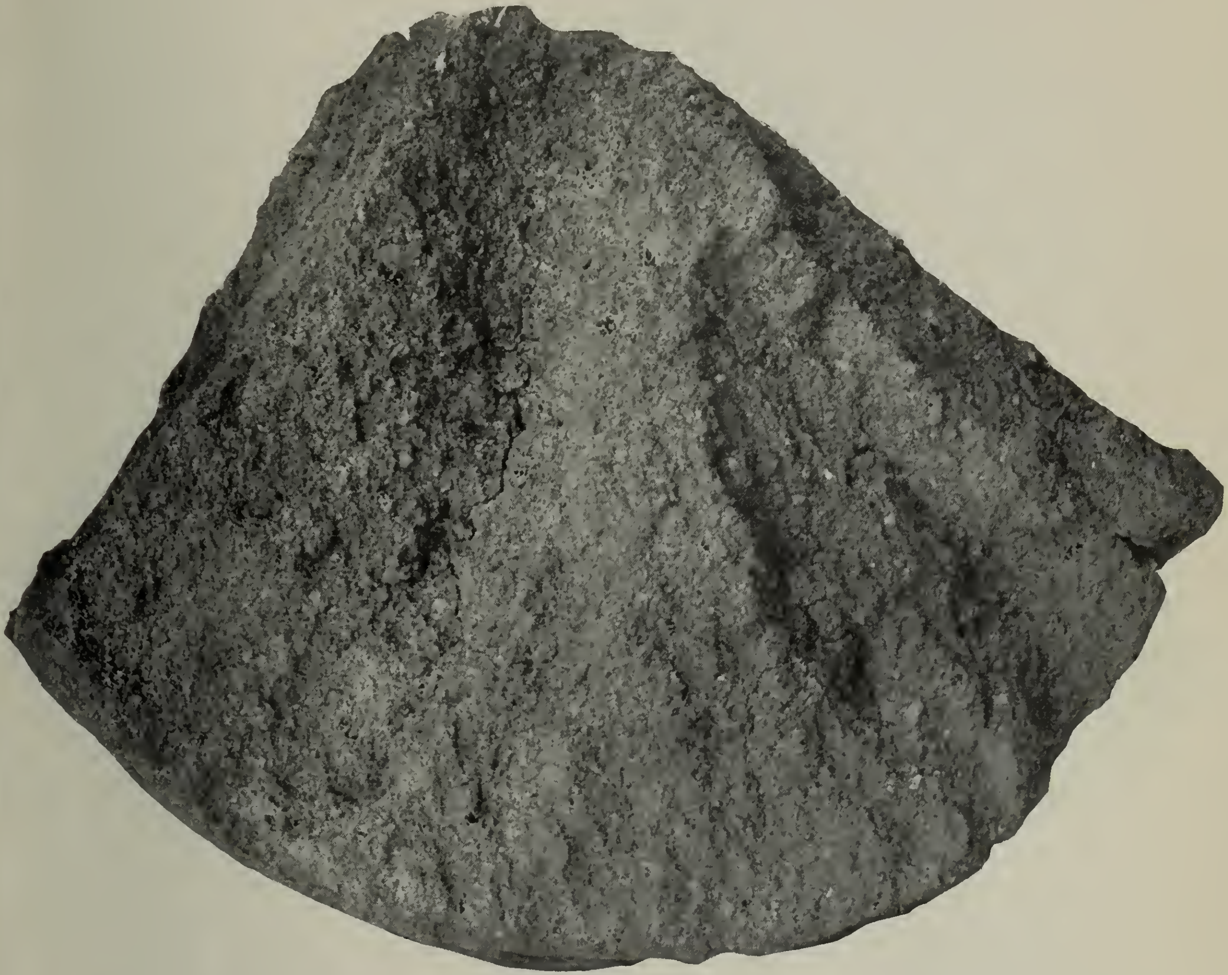

Fig. 7. B-18. Coke from Williamson County Coal Mixed with Coke Dust

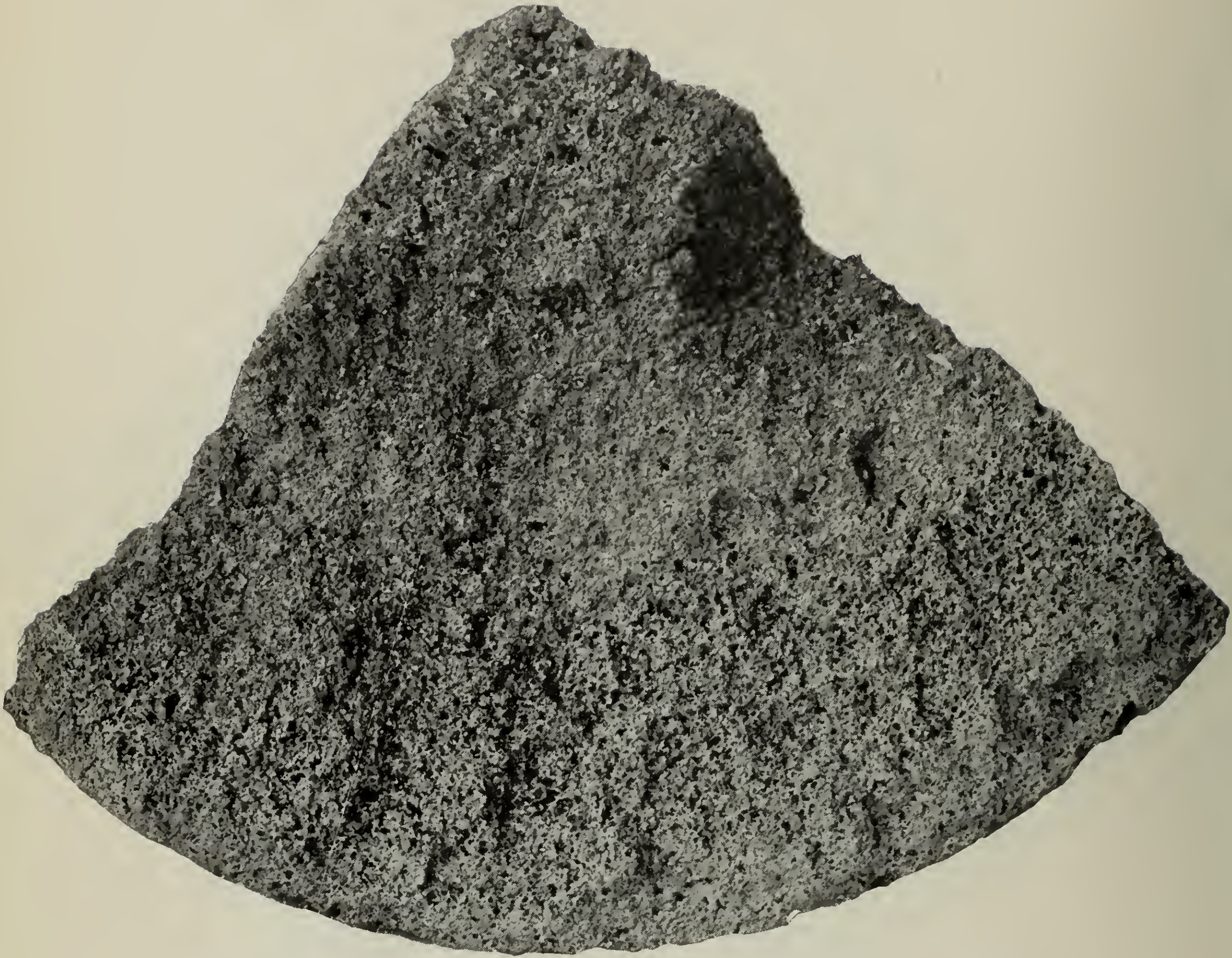

Fig. 8. B-21. Coke From Equal Parts of Vermilion County Coal and Coke Dust

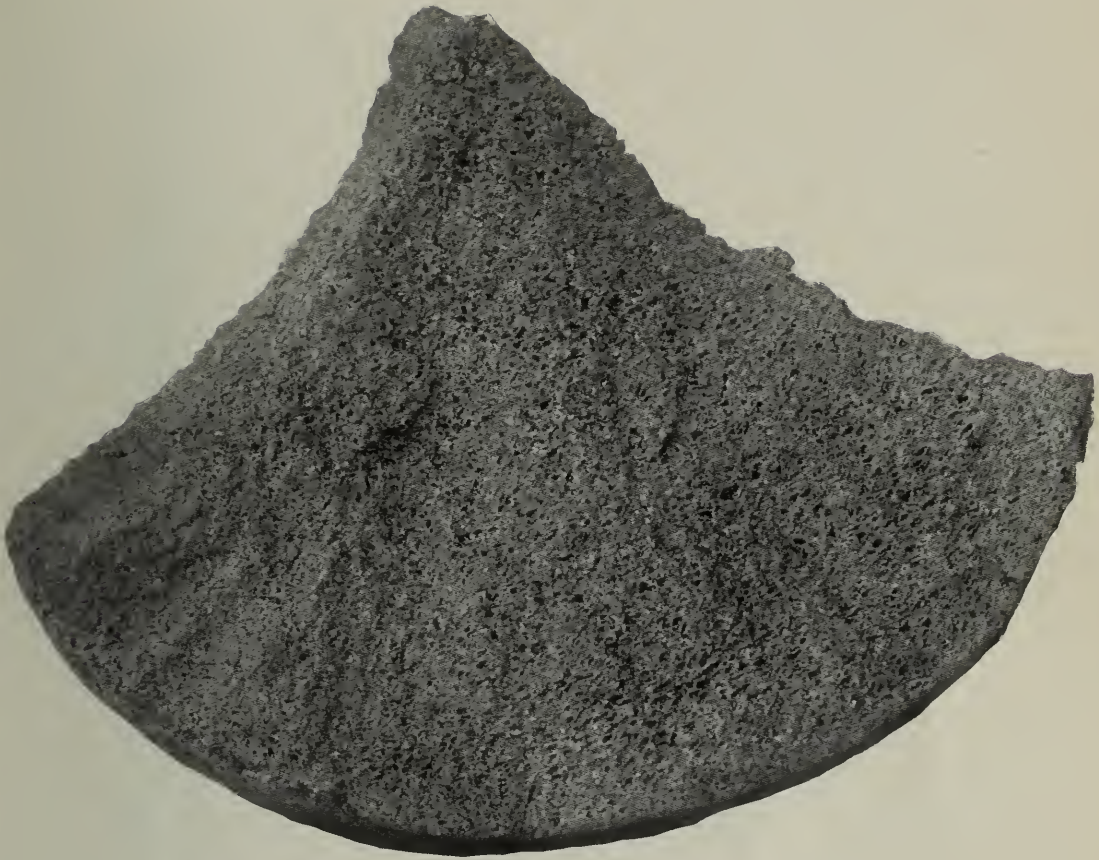

FIG. 9. B-22. COKE FROM VERMILION COUNTY COAL AND COKE DUST

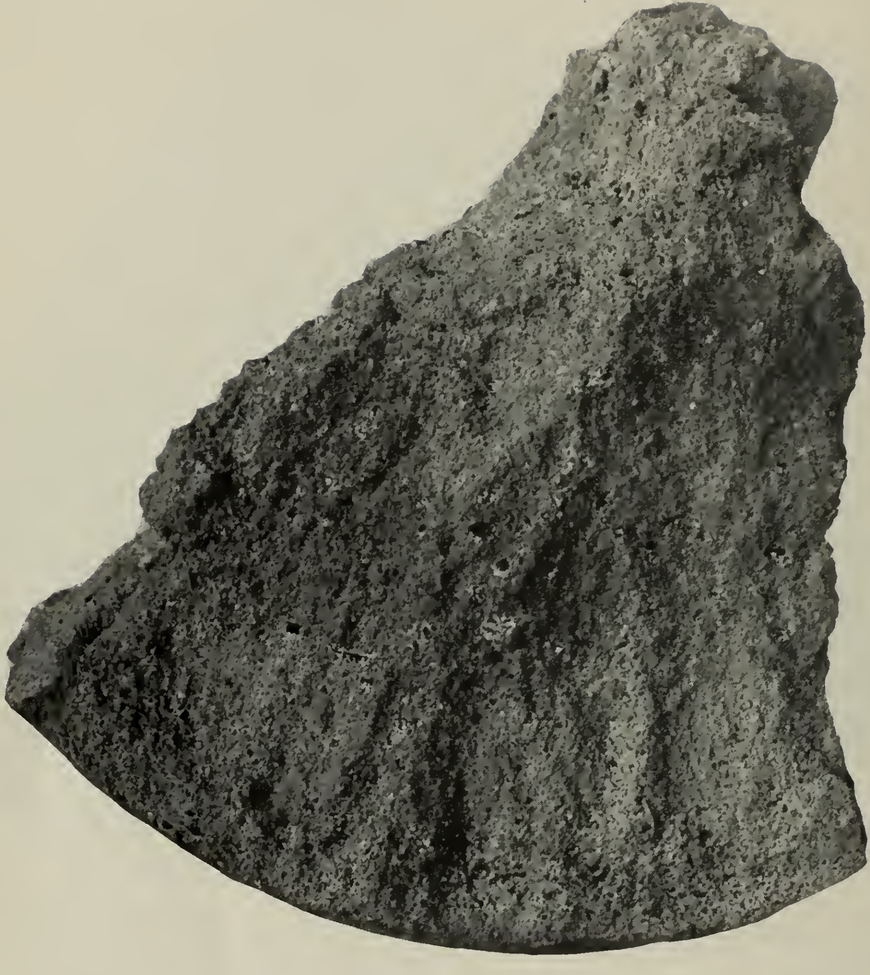

FIG. 10. B-25. COKE FROM WILLIAMSON COUNTY COAL AND COKE DUST

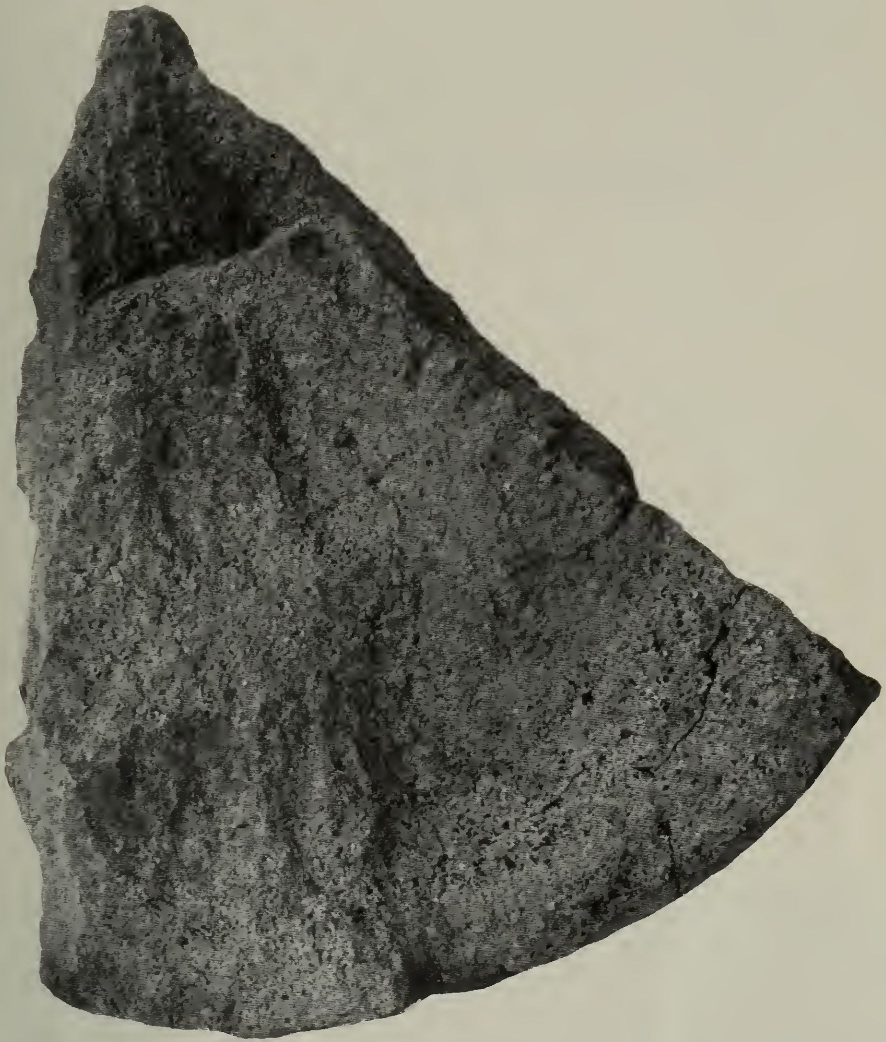

Fig. 11. B-13. Coke From Saline County Coal and Coke Dust

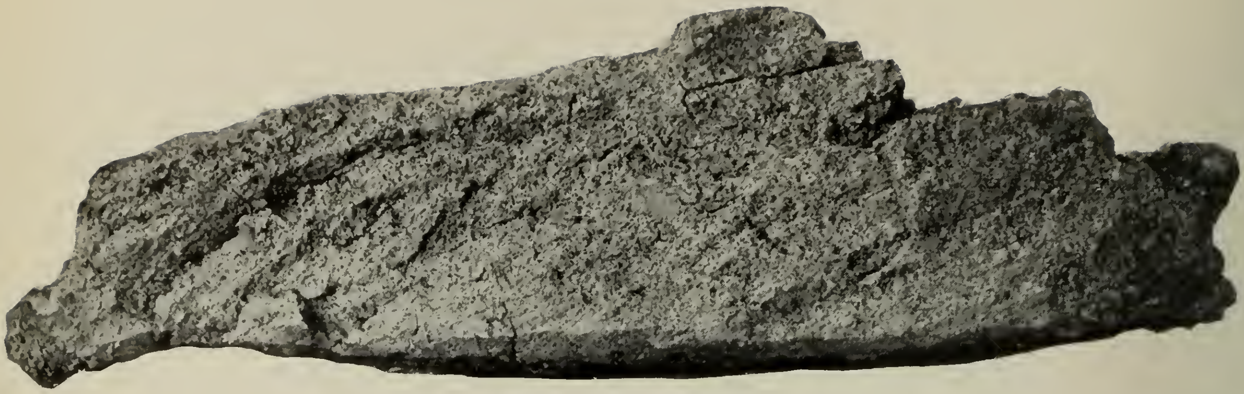

Fig. 12. B-29. 16-hour Solvay Coke

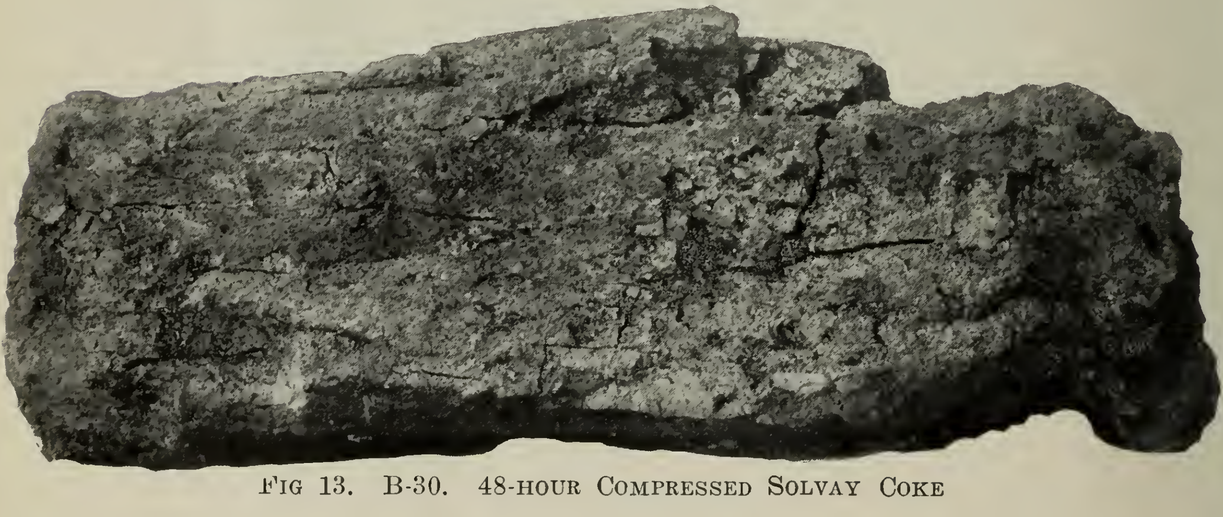

Fig 13. B-30. 48-hour Compressed Solvay Coke

few of the most important substances which are of interest because of their practical value and not with those which are of interest from the scientific standpoint only.

The phenols and other acid substances of the light oil were separated from the neutral and basic constituents by shaking the fraction in a separatory funnel with half its volume of 10 per cent caustic soda solution. After this treatment had been repeated until no further reduction in volume of the residue took place, this caustic solution was drawn off and acidified with sulphuric acid and the whole extracted with ether. The ether extract, when evaporated down, yielded crude phenol and its homologues.

The amines were removed from the oily residue after the phenol extraction by shaking it with dilute sulphuric acid, of sp. gr. 1.15, with warming. The solution was separated as before, neutralized with caustic soda, and the bases extracted with ether.

The method for separating paraffins and benzenes is based upon the familiar principle that the hydrocarbons of the paraffin series are practically inert toward sulphuric acid, while those of the aromatic series react with comparative ease, forming sulphonic acids, in which sulpho groups replace hydrogen of the benzene ring. These acids are very soluble in water and are therefore easily separated from the insoluble oily paraffin residues.

The process used was that adopted by the Forest Products Laboratory.* Ten cubic centimeters of the fraction to be tested were measured into a Babcock milk bottle. To this was added 40 cc. of 37 N sulphuric acid (made by adding the calculated amount of fuming sulphuric acid to the ordinary acid, sp. gr. 1.84) 10 cc. at a time. The mixture after being kept at 100° C for an hour was cooled and then whirled for five minutes in a Babcock separator. The volume of the unsulphonated or paraffin residue was read off directly. Equally good results are obtained, however, by pouring the mixture into a large volume of water and separating by means of the funnel.

As a step toward identifying those constituents which do not react with acids and bases, a sample of the oil from which the phenols and pyridines had been removed but which had not been sulphonated, was subjected to a second redistillation, with the measurement of the volumes going over between comparatively narrow limits of temperature.

A summary of the results obtained from the various tests and analyses follows:

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*Bateman, E. Modification of the Sulphonation Test for Cresote. Forest Service, Circular 191, U. S. Dept. of Agriculture.

## TABLE 9.

### LIGHT OIL FRACTION (TO 210°C).

Specific gravity .966

| | Percentage on Basis of Crude Tar | Percentage on Basis of Light Oil Fraction |
|---|---|---|
| Light oil fraction | 17.2 | 100.0 |
| Phenols (and other acid substances | 5.7 | 33.0 |
| Amines (and other bases) | 0.9 | 5.3 |
| Paraffins | 3.12 | 18.1 |

The results of the refractionation of the light oil from which the acids and bases had been removed are given in the following table:

## TABLE 10.

### DISTRIBUTION OF NEUTRAL SUBSTANCES IN LIGHT OIL.

| Fraction | Percentage on Basis of Light Oil Fraction | Percentage on Basis of Crude Tar |
|---|---|---|
| 20°–75° low boiling bodies | 5.26 | .25 |
| 75°–95° crude benzol (90%) | 2.10 | .38 |
| 95°–125° crude toluol | 7.36 | 1.33 |
| 125°–170° crude solvent naptha | 26.30 | 4.77 |
| 170°–200° crude heavy naptha | 14.73 | 2.67 |
| 200°–210° crude heavy naptha | 7.30 | 1.32 |

The classification as made in Table 10 is based on the outline for the examination of light oil as given by F. E. Dodge in his article on "Coal Tar and Its Distillation Products."* It is understood that this is a generalization only, and that it shows composition by classes of compounds rather than by individuals. However, since the different homologues of a class within rather close limits of the boiling point usually differ little in character, a fair idea of the composition of the mixture may be gained without further separation.

Fraction No. 1, besides some benzene, contains also pentanes and hexanes of the paraffin series, ranging in boiling point from 31° C to 64° C. No. 2 includes some of the heptanes boiling in the nineties; No. 3, according to tables complied from data† obtained by distilling known mixtures, consists of benzene and toluene, the latter predominating, while fraction No. 4 includes the xylenes boiling from 138° C to 143° C, with perhaps some mesitylene, b.p. 164°C. Likewise the part reported as phenols contains besides, one or all of the three cresols, although the close range of boiling points (190°, 201°, and 202°) made a quantitative fractionation of the crude extract impracticable.

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*Industrial Chemistry. Rogers and Aubert, 1912. p. 492.

†G. E. Davis. Industrial Chemistry Rogers and Aubert. p. 499.

Finally, the rapid darkening of both the neutral and the active fractions on even short time exposures to air points to the presence of various unsaturated substances, difficult to isolate and probably of no great practical importance.

13. *Examination of Heavy Oil Fraction, 210°C to 325°C.*—The heavy oil obtained from the first distillation to which was added the residue boiling above 210°, from the redistillation of the light oil, is a thick viscous liquid of a rich brown color. After standing for a time a non-crystalline sediment is formed.

Tar acids were determined by the methods used in the examination of the light oils except that after the addition of the caustic soda it was necessary to warm the mixture in order to facilitate the separation, while the paraffin and aromatic content of substances was found by sulphonation as before. Likewise a sample from which the acid constituents had been removed was refractionated and the volumes of distillate given off between close temperature limits were noted.

The napthalene content was determined by cooling, with a freezing mixture, the heavy oil sample from which the acids had been extracted, in order to crystallize out any of this substance which might be present. The test gave zero results; the conclusion is that no napthalene was present. This result is confirmed by observations made when the original tar was distilled from the coal, for at no time did the tar condenser become clogged as it would if napthalene were going over even in small quantities, nor did the pungent odor of its vapors ever become noticeable.

The method for the quantitative assay of anthracene as given by Allen* was used for this material. The sample, dissolved in boiling glacial acetic acid, was treated with chromic acid slowly dropped through a reflux condenser to oxidize any anthracene present to anthraquinone. The results showed that the amount present was negligible.

TABLE 11.

COMPOSITION OF HEAVY OIL (210°C–325°C).

Specific gravity 1.032

| Fraction | Percentage on Basis of Crude Tar | Percentage on Basis of Heavy Oil Fraction |
|---|---|---|
| Heavy oil fraction | 52.7 | 100.0 |
| Tar acids | 22.2 | 42.13 |
| Paraffins | 6.2 | 32.66 |
| Napthalene | .0 | .0 |
| Anthracene | .0 | .0 |

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*Commercial Organic Analysis. Vol. II, part 2, p. 229.

Distillation of the acid free samples gave the following results.

### TABLE 12.

### DISTRIBUTION OF NEUTRAL SUBSTANCES IN HEAVY OIL.

| Fraction | Percentage on Basis of Crude Tar | Percentage on Basis of Heavy Oil Fraction |
|---|---|---|
| 210° to 250° | 2.87 | 10.3 |
| 250° to 270° | 13.55 | 48.5 |
| 270° to pitch | 11.53 | 41.2 |

In attempting to identify the various substances of the heavy oil fraction the worker finds the problem even more difficult than in the case of the light oil because the higher boiling points and molecular weights of the compounds make them more difficult to separate. A partial purification of the acid bearing extract of this part of the tar showed by the boiling points that a high percentage of creosols was present. The higher boiling members were not identified. It is possible that they are polyhydric phenols, such as Lewes mentions as being present in the tar from coalite and which he says form resinous masses difficult to investigate.

Fraction No. 2 is the neutral part of the cut in tar distilling known as creosote oils. Emmet and Reingruber* say of this class of substances that after removing basic, oxygenated, and crystallizable bodies, there remain several isomeric dimethylnapthalenes, the separation of which has proved unmanageable, and which constitute the major portion of the fraction.

14. *Pitch.*—The residuum from the original distillation of the crude tar is a hard, black substance, rather brittle, breaking with a bright fracture. As indicated by the table of its properties it is a hard pitch, as contrasted with soft pitch having a melting point of about 75° C.

### TABLE 13.

### PITCH FRACTION (ABOVE 325° C).

| | |
|---|---|
| Per cent on basis of crude tar | 30.1 |
| Melting point | 110° C |
| Specific gravity | 1.27 |
| Free carbon | 12% |

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*Annalen 211, 365.

15. *Oxygen Absorbing Power of Tar.*—In the publication of the results of the preliminary studies of low-temperature tar,* attention was called to the fact that both the light and the heavy fractions were readily oxidizable and the question was raised whether these oils might not serve as drying bodies or paint vehicles by forming coatings or films on oxidation. In an attempt to approximate a quantitative measurement of this capacity for absorbing oxygen, a series of iodine absorption determinations was made, with rather variable results.

Further investigations of the kind have proved what was conceded as a probability at the time, that other reactions than those of simple saturation take place, chief among which is substitution with the formation of hydriodic acid. While fairly consistent results have been obtained in the supplementary work under strictly standard conditions, slight changes in the concentration of the iodine solution, in the time of digestion and particularly in the temperatures of the reacting substances so varied the values found that the method is deemed unreliable. Bromine is even more uncertain. Among the substances present in the tar from which hydrogen is easily displaced by the halogens are the phenols, the reaction taking place almost immediately even in the cold.

The only reliable criterion of its value as a paint drier is perhaps the actual test of its behavior when exposed to air. The light oil fraction forms a thin film when spread on a glass plate inclined to 45 degrees. Using good linseed oil as a standard for reference, it is found that the tar oil films are decidedly thinner, i.e., the oil flows more before thickening, and that the time of maximum drying is from two to three times that observed in the case of the linseed oil.

## III. Appendix of Supplementary Results.

At the request of S. R. Church, head of the Research Department of the Barrett Manufacturing Company of New York, a small sample of the crude tar remaining after the conclusion of the work described, was sent to the New York Laboratories for examination. Mr. Church, in a communication commenting on it briefly, says, "This tar in its characteristics, resembles somewhat the Scotch blast furnace tars although it is higher in tar acid content than the Scotch tars, and apparently not quite so high in paraffin-like bodies. The most interesting feature to us is its exceptionally high content of oxygenized compounds of phenoloid character."

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*Parr and Olin. Bulletin No. 60, University of Illinois Eng. Exp. Sta. p. 13.

"Should tar of this nature become a commercial product, it would undoubtedly have a certain value to the tar distillers although it would, of course, have to be handled in an entirely different manner from the ordinary coal tars."

The outline of his results is given in the following table.

Table 14.

Tests of Tar Derived from the Distillation of Coal from Vermilion Co., Illinois. (S. R. Church.)

| | |
|---|---|
| Water, per cent | 21.7 |
| Tests on Dry Tar: | |
| Specific gravity at 15.5°C | 1.072 |
| Free carbon, per cent | 1.3 |
| Light Oil (to 210°) per cent by vol. | 15.2 |
| Heavy Oil (to pitch) per cent by vol. | 40.2 |
| Light Oil: | |
| Tar acids, per cent | 30.0 |
| Sulphonation residue, per cent | 15.0 |
| Heavy Oil: | |
| Standard retort distillation: | |
| Total per cent to 170°C | 0.5 |
| 200° | 2.0 |
| 210° | 6.6 |
| 235° | 33.6 |
| 270° | 61.0 |
| 315° | 65.8 |
| 355° | 94.9 |
| Tar acids in total distillation | 50.0 |
| Sulphonation residues on fractions | 4 to 5 per cent |
| Pitch | |
| Melting point, | 76°C |
| Free carbon, per cent | 9.0 |

Of equal interest is the supplementary report furnished by Mr. E. B. Fulks of the American Creosoting Company of Louisville, Kentucky, who visited the laboratory early in the year 1913 and obtained a sample of a tar similar to the one that has been discussed. He writes: "This tar is quite different from ordinary coal tar, in that it is thinner, has a lower specific gravity and much smaller percentage of pitch. The principal difference however, is in the high percentage of tar acids and in the presence of considerable quantities of paraffinoid bodies. These differences probably would make it necessary to work this tar somewhat differently from the method employed for ordinary tars but undoubtedly it would have considerable commercial value."

"The quantity at our disposal was so small that we were unable to separate an amount large enough to test the preservative qualities of that proportion which would be used for this purpose. Apparently, however, this tar would produce from 30 to 40 per cent of oil of fair preservative value. The high percentage of tar acids would make it a valuable source of material for the manufacture of antiseptic solutions, sheep dip, etc. The pitch contains a small proportion of free carbon and probably could be used for roofing purposes and paving filler."

The report of his analysis is given in the following table:

TABLE 15.

OIL FROM LOW TEMPERATURE COKING EXPERIMENTS
UNIVERSITY OF ILLINOIS.

Description: Thick, dark brown oil, having a very disagreeable odor resembling Pintsch gas; liquid at ordinary temperature.

| | | | |
|---|---|---|---|
| Specific gravity at 38° C. | | | 1.041 |
| Water | | | Trace |
| Tar acids by volume | | | 27.0% |
| Fractions | | | |
| 210 ° | 10.6% | Liquid | |
| 235° | 8.9% | " | |
| 270° | 12.5% | " | |
| 315° | 13.1% | " | |
| 355° | 14.1% | " | |
| Res. | 40.4 | | |
| Hard, black, brittle, bright fracture. | | | |

Paraffin Oils:

| | Fractions | Per cent of Fraction | Per cent of Straight Oil |
|---|---|---|---|
| 210° C | 9.7 | 25.0 | 2.4 |
| 235° | 9.0 | 10.0 | 0.9 |
| 270° | 12.3 | 10.0 | 1.2 |
| 315° | 13.7 | 10.0 | 1.4 |
| 355° | 15.0 | 15.0 | 2.3 |
| | 59.8 | | 8.2 |
| Total per cent paraffin in fractions below 355° C. | | | 13.7% |
| Total per cent paraffin in fractions below 355° C. based on whole oil | | | 8.2% |

## IV. SUMMARY.

1. Coke of good density and hardness may be made by mixtures of semi-coke and raw coal if both are finely divided and evenly mixed. A variation is noticeable in the quantity of such non-coking material which may be incorporated with different coals. For example, fresh coal from Vermilion County will carry such an addition of 100 per cent of its weight to advantage. Coals from Saline and Williamson Counties give coke of the highest density when mixed in the proportion of 50 per cent of their weight with semi-coke.

2. The coke resulting from the low temperature process has from 18 to 22 per cent of volatile matter remaining, but since it has been heated above 400° there should be none of the tar constituents remaining. The most convincing test on this point as also the best method of arriving at a conclusion as to its adaptability for such work was to try out the material in a suction gas producer. The results indicated that

no clogging effect whatever results, thus showing the absence of tar bodies. The physical operation of the producer as well as the grade of the gas produced was fully equal if not superior to the performance of the outfit when anthracite was used.

3. The semi-coke has such an amount of volatile matter remaining, together with the right degree of coherence as to make it especially well adapted to household use. It is clean to handle, free from dust, and burns without smoke or the formation of soot. Especially to be noted in this connection is its ability to retain a fire without undue attention as to drafts, etc.

4. The average specific gravity of the tar is 1.069. It is rich in low boiling distillate passing over at 210°. This product averages 18 per cent of the total. The pitch residue amounts to approximately 30 per cent and is remarkably free from precipitated carbon.

5. The adaptability of the tar for wood preservation processes seems to be indicated by the high percentage of tar acids. These constituents make up from 28 to 30 per cent of the crude material. The larger part, about 22 per cent is found in the second distillate (210°–325°), only about 7 per cent coming over below 210°.

6. Approximately 10 per cent of the crude tar is found to be low boiling distillate free from the tar acids and suitable for use in internal combustion engines.

7. Naphthalene is absent. The free carbon in the crude tar is less than 2 per cent and the residual product after the light distillate and heavy oils are removed would be classed as hard pitch.

8. A principal feature results from this study of these various substances, namely, that all three of the general divisions of coke, tar, and gas have specific properties of an especially valuable sort which would indicate that the process of coking at low temperatures could be established successfully on a commercial basis.

## V. Review of Literature.

In the effort to solve the smoke problem which arises wherever bituminous coal is extensively used, numerous attempts have been made to modify the raw fuel in various ways in order to obtain a product which should be more or less free from smoke producing constituents. Among the first to make the attempt was Col. Scott-Moncrieff,* who proposed to subject coal to the coking process in the ordinary gas re-

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*Jour. of Gas Lighting, **101**, 823.

torts at the usual temperature until one-half of the volume of gas usually obtained was driven off. The charge was then drawn and quenched.

The scheme resulted in failure for very obvious reasons. On account of the porosity of coke it is a very poor heat conductor and so the interior of the charge received an insufficient amount of heat to drive off the tar and smoke producing substances while on the other hand the exterior was heated to such a degree that it became essentially hard gas-coke which ignited with difficulty and burned slowly. A mass consisting of a soft uncarbonized interior and a hard shell was the result, a type of fuel which possessed no good qualities and which gained no popularity. At the same time the manufacturer lost one-half of the gas and much of the ammonia that would be produced in ordinary practice.

*In 1907 a radical change in the method of heating was made by W. Parker and the product obtained was extensively advertised under the trade name of "Coalite." This was the coke made by the partial distillation of slack coal in vertical oblong cast iron boxes or stills about ten or twelve feet high and ten by forty-eight inches in cross section. Carbonization was carried on at temperatures near 450° C, yielding a coke having about 10 per cent of volatile matter and 80 per cent of fixed carbon.

The following table shows the composition of the gas obtained in the distillation.

TABLE 16.

COMPOSITION OF GAS OBTAINED IN COALITE MANUFACTURE.

| | | | |
|---|---|---|---|
| $H_2S$ | 1.0 | $H_2$ | 14.30 |
| $CO_2$ | 3.45 | $CH_4$ | 61.00 |
| $O_2$ | .72 | $N_2$ | 9.28 |
| Illum. | 4.19 | CO | 6.06 |

Coalite has not proved to be the commercial success that its promoters hoped to make it, but experimental work is still being carried on with that end in view.

The Premier Tarless Fuels Company† of Battersea, England, have lately installed a plant for the production of smokeless fuel by low temperature distillation. The distinctive features are the use of annular retorts which allow the thickness of the charge to be reduced to 2½ inches and the coking time to be shortened proportionately, and

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*Gas World, June 8, 1907. p. 715.

†Jour. of Gas Light & Water Supply. 122, 514.

the maintainance of a reduced pressure of 27 inches in the retorts. Temperatures of about 500° C are used and the charge is drawn at the end of four or five hours.

A sample yield from a Lancashire coal is given below.

TABLE 17.

YIELD OF PRODUCTS FROM VACUUM PROCESS.

| | |
|---|---|
| Tarless fuel | 78.0 per cent |
| Tar (per ton of coal) | 20.19 gal. |
| Ammonium sulphate (per ton) | 45 lb. |
| Approximate Composition of Fuel | |
| Fixed carbon | 92.86 |
| Volatile matter | 3.86 |
| Ash | 3.28 |

This fuel is said to work well in the gas producer. Another sample with 8.26 per cent of volatile matter gives good results when used for domestic purposes, burning with a free flame, giving out intense heat and little smoke.

In considering the subject of tars and gases and their relationship to the matter of economy in coal carbonization no attempt will be made to review the great mass of literature relating to them, but merely to call attention to results which show causes for variation in the quality and yield, chief of which are range of distillation temperatures and differences in the sizes and shapes of the retorts.

The results obtained by Lewis P. Wright,* published twenty-five years ago, show the effect of increase in heat intensity in a striking manner. He does not state definitely the temperatures at which the different tars were produced but the following table compiled from his data indicates clearly that the gradations were marked.

TABLE 18.

RESULTS OF FOUR RUNS (2cwt. charges).

| Sp. Gr. Tar | Duration of Distillation | Gas Yield per Ton of Coal, Cu. Ft. | Free Carbon in Tar, Per Cent |
|---|---|---|---|
| (1) 1.084 | 8 hrs. | 6600 | 8.69 |
| (2) 1.103 | 7 hrs. | 7200 | 11.92 |
| (3) 1.149 | 6 hrs. | 8900 | 15.53 |
| (4) 1.204 | 5 hrs. | 11700 | 24.67 |

His analyses of the tars obtained show the characteristic increase in volatile constituents with rise in distillation temperature.

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*Jour. Soc. Chem. Ind. 7, 59.

TABLE 19.

PERCENTAGE COMPOSITION BY WEIGHT OF TARS.

| | No. 1 | No. 2 | No. 3 | No. 4 |
|---|---|---|---|---|
| Crude naptha | 9.17 | 9.05 | 3.73 | .99 |
| Light oil | 10.50 | 7.46 | 4.47 | .57 |
| Creosote oil | 26.45 | 25.83 | 27.29 | 19.44 |
| Anthracene oil | 20.32 | 15.57 | 18.13 | 12.28 |
| Pitch | 28.89 | 36.80 | 41.80 | 64.08 |
| Paraffin contents of crude naptha (by volume) | 5.0 | 4.0 | 1.5 | 1.0 |

There is, likewise, a consistent fall in the percentages of the acid in the liquid fraction as the following table shows.

TABLE 20.

PERCENTAGE OF TAR ACIDS.

| | In Crude Naptha | In Light Oil | Jn Creosote |
|---|---|---|---|
| No. 1 | 13 | 34 | 35 |
| No. 2 | 9 | 35 | 29 |
| No. 3 | 8 | 29 | 28 |
| No. 4 | 6 | 22 | 20 |

Napthalene and anthracene become prominent in the heavier tars according to his investigations, the former appearing in considerable quantity in No. 4 and the latter being at its maximum in No. 3.

As proof that the tendency of increased temperature in the distillation of coal is to destroy, preferably, the phenol bearing light oils intermediate between the crude naptha and the creosote, he cites the case of a tar of specific gravity 1.33, distilled at high temperatures, which showed practically no light oils but which yielded solid napthalene immediately after the naptha had come over. Watson Smith, in an article on "Variation in the Products of the Destructive Distillation of Coals,"* confirms these results, by stating that tars which were produced at very high temperatures in Simon-Carves ovens and examined by him, showed a conspicuous lack of the middle oil fraction which contains the phenols and that the creosote and anthracene oils were semi-solid, the former being thick with napthalene.

He calls attention to a theory of Schulze† that the primary products of the dry distillation of coals are phenols. These phenols are then at the higher temperatures of the retorts split up so as to yield water and high boiling hydrocarbons, or finally, with more entire decomposition, into illuminating gas.

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*Jour. Soc. Chem. Ind. 8, 952.

†Annalen, 227, 143.

An interesting table showing the effect of change of temperature on the yield of gas and tar is given by V. B. Lewes.*

### TABLE 21.

### VARIATION OF GAS AND TAR YIELD WITH TEMPERATURE RANGE.

| Temp. (C) | Gas yield per ton, cu. ft. | Tar gallons | Sp. gr. of Tar |
|---|---|---|---|
| 900° | 11 000 | 9 | 1.200 |
| 800° | 10 000 | 12 | 1.170 |
| 700° | 9 000 | 15 | 1.140 |
| 600° | 7 750 | 18 | 1.115 |
| 500° | 6 400 | 21 | 1.087 |
| 400° | 5 000 | 23 | 1.060 |

Lewes points out that tars produced at temperatures between 400° C and 500° C contain relatively low percentages of aromatic substances and high percentages of the paraffin series which are adapted to use as motor fuels. The middle oils are free from napthalene and yield excellent enriching oils. The pitch having practically no free carbon, he says, forms an ideal electric insulating material.

Variations in the quality and composition of the gas are shown in the following table.

### TABLE 22.

### COMPOSITION OF GASES PRODUCED AT DIFFERENT TEMPERATURES.

| | 400° | 500° | 600° | 700° | 800° | 900° |
|---|---|---|---|---|---|---|
| Hydrogen | 21.2 | 28.3 | 33.8 | 41.6 | 48.2 | 54.5 |
| Saturated hydrocarbons | 60.1 | 56.2 | 30.7 | 45.0 | 39.1 | 34.2 |
| Unsaturated hydrocarbons | 6.3 | 5.8 | 5.0 | 4.4 | 3.8 | 3.5 |

The gas produced at the lowest temperatures quoted has a calorific value of 750 B.t.u. and measures 20 candle power.

Dr. H. W. Jayne, in a paper† before the Fifth International Congress of Applied Chemistry at Berlin in 1903, says in regard to coal tars:

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*Jour. of Gas Light. 101, 823.

†The Coal Tar Industry in the U. S. Report of the Fifth International Congress of Applied Chemistry, Section IVa, Volume II, p. 721.

"The influence of the temperature in carbonizing is strikingly shown by the test of two tars, both from the same coal, and made in the same kind of ovens. One plant was producing gas as its main object. The tar from this source had a gravity of 1.21 and tested 17.5 per cent of free carbon; the light oil fraction was 2.2 per cent of a gravity 0.979, testing 23 per cent to 170 deg.; the pressed napthalene yield was 7.4 per cent. In the second tar, in which coke was the main object, evidently much lower heats were used, the tar having a gravity of 1.137, and testing 3.2 per cent of free carbon; the light oil amounted to 11.9 per cent, and had a gravity of .970, testing 28 per cent to 170 deg., or six times more crude naptha than in the first; the total tar acids were 12.48 per cent, while the pressed napthalene fell to 1.2 per cent." It is evident, he thinks, that in the first tar the light hydrocarbons and tar acids had been destroyed by the temperature employed, with formation of napthalene.

R. P. Perry* states that "Depending upon the coal used and more particularly upon the size and shape of retorts, the travel which the gases make over the hot surfaces, and the temperatures to which the gas is subjected, the coal tars vary within wider limits. In general with the vapors subjected to the high temperatures usually characteristic of gas works as compared with coke works, there is an increase in specific gravity and free carbon. For example the tar from many gas works will average about 1.24 specific gravity at 15.5°C. and about 25 per cent to 30 per cent free carbon by weight, whereas from the by-product coke ovens the specific gravity would average more nearly 1.19 and the free carbon would vary about 5 per cent to 18 per cent, the average being less than 10 per cent."

This so-called free carbon represents a finely divided inert material, largely carbon, which is the portion of tar insoluble in benzol and toluol. Perhaps a very small part is carried over mechanically in the coking process, but for the most part it is due to the cracking of the hydrocarbons by exposure to heat, and the higher percentage usually found in gas works tars is from this cause."

S. R. Church of the Barrett Manufacturing Company, in an interesting article on "Tar and Its By-Products,"† gives a table of results of analyses which were made in his laboratory, of the typical coke-oven tars produced in the United States. It is to be noted that the different

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*R. P. Perry. Tar Distillation in the U. S. Eighth International Congress of Applied Chemistry. **10**, 233.

†"Tar and Its By-Products," Gas Age, May 15, 1913.

gas retorts and ovens used vary in size and shape and that the former are, of course, heated to a much higher temperature than the latter.

### TABLE 23.

### ANALYSES OF TYPICAL TARS; DRY TAR.

| | Gas Retort | | | | Coke Oven | | | |
|---|---|---|---|---|---|---|---|---|
| | Horizontal | | Inclined | Vertical | United Otto | Semet-Solvay | Koppers | Water gas Eastern |
| Specific gravity at 15.5° C. | 1.266 | 1.293 | 1.238 | 1.153 | 1.207 | 1.188 | 1.186 | 1.083 |
| Free carbon (insoluble in $C_6H_6$) | 28.8 | 37.5 | 24.3 | | 13.3 | 10.7 | 6.8 | 0.1 |
| Specific viscosity (Engler at 100°C) | 21.8 | 30 | 14.9 | 2.1 | 3.4 | 3 | 2.1 | 1.6 |
| Oil to soft pitch 60°C. (Per cent by volume) | 13.2 | | 14.3 | 28.8 | 21.2 | 21.8 | 35.3 | 43.1 |
| Pitch residue 60°C. mp (Per cent by volume) | 86.8 | | 85.7 | 71.2 | 78.8 | 78.2 | 64.7 | 56.9 |
| Refractive indices at 60° C. (taken on oil) | 1.5932 | | 1.5807 | 1.5755 | 1.5987 | 1.6122 | 1.6139 | 1.5678 |
| Sulphonation residue (Per cent taken in oil) | 0.4 | | 2.4 | 4.3 | | | | 8 |
| Tar acids (Per cent taken in oil) | 14. | | 21.0 | 29. | 12 | 4 | | |

The figures given here are especially significant when compared with those of low temperature tars.

Vivian B. Lewes* in the series of articles mentioned earlier says of the tar produced in the manufacture of coalite, distilled at 400° to 500° C.: "The low-temperature tar is distinctive in its characteristics. It has a specific gravity of 1.075, is very liquid, and contains an abundance of light solvent oils, very low aromatic hydrocarbons, very little phenol but large quantities of cresol, no napthalene, and very little anthracene, while the free carbon is as a rule below two per cent."

The very low percentage of benzene in the light oils, is made up for by the presence of paraffins, such as hexane, heptane and octane, while there are also present considerable quantities of that curious group of hydrocarbons known as napthenes or hexahydro-benzenes, which play so important a part in Russian petroleum.

As before mentioned, carbolic acid occurs in small quantities, but its higher homologues, such as cresylic acid, etc., occur in much larger quantities than in coal tar, and there are also present quantities of polydydric phenols, or other esters of the type met with in coal tar which form resinous masses difficult to analyze. He states that coals rich in oxygen (10 to 11 per cent) and hydrogen (above 5 per cent) and which, therefore, have large percentages of resinic bodies, on being carbonized at low temperatures yield tars rich in phenol and cresol. These results are confirmed by Bornstein. The pitch left as a residue amounts to about 40 per cent of the tar, and is of fine quality owing to the practical absence of free carbon. The ammonium sulphate amounted to only 12 lb. per ton of coal, the temperature being too low for large production.

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*The Carbonization of Coal. Lecture IV. Jour. Roy. Soc. Arts. 60, 216.

The tar yield varies with the coal used and in most cases averages twenty gallons per ton of coal.

## TABLE 24.

## COMPOSITION OF TAR FROM COALITE.

| | |
|---|---|
| Specific Gravity | 1.073 |
| Distillation on 2,274 cc. | (0.5 gallon) |

| | Temp. | By volume on tar | Sp. gr. | Volume of hydrocarbons | Tar acids |
|---|---|---|---|---|---|
| Water | | 2 64 | | | |
| Light Oil | 170° | 3.10 | .844 | 3.10 | |
| Carbolic Oil | 170°–225° | 13.72 | .959 | 8.62 | 4.80 |
| Creosote Oil | 225°–240° | 8.35 | .988 | 4.64 | 3.10 |
| Creosote Oil | 240°–270° | 8.35 | .992 | 5.45 | 2.55 |
| Anthracene Oil | 270°–300° | 8.80 | 1.029 | 6.60 | 1.76 |
| Anthracene Oil | 300°–320° | 12.31 | 1.033 | 8.80 | 3.10 |
| | Pitch by weight on volume | | | 41.71% | |
| | Bases | | | 1.32% | |

| Percentage distilling below | Light Oils | Calculated on Tar |
|---|---|---|
| 100° C | 15.6 by vol. | .55 |
| 120° | 31.2 | 1.09 |
| 140° | 54.7 | 1.91 |
| 170° | 82.8 | 2.90 |
| over 170° and loss | 17.2 | .20 |

Prevost Hubbard* has lately published a similar table of his own results obtained from the analyses of tars from twenty-six of the thirty-one by-product coke ovens operating in this country in 1910. His figures may be considered as showing authoritatively the character of the 67,000,000 gallons of tar produced from these sources in that year.

The specific gravities of the samples ranged from 1.133 to 1.214, the majority being lower than 1.200 and indicating low percentages of free carbon. The minimum percentage of free carbon was 2.73, the maximum 16.80, and the average for the 26 samples was 8.38. Eighteen samples contained less than 10 per cent of free carbon, and eight samples more than 10 per cent. About two-thirds of these products might, therefore, be considered as low-carbon tars and the other third, as medium carbon tars.

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*Coke-Oven Tars of the United States, Circular No. 97. Office of Public Roads, U. S. Dept. of Agriculture.

In accordance with the type of oven in which these products are made, there is a considerable variation in composition, even though temperatures differ but little. It seems probable, therefore, that high pitch and free carbon contents are not simple functions of temperature alone but that other factors complicate the problem.

TABLE 25.

COMPARISON OF TARS FROM VARIOUS TYPES OF OVENS.

| Kind of Oven | Maximum Temperature of Retorts (C) | Per Cent Free Carbon | Per Cent Pitch Residue |
|---|---|---|---|
| Koppers | 1100°–1444° | 3.38 | 70 |
| Semet-Solvay | 1050°–1450° | 6.74 | 63–79 |
| United Otto | 1220°–1660° | 9.00 | 69–78 |
| Otto Hoffman | 1000°–1100° | 12.16 | 60–78 |
| Otto Hoffman and United Otto (mixed) | 1000°–1200° | 12.51 | 76 |
| United Otto and Rothenberg (mixed) | 1000° | 17.17 | 76 |

Furthermore, the tars from different ovens of the same type show marked inconsistencies in many respects. For instance, twelve tars from as many different Semet-Solvay plants in which retort firing temperatures were in each case 1050°–1450° C, and maximum temperatures to which coal was brought, 950°–1150° C, show amounts of free carbon ranging from 4 per cent to 9 per cent; of pitch, from 63 per cent to 79 per cent; and of anthracene oils, from 5.5 per cent to 11 per cent.

Lunge,* the authority on coal tars, discusses only very briefly the effect of temperature range on the quality of tar. Among others, he quotes Behrens,† who found that the tar obtained in the distillation of coal in the ordinary fire-clay gas-retorts was much richer in benzene, tolene, and napthalene than that made from the same kind of coal in coke ovens operated at a lower temperature.

In considering and comparing the foregoing reports it should be borne in mind that as yet no standard or official methods for tar analysis and testing have been adopted by the industry as a whole. It is true, that while the manufacturers of special products such as creosote and road material, each in his own particular field has defined, more or less, his standards, the results obtained are not always comparable, and this is especially true when dealing with such complex hydrocarbons as coal tars. In spite of this difficulty, however, there is sufficient unanimity among the authorities to establish the fact fully confirmed by the worker in organic chemistry that rise of temperatures tend to produce polymerization with the formation of more complex substances of higher molecular weight, and that comparatively valueless products such as naphthalene and heavy pitch are produced in increasing quantities. In other words, the lighter and more valuable hydrocarbons are being destroyed in favor of the heavier ones which find little industrial use.

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*Lunge, Coal Tars and Ammonia, 4th ed. 1909.

†Dingler's Polytech. Jour. **208**, 362.

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 80 JUNE, 1915

# WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS.

BY W. M. WILSON, ASSISTANT PROFESSOR OF STRUCTURAL ENGINEERING, AND G. A. MANEY, FORMERLY RESEARCH FELLOW IN DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS.*

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*While the bulletin is the result of the joint efforts of the two writers, certain parts are so distinctly the work of one that it seems desirable to make a statement relative to the part which should be credited to each. The method of making the analysis involving the use of the slope-deflection equations should be credited to G. A. Maney. The conception of the bulletin as a whole, the method of presenting the results, and the authorship of the text should be credited to W. M. Wilson. The numerical calculations of Tables 11 to 22, inclusive, were made by both writers.—The Editor.

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# WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS

## I. Introduction.

1. *Preliminary.*—The increase in the price of land in large cities has made it necessary to build high buildings in order to get a large rentable floor space on a small parcel of land. The type of building generally used is known as the steel-skeleton building. In this type of building the live and dead loads, including the weight of the walls, are carried by a system of beams and girders to columns and are carried by the columns to the footings.

In high buildings the horizontal shear due to the wind load is very large; and, since it is usually impracticable to put diagonal braces between the columns, it is customary to make the steel frame rigid enough to resist the horizontal shear by virtue of the stiffness of the columns and girders. The exact determination of the stresses in a steel frame due to a horizontal shear is one of the problems of structural engineering which remains to be solved. While the writers realize that the method of determining these stresses presented in this bulletin is based upon assumptions which are not exactly true, they believe that the method is more accurate than the methods ordinarily used.

2. *Acknowledgment.*—Messrs. Anderson, Becker, Gomez, and Richart, graduate students in the College of Engineering of the University of Illinois, calculated the moments in Table 23, and the moments as determined by methods I, II, and III in the keys to the diagrams in Figs. 10, 11, and 12, and assisted with the calculations necessary to determine the curves shown in these figures. Professor Ira O. Baker and Professor C. A. Ellis rendered valuable assistance, criticizing the bulletin during its prepartion. The writers gratefully acknowledge indebtedness to these men.

## II. Present Methods of Calculating Wind Stresses in Office Buildings.

3. *Classification of Methods.*—Methods of calculating wind stresses in the steel frames of office buildings may be divided into two classes, viz.: (1), those used in the actual design of buildings and (2), those

which have resulted from an effort to make an exact analysis of the stresses. In the methods of the first class, accuracy has been sacrificed to shorten and simplify the calculations. In the methods of the second class, the aim has been to make an exact analysis rather than an analysis that can be used in the actual design of a building.

For the sake of convenience in reference, methods of the first class are designated as *approximate methods,* and those of the second class are designated as *exact methods.*

4. *Approximate Methods.*—(a) Fleming's Methods. Mr. R. Fleming, in an article in *Engineering News,* describes three methods, which are in current use.* These methods are designated as methods I, II, and III. The three methods, as applied to a building in which all columns of a story have the same section, are based upon the following assumptions:

Assumptions in Method I.

1. A bent of a frame acts as a cantilever.
2. The point of contra-flexure of each column is at mid-height of the story.
3. The point of contra-flexure of each girder is at its mid-length.
4. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent.

Assumptions in Method II.

1. A bent of a frame acts as a series of portals.
2. The point of contra-flexure of each column is at mid-height of the story.
3. The shear is the same on all columns of a story.
4. Each pair of adjacent columns of a bent acts as a portal, and each interior column is a member of two adjacent portals. The direct stress in an interior column, when the column is considered as a member of the portal on one side, is of opposite sign from the direct stress in the same column when considered as a member of the portal on the opposite side and the resultant direct stress is equal to zero.

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*Wind Bracing without Diagonals for Steel-Frame Office Buildings, Engineering News, March 13, 1913.

Assumptions in Method III.

1. A bent of a frame acts as a continuous portal.
2. The point of contra-flexure of each column is at mid-height of the story.
3. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent.
4. The shear is the same on all columns of a story.

(b) Smith's Methods. Professor Albert Smith, in a paper before the Western Society of Engineers, describes a method which he has used in his classes in Structural Engineering at Purdue University.‡ This method is here designated as Method IV.

Assumptions in Method IV.

1. The point of contra-flexure of each column is at mid-height of the story.
2. The point of contra-flexure of each girder is at its mid-length.
3. The shears on the internal columns are equal and the shear on each external column is equal to one-half of the shear on an interior column.

If all of the assumptions of any of these methods are accepted, the stresses in a frame may be determined by applying the fundamental equations of static equilibrium. It is apparent from the assumptions that the results obtained by these methods are radically different.

5. *Exact Methods.*—(a) Melick's Method. Dr. Cyrus A. Melick* used a method which takes into account the form of the elastic curves and the deflections and changes in length of the members. The method is so long that, when applied to a building only four stories high, the amount of work required is almost prohibitive. To apply it to a building twenty stories high would be impracticable if not, in fact, impossible.

(b) Jonson's Method. Mr. Ernest F. Jonson suggested a method which takes into account the deflections of the columns and the changes in the slopes of the tangents to the elastic curves of the columns and girders at the points where they intersect.†

If the method which he suggests were used, it would give the stresses with a fair degree of accuracy; but his method involves so many unknowns that its use would not be practicable in the actual design of buildings.

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‡Wind Stresses in the Frames of Office Buildings, by Albert Smith, Journal Western Society of Engineers, Vol. XX, No. 4, p. 341.

*Stresses in Tall Buildings, by Cyrus A. Melick. Bulletin No. 8, College of Engineering, University of Ohio.

†The Theory of Frameworks with Rectangular Panels and Its Application to Buildings which Have to Resist Wind, by Ernst F. Jonson, Tran. Am. Soc. C. E., Vol. 55, p. 413.

(c) Method of Least Work. Professor Albert Smith has determined the wind stresses in symmetrical bents having two, three, and four spans by the method of least work.* This method is exact, but, of course, is extremely long.

## III. OUTLINE OF THE PROPOSED ANALYSIS.

6. *Outline of the Method.*—In making an analysis of the stresses, the writers made certain assumptions and applied certain fundamental principles of mechanics and obtained equations from which the stresses in a frame can be determined. The assumptions which have been made are stated in Section IV and the derivation of the equations is given in Sections V and VI.

It can be proven that the moment at an end of a member of a frame is a function of the changes in the slopes of the tangents to the elastic curve of the member at its ends and of the deflection of one end of the member relative to the other end (see equation A, page 13).

In the strained position, all the columns and girders which intersect at one point have been subjected to the same change in slope (see assumption 1, Section IV); the vertical deflections of the ends of all girders are equal to zero; and the horizontal deflections of the tops of all columns of a story are equal (see assumption 2, Section IV).

Consider any story of a bent. Take the point of intersection of the neutral axes of a column and a girder as a free body. It is in equilibrium under the action of the moments at the extremities of the columns and girders which intersect at the point. Each of the moments may be expressed in terms of the changes in the slopes at the extremities of the member, and the deflection of one end of the member relative to the other. A moment equation can therefore be written for each point where the columns and girders intersect, and the only unknown quantities will be the changes in the slopes at the extremities of the columns and the horizontal deflections of the columns in a story.

If all the columns of a story be taken together as a free body, the sum of the moments at the two extremities of all the columns will be balanced by a couple whose moment is equal to the total shear on the story multiplied by the story height. The shear and the height of the story are known, and the moments in the columns can be expressed in terms of the slopes and the deflections at their extremities the same as in the previous equations. It is therefore possible to write as many equations for each story as there are columns in the story, plus one. As the only unknown quantities in these equations are the changes in the slopes at the extremities of the columns and the deflection in a story common to all columns, there are as many equations per story as there are unknowns. By solving these equations the slopes and the deflections can be determined. Knowing the slopes and the deflections, the moments can be computed.

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*Wind Stresses in the Frames of Office Buildings, by Albert Smith, Journal Western Society of Engineers, Vol. XX, No. 4, p. 341.

The product of the shear on a member and the length of the member is equal to the algebraic sum of the moments at the extremities of the member. Since the moments and the length of the member are known the shear can be computed.

With the shears in the girders known, the direct stress in any column can be determined by taking the column as a free body and equating the sum of the vertical forces to zero.

The direct stress in a girder may be determined in a similar manner.

The method just described is based upon the proposition in mechanics used by Mr. Jonson, but the method which the writers have developed differs from the one used by him in that the changes in the slopes and the deflections have been used as the unknown quantities instead of the direct stresses and the moments. Four members, two columns and two girders, intersect in a point. Each member is subjected to a different direct stress and a different moment, whereas all of the members are subjected to the same change in slope, and all of the columns in a story are subjected to the same deflection. It is therefore apparent that there are fewer unknown slopes and deflections than moments and direct stresses. The large reduction in the number of unknowns very much simplifies the solution of the equations.

## IV. Assumptions Upon Which the Analysis Is Based.

7. *Statement of Assumptions.*—The proposed analysis is based upon the following assumptions:

1. The connections between the columns and girders are perfectly rigid.
2. The change in the length of a member due to the direct stress is equal to zero.
3. The length of a girder is the distance between the neutral axes of the columns which it connects and the length of a column is the distance between the neutral axes of the girders which it connects.
4. The deflection of a member due to the internal shearing stresses is equal to zero.

5. The wind load is resisted entirely by the steel frame.

These assumptions will be discussed in Section XI.

## V. Fundamental Equation.

8. *Fundamental Proposition.*—The fundamental equation used in this analysis is based upon the following proposition:

*When a member is subjected to flexure, the deflection of any point in the neutral axis from the tangent to the elastic curve at any other point is equal to the moment of the area of the* $\frac{M}{EI}$ *diagram for the portion of the member between the two points, about the point where the deflection is measured.*

9. *Proof of the Proposition.*—The line *AB*, Fig. 1, represents the neutral axis of a member subjected to flexure. The deflection of the member is very much exaggerated in the figure. The actual deflection is so small that the length of the curve may be considered equal to its horizontal projection. It is required to prove that the deflection of any point *P* from the tangent to the line *AB* at any other point *Q* is equal to $\int_Q^P \frac{M}{EI}\, dx{\cdot}x.$

Extend the tangents at the extremities of an element of the curve until they intersect the vertical line through *P*. The intercept on this vertical line between two consecutive tangents is equal to $x{\cdot}d\theta$. The total deflection of *P* from the tangent at *Q* is equal to the algebraic sum of these intercepts for the elements of the elastic curve between the points *Q* and *P*. That is, $y = \int_Q^P x d\theta$. The equation of the elastic curve may be written $\frac{d\theta}{dx} = \frac{M}{EI}$. Substituting the value of $d\theta$ from this equation in the expression for $y$, gives $y = \int_Q^P \frac{M}{EI}\, dx{\cdot}x.$ The quantity $\frac{M}{EI}dx$ can be considered as an elementary area of the $\frac{M}{EI}$ diagram. The quantity $\int_Q^P \frac{M}{EI} dx{\cdot}x$ can therefore be considered as the moment of the $\frac{M}{EI}$ diagram about the point *P*.

10. *Derivation of Fundamental Equation.*—Consider a member which is not acted upon by any external forces or couples except at the ends. The line $AB$ in Fig. 2 represents the neutral axis of such a member. The moment at $A$ is represented by $M_{AB}$ and at $B$ by $M_{BA}$. The change in the slope of the elastic curve at $A$ due to the external forces is represented by $\theta_A$, and at $B$ by $\theta_B$. The deflection of $A$ from its original position $A'$, is $d$. The deflection of $A$ from the tangent at $B$ is represented by $(d - l{\cdot}\theta_B)$; and the deflection of $B$ from the tangent at $A$ is represented by $(d - l{\cdot}\theta_A)$. The $\frac{M}{EI}$ diagram is shown in Fig. 3.

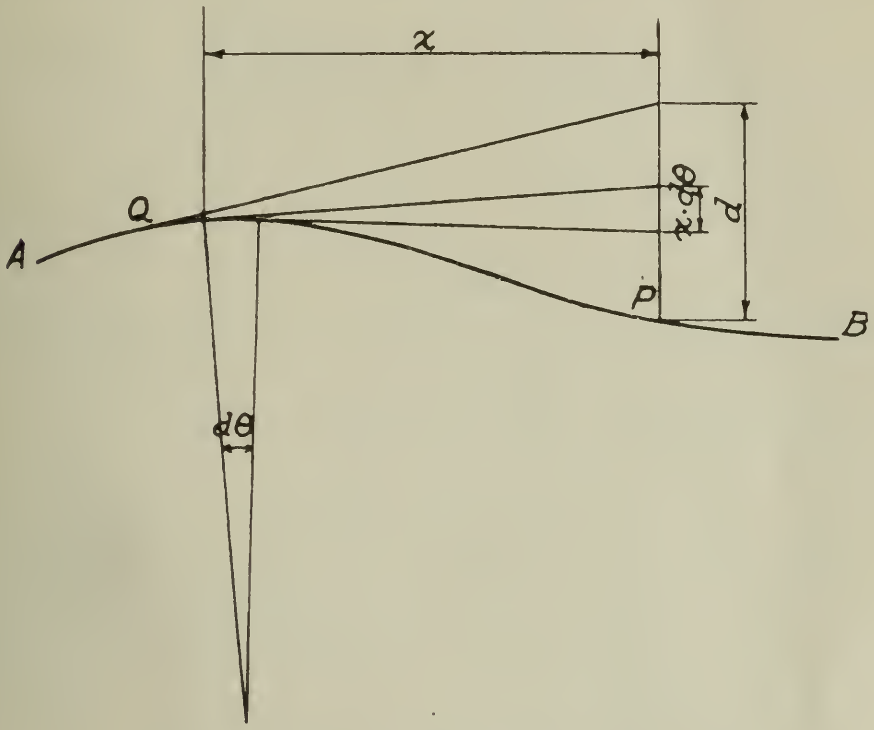

FIG. 1. PROOF OF FUNDAMENTAL EQUATION.

The signs of the quantities are determined by the following rules:

The change in the slope, or the angular deformation, is positive (+) when the tangent to the elastic curve of the member is turned in a clockwise direction.

Distances and deflections are positive when they are measured in the same direction from the base line as are positive slopes.

The moment acting upon a member at the section where the deflection is measured is positive (+) when it produces a clockwise rotation.

Substituting $(d - l{\cdot}\theta_B)$, the deflection of $A$ from the tangent at $B$, for $y$ in the equation $y = \int \frac{M}{EI} dx{\cdot}x$, gives

$$(d - l{\cdot}\theta_B) = \int_A^B \frac{M}{EI} dx{\cdot}x.$$

Substituting the value of $M$ from the equation

$$M = M_{AB} + \left[\frac{M_{BA} - M_{AB}}{l}\right]x, \text{ gives}$$

$$(d - l{\cdot}\theta_B) = \int_A^B \frac{M_{AB}}{EI} xdx + \left[\frac{M_{BA} - M_{AB}}{EI{\cdot}l}\right] x^2{\cdot}dx.$$

If the material is homogeneous and the section uniform, $E$ and $I$ are constants. Performing the indicated integration, gives

$$(d - l{\cdot}\theta_B) = \frac{M_{AB}}{EI}\frac{l^2}{2} + \frac{M_{BA}}{EI}\frac{l^2}{3} - \frac{M_{AB}}{EI}\frac{l^2}{3}, \text{ or}$$

$$d = l{\cdot}\theta_B + \frac{l^2}{6EI}\left[2M_{BA} + M_{AB}\right] \dots\dots\dots\dots (1)$$

Substituting $(d - l{\cdot}\theta_A)$, the deflection of $B$ from the tangent at $A$, for $y$ in the equation, $y = \int \frac{M}{EI} dx{\cdot}x$, gives

$$(d - l{\cdot}\theta_A) = \int_B^A \frac{M}{EI} dx{\cdot}x.$$

The moment at any section in the member, when considering the deflection at $B$, is of opposite sign from the moment at the same section when considering the deflection at $A$. (See preceding rule for determining the sign of the moment.)

Substituting the value of $M$ from the equation

$$M = -M_{BA} - \left[\frac{M_{AB} - M_{BA}}{l}\right]x, \text{ gives}$$

$$(d - l{\cdot}\theta_A) = \int - \frac{M_{BA}}{EI} xdx - \left[\frac{M_{AB} - M_{BA}}{l}\right]x^2 \, dx \text{ or}$$

$$d = l{\cdot}\theta_A - \frac{M_{BA}}{EI}\frac{l^2}{2} - \frac{M_{AB}}{EI}\frac{l^2}{3} + \frac{M_{BA}}{EI}\frac{l^2}{3}$$

$$d = l{\cdot}\theta_A + \frac{l^2}{6EI}\left[-M_{BA} - 2M_{AB}\right] \dots\dots\dots\dots\dots (2)$$

Multiplying equation (2) by 2, gives

$$2d = 2l{\cdot}\theta_A + \frac{l^2}{6EI}\left[-2M_{BA} - 4M_{AB}\right] \dots\dots\dots\dots (3)$$

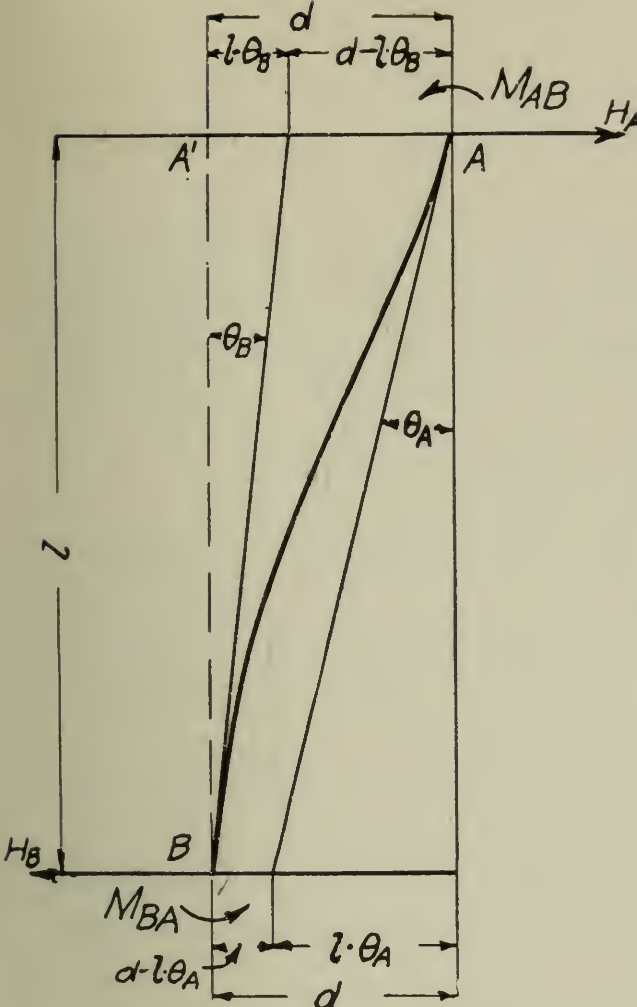

FIG. 2. DERIVATION OF FUNDAMENTAL EQUATION.

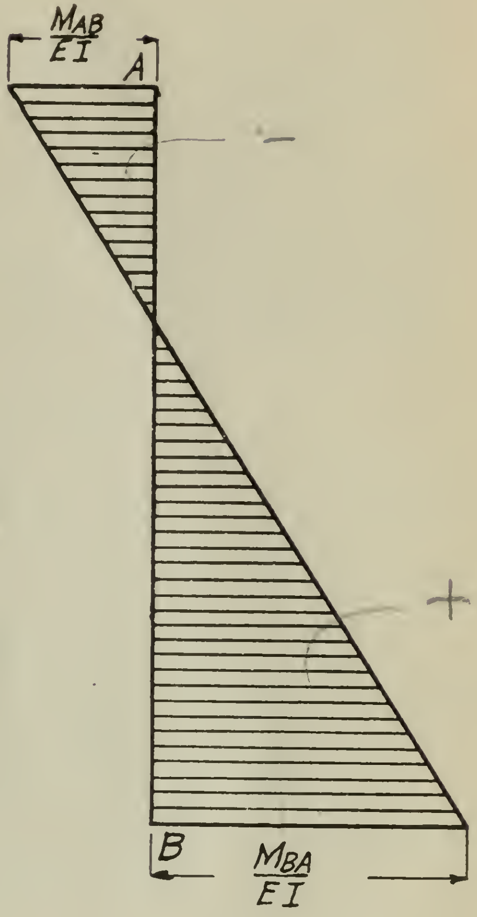

FIG. 3. $\frac{M}{EI}$ DIAGRAM.

Adding equations (1) and (3) gives

$$3d = 2l\cdot\theta_A + l\cdot\theta_B + \frac{l^2}{6EI}\left[-3M_{AB}\right].$$

Substituting $K$ for $\frac{I}{l}$ and $R$ for $\frac{d}{l}$ and solving for $M_{AB}$, gives,

$$M_{AB} = 2\,EK\ \ [2\,\theta_A + \theta_B - 3R] \ldots\ldots\ldots\ldots\ldots (A)$$

When $d = 0$, equation (A) becomes

$$M_{AB} = 2\,EK\ \ (2\,\theta_A + \theta_B) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (B)$$

Equation (A) is general and may be applied to any length of any member in bending provided the length considered has no intermediate external force applied to it. That is, one or more of the quantities

$\theta_A$, $\theta_B$, and $d$ may be negative and equation (A) will still give the moment at the point $A$ in both magnitude and sign. Equation (A) is the fundamental equation upon which the analysis which follows is based. Equation (B) is a special form of equation (A). Equation (A) may be expressed as follows:

*The moment at the end of any member is equal to 2EK times the quantity: Two times the change in the slope at the near end plus the change in the slope at the far end minus three times the deflection divided by the length. E is the modulus of elasticity of the material and K is the ratio of the moment of inertia to the length of the member.*

## VI. DERIVATION OF GENERAL EQUATIONS.

11. *Notation.*—The following notation has been used:

$A$, $B$, $C$, etc. = the columns of a bent, beginning at the right and reading toward the left.

$a$, $b$, $c$, etc.= the bays of a bent, beginning at the right and reading toward the left. The girder in bay $a$ is designated as girder $a$, in bay $b$ as girder $b$, in bay $c$ as girder $c$, etc.

$A_1$, $A_2$, $A_3$, etc. = the intersections of the neutral axes of the girders at the tops of the first, second, third, etc., stories with the neutral axis of column $A$.

$B_1$, $B_2$, $B_3$, etc. = the intersections of the neutral axes of the girders at the tops of the first, second, third, etc., stories with the neutral axis of column $B$.

$d$ = deflection of the columns in a story height.

$E$ = modulus of elasticity of the material.

$h$ = length of a column measured from neutral axis to neutral axis of the girders.

$I$ = moment of inertia of the girder and column sections.

$J = 2\sum\left(\frac{I}{l}+\frac{I}{h}\right)$ for all columns and girders which intersect at a point.

$K = \frac{I}{l}$ for girders and $\frac{I}{h}$ for columns.

$l$ = length of a girder measured from neutral axis to neutral axis of the columns.

$M$ = bending moment.

$N = 2\sum\left(\frac{I}{h}\right)$ for all columns in a story.

$R = \frac{d}{h}$

$W$ = total horizontal shear in a bent at any story.

$w$ = increment of the horizontal shear in a bent in a story height.

$\theta$ = change in the slope of the tangent to the elastic curve.

Subscripts are added to the letters to indicate the particular part of a bent to which a given symbol applies. For example, referring to Fig. 4, girder $b_1$ is the girder in bay $b$ at the top of the first story, $A_1$ is the intersection of the girder at the top of the first story with column $A$, $\theta_{A_1}$ is the slope of the tangent to the elastic curve at the point $A_1$; $d_3$ is the deflection of the columns in the third story; $J_{B_4}$ is the $J$ at the point $B_4$; $K_{A_2}$ is the $K$ of column $A$ in the second story. The moment at the right end of the girder in bay $a$ at the top of the second story is designated as $M_2^{AB}$, and the moment at the left end of the same girder is designated as $M_2^{BA}$. The moment at the top of the column $B$ in the third story is designated as $M_B^{32}$, and the moment at the bottom of the same column is designated as $M_B^{23}$.

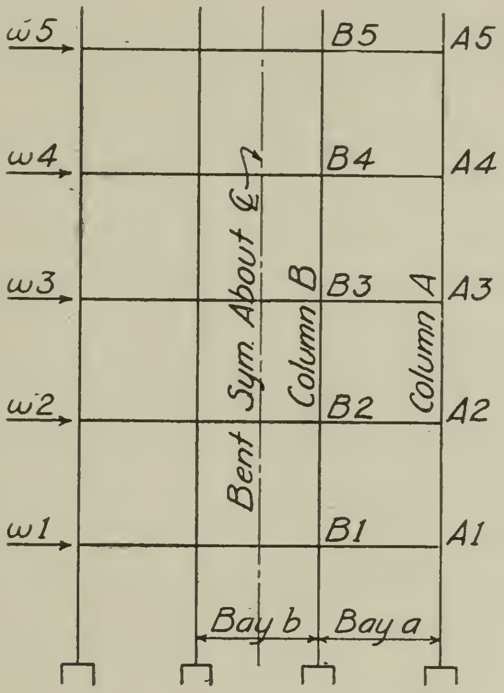

FIG. 4. BOTTOM FIVE STORIES OF A SYMMETRICAL THREE-SPAN BENT.

12. *Derivation of Equations.*—Fig. 4 shows the bottom five stories of a symmetrical three-span bent. It is required to find the stresses in all of the members.

Consider the columns of the third story acting together as a free body. (Any other story could have been used.) The algebraic sum of all of the moments at the tops and the bottoms of all the columns plus the product of the total shear in the story and the story height, is equal to zero. That is, $2\,(M_A{}^{32} + M_A{}^{23} + M_B{}^{32} + M_B{}^{23}) + W_3h_3 = 0$

Substituting the values of the moments as given by equations A and B of Section V gives

$$2\left[\,2EK_{A3}\,(2\theta_{A3} + \theta_{A2} - 3R_3) + 2EK_{A3}\,(2\theta_{A2} + \theta_{A3} - 3R_3) + 2EK_{B3}\,(2\theta_{B3} + \theta_{B2} - 3R_3) + 2EK_{B3}\,(2\theta_{B2} + \theta_{B3} - 3R_3)\,\right] + W_3h_3 = 0.$$

Letting $N = 2\sum\left(\dfrac{I}{h}\right)$ for all of the columns in a story, and collecting, gives

$$2K_{A3}\,\theta_{A2} + 2K_{B3}\,\theta_{B2} - N_3R_3 + 2K_{A3}\,\theta_{A3} + 2K_{B3}\,\theta_{B3} = -\,\frac{W_3h_3}{6E}\,\ldots\ldots(1)$$

Consider the point $A_3$ as a free body. Taking $\sum M = 0$, gives $M_A{}^{34} + M_A{}^{32} + M_3{}^{AB} = 0$. Substituting the values of these moments as given by equations A and B gives,

$$2EK_{A4}\,(2\theta_{A3} + \theta_{A4} - 3R_4) + 2EK_{A3}\,(2\theta_{A3} + \theta_{A2} - 3R_3) + 2EK_{a3}\,\,(2\theta_{A3} + \theta_{B3}) = 0.$$

Combining the coefficients of the unknowns and cancelling the common factor $2E$ gives,

$$\theta_{A2}K_{A3} + \theta_{A3}\,(2K_{A4} + 2K_{A3} + 2K_{a3}) + \theta_{A4}\,K_{A4} + \theta_{B3}K_{a3} - R_3 3K_{A3} - R_4 3K_{A4} = 0.$$

Substituting $J_{A3}$ for $2\sum\left(\dfrac{I}{h} + \dfrac{I}{l}\right)$ of all the members that intersect at $A_3$, the equation becomes:

$$K_{A3}\,\theta_{A2} - 3K_{A3}\,R_3 + J_{A3}\,\theta_{A3} + K_{a3}\,\theta_{B3} - 3K_{A4}R_4 + K_{A4}\theta_{A4} = 0\ldots(2)$$

The point $B_3$ is in equilibrium under the action of the four moments $M_3{}^{BA}$, $M_3{}^{BB}$, $M_B{}^{34}$, and $M_B{}^{32}$. Equating the sum of these four moments to zero gives,

$$2EK_{a3}\,(2\theta_{B3} + \theta_{A3}) + 2EK_{b3}\,(2\theta_{B3} + \theta_{B3}) + 2EK_{B4}\,(2\theta_{B3} + \theta_{B4} - 3R_4) + 2EK_{B3}\,(2\theta_{B3} + \theta_{B2} - 3R_3) = 0.$$

Combining the coefficients for the unknowns and cancelling the common factor $2E$, gives,

$$2\theta_{B3}\,(K_{a3} + K_{b3} + K_{B4} + K_{B3}) + \theta_{A3}K_{a3} + \theta_{B3}K_{b3} + \theta_{B4}K_{B4} + \theta_{B2}K_{B3} - 3R_4K_{B4} - 3R_3K_{B3} = 0.$$

Substituting $J_{B3}$ for $2\sum\left(\dfrac{I}{h} + \dfrac{I}{l}\right)$ of all the members intersecting at $B_3$, the equation becomes,

$$K_{B3}\,\theta_{B2} - 3K_{B3}R_3 + K_{a3}\,\theta_{A3} + (K_{b3} + J_{B3})\,\theta_{B3} - 3K_{B4}R_4 + K_{B4}\theta_{B4} = 0 \quad \ldots\ldots (3)$$

The three equations, 1, 2, and 3, can be written for any story by making the proper changes in the subscripts. As there are only three unknowns, $\theta_A$, $\theta_B$, and $R$ for each story, as many equations can be written for a bent as there are unknowns to be determined. It is possible to solve these equations for the unknown quantities algebraically, but the large number of equations involved makes the work very difficult. It is simpler to substitute the numerical values of the coefficients in the equations and solve for the numerical values of the unknown quantities by the process of elimination explained in Section VII.

For convenience in the application of this process of elimination, the equations are written in tabular form as shown in Table 3 in which the unknown changes in the slopes and the ratios of deflection to story height are written at the tops of the columns and the coefficients of these unknowns are written below. For example, in writing equation A of Table 3, which is $-N_1R_1 + 2K_{A1}\theta_{A1} + 2K_{B1}\theta_{B1} = -\frac{W_1h_1}{6E}$ $-N_1$, the coefficient of $R_1$, is placed in the column under $R_1$; $2K_{A1}$, the coefficient of $\theta_{A1}$, is placed in the column under $\theta_{A1}$; $2K_{B1}$, the coefficient of $\theta_{B1}$, is placed in the column under $\theta_{B1}$; and $-\frac{W_1h_1}{6E}$ is placed in the column headed "Right-Hand Member of Equation." Having the equations written in this form, it is unnecessary to repeat the unknown quantities when eliminating them from the equations by the method used in Table 14.

Table 3 contains the general equations to be used in determining the slopes and the deflections in a symmetrical three-span bent any number of stories high. The subscripts 1, 2, and 3 refer to the first, second, and third stories respectively, and the subscripts $x$, $y$, and $z$ refer to the second from the top, the next to the top, and the top stories respectively. The equations for the intervening stories are of the same form, but have different subscripts.

By using the general method outlined above, that is, by writing an equation similar to equation 1 for each story and an equation similar to equations 2 and 3 for each intersection of a column with a girder for each story, as many equations as there are unknowns can be written for a bent containing any number of spans and any number of stories. General equations for *symmetrical* bents of from one to five spans are given in Tables 1 to 5 and similar equations for *unsymmetrical* bents of from one to five spans are given in Tables 6 to 10.

In order to check the equations of Tables 1 to 10, a model of a

## TABLE 1.

## GENERAL EQUATIONS FOR A SYMMETRICAL SINGLE-SPAN BENT ANY NUMBER OF STORIES HIGH.

| Equation | Left-Hand Member of Equation | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | Story No. 2 | | Story No. 3 | | Intervening Stories | Story No. X (Second from Top) | | Story No. Y (Next to Top) | | Story No. Z (Top) | | |
| | $R_1$ | $\theta_{A1}$ | $R_2$ | $\theta_{A2}$ | $R_3$ | $\theta_{A3}$ | | $R_X$ | $\theta_{AX}$ | $R_Y$ | $\theta_{AY}$ | $R_Z$ | $\theta_{AZ}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | |
| A | $-N_1$ | $2K_{A1}$ | | | | | | | | | | | | $-W_1h_1 \div 6E$ |
| B | $-3K_{A1}$ | $J_{A1}+K_{a1}$ | $-3K_{A2}$ | $K_{A2}$ | | | | | | | | | | O |
| C | | $2K_{A2}$ | $-N_2$ | $2K_{A2}$ | | | | | | | | | | $-W_2h_2 \div 6E$ |
| D | | $K_{A2}$ | $-3K_{A2}$ | $J_{A2}+K_{a2}$ | $-3K_{A3}$ | $K_{A3}$ | | | | | | | | O |
| Similar equations for intervening stories | | | | | | | | | | | | | | |
| W | | | | | | | | | $2K_{AY}$ | $-N_Y$ | $2K_{AY}$ | | | $-W_Yh_Y \div 6E$ |
| X | | | | | | | | | $K_{AY}$ | $-3K_{AY}$ | $J_{AY}+K_{aY}$ | $-3K_{AZ}$ | $K_{AZ}$ | O |
| Y | | | | | | | | | | | $2K_{AZ}$ | $-N_Z$ | $2K_{AZ}$ | $-W_Zh_Z \div 6E$ |
| Z | | | | | | | | | | | $K_{AZ}$ | $-3K_{AZ}$ | $J_{AZ}+K_{aZ}$ | O |

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## TABLE 2.

### GENERAL EQUATIONS FOR A SYMMETRICAL TWO-SPAN BENT ANY NUMBER OF STORIES HIGH.

| Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | | Story No 2 | | | Story No. 3 | | | Intervening Stories | Story No. X (Second from Top) | | | Story No. Y (Next to Top) | | | Story No. Z (Top) | | | |
| | $R_1$ | $\theta_{A1}$ | $\theta_{B1}$ | $R_2$ | $\theta_{A2}$ | $\theta_{B2}$ | $R_3$ | $\theta_{A3}$ | $\theta_{B3}$ | | $R_X$ | $\theta_{AX}$ | $\theta_{AY}$ | $R_Y$ | $\theta_{AY}$ | $\theta_{BY}$ | $R_Z$ | $\theta_{AZ}$ | $\theta_{BZ}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | | | | | |
| A | $-N_1$ | $2K_{A1}$ | $K_{B1}$ | | | | | | | | | | | | | | | | | $-W_1h_1 \div 6E$ |
| B | $-3K_{A1}$ | $J_{A1}$ | $K_{a1}$ | $-3K_{A2}$ | $K_{A2}$ | | | | | | | | | | | | | | | 0 |
| C | $-3K_{B1}$ | $2K_{a1}$ | $J_{B1}$ | $-3K_{B2}$ | | $K_{B2}$ | | | | | | | | | | | | | | 0 |
| D | | $2K_{A2}$ | $K_{B2}$ | $-N_2$ | $2K_{A2}$ | $K_{B2}$ | | | | | | | | | | | | | | $-W_2h_2 \div 6E$ |
| E | | $K_{A2}$ | | $-3K_{A2}$ | $J_{A2}$ | $K_{a2}$ | $-3K_{A3}$ | $K_{A3}$ | | | | | | | | | | | | 0 |
| F | | | $K_{B2}$ | $-3K_{B2}$ | $2K_{a2}$ | $J_{B2}$ | $-3K_{B3}$ | | $K_{B3}$ | | | | | | | | | | | 0 |
| Similar equations for intervening stories | | | | | | | | | | | | | | | | | | | | |
| U | | | | | | | | | | | | $2K_{AY}$ | $K_{BY}$ | $-N_Y$ | $2K_{AY}$ | $K_{BY}$ | | | | $-W_Yh_Y \div 6E$ |
| V | | | | | | | | | | | | $K_{AY}$ | | $-3K_{AY}$ | $J_{AY}$ | $K_{aY}$ | $-3K_{AZ}$ | $K_{AZ}$ | | 0 |
| W | | | | | | | | | | | | | $K_{BY}$ | $-3K_{BY}$ | $2K_{aY}$ | $J_{BY}$ | $-3K_{BZ}$ | | $K_{BZ}$ | 0 |
| X | | | | | | | | | | | | | | | $2K_{AZ}$ | $K_{BZ}$ | $-N_Z$ | $2K_{AZ}$ | $K_{BZ}$ | $-W_Zh_Z \div 6E$ |
| Y | | | | | | | | | | | | | | | $K_{AZ}$ | | $-3K_{AZ}$ | $J_{AZ}$ | $K_{aZ}$ | 0 |
| Z | | | | | | | | | | | | | | | | $K_{BZ}$ | $-3K_{BZ}$ | $2K_{aZ}$ | $J_{BZ}$ | 0 |

## TABLE 3.

## GENERAL EQUATIONS FOR A SYMMETRICAL THREE-SPAN BENT ANY NUMBER OF STORIES HIGH.

| Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | | Story No. 2 | | | Story No. 3 | | | Intervening Stories | Story No. X (Second from Top) | | | Story No. Y (Next to Top) | | | Story No. Z (Top) | | | |
| | $R_1$ | $\theta_{A1}$ | $\theta_{B1}$ | $R_2$ | $\theta_{A2}$ | $\theta_{B2}$ | $R_3$ | $\theta_{A3}$ | $\theta_{B3}$ | | $R_X$ | $\theta_{AX}$ | $\theta_{BX}$ | $R_Y$ | $\theta_{AY}$ | $\theta_{BY}$ | $R_Z$ | $\theta_{AZ}$ | $\theta_{BZ}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | | | | | |
| A | $-N_1$ | $2K_{A1}$ | $2K_{B1}$ | | | | | | | | | | | | | | | | | $-W_1 h_1 \div 6E$ |
| B | $-3K_{A1}$ | $J_{A1}$ | $K_{a1}$ | $-3K_{A2}$ | $K_{A2}$ | | | | | | | | | | | | | | | 0 |
| C | $-3K_{B1}$ | $K_{a1}$ | $J_{B1}+K_{b1}$ | $-3K_{B2}$ | | $K_{B2}$ | | | | | | | | | | | | | | 0 |
| D | | $2K_{A2}$ | $2K_{B2}$ | $-N_2$ | $2K_{A2}$ | $2K_{B2}$ | | | | | | | | | | | | | | $-W_2 h_2 \div 6E$ |
| E | | $K_{A2}$ | | $-3K_{A2}$ | $J_{A2}$ | $K_{a2}$ | $-3K_{A3}$ | $K_{A3}$ | | | | | | | | | | | | 0 |
| F | | | $K_{B2}$ | $-3K_{B2}$ | $K_{a2}$ | $J_{B2}+K_{b2}$ | $-3K_{B3}$ | | $K_{B3}$ | | | | | | | | | | | 0 |
| Similar equations for intervening stories | | | | | | | | | | | | | | | | | | | | |
| U | | | | | | | | | | | | $2K_{AY}$ | $2K_{BY}$ | $-N_Y$ | $2K_{AY}$ | $2K_{BY}$ | | | | $-W_Y h_Y \div 6E$ |
| V | | | | | | | | | | | | $K_{AY}$ | | $-3K_{AY}$ | $J_{AY}$ | $K_{aY}$ | $-3K_{AZ}$ | $K_{AZ}$ | | 0 |
| W | | | | | | | | | | | | | $K_{BY}$ | $-3K_{BY}$ | $K_{aY}$ | $J_{BY}+K_{bY}$ | $-3K_{BZ}$ | | $K_{BZ}$ | 0 |
| X | | | | | | | | | | | | | | | $2K_{AZ}$ | $2K_{BZ}$ | $-N_Z$ | $2K_{AZ}$ | $2K_{BZ}$ | $-W_Z h_Z \div 6E$ |
| Y | | | | | | | | | | | | | | | $K_{AZ}$ | | $-3K_{AZ}$ | $J_A$ | $K_{aZ}$ | 0 |
| Z | | | | | | | | | | | | | | | | $K_{BZ}$ | $-3K_{BZ}$ | $K_{aZ}$ | $J_{BZ}+K_{bZ}$ | 0 |

# TABLE 4.

## GENERAL EQUATIONS FOR A SYMMETRICAL FOUR-SPAN BENT ANY NUMBER OF STORIES HIGH.

| Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | | | Story No. 2 | | | | Story No. 3 | | | | Intervening Stories | Story No. X (Second from Top) | | | | Story No. Y (Next to Top) | | | | Story No. Z (Top) | | | | |
| | $R_1$ | $\theta_{A1}$ | $\theta_{B1}$ | $\theta_{C1}$ | $R_2$ | $\theta_{A2}$ | $\theta_{B2}$ | $\theta_{C2}$ | $R_3$ | $\theta_{A3}$ | $\theta_{B3}$ | $\theta_{C3}$ | | $R_X$ | $\theta_{AX}$ | $\theta_{BX}$ | $\theta_{CX}$ | $R_Y$ | $\theta_{AY}$ | $\theta_{BY}$ | $\theta_{CY}$ | $R_Z$ | $\theta_{AZ}$ | $\theta_{BZ}$ | $\theta_{CZ}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | $-N_1$ | $2K_{A1}$ | $2K_{B1}$ | $K_{C1}$ | | | | | | | | | | | | | | | | | | | | | | $-W_1 h_1 \div 6E$ |
| B | $-3K_{A1}$ | $J_{A1}$ | $K_{a1}$ | | $-3K_{A2}$ | $K_{A2}$ | | | | | | | | | | | | | | | | | | | | O |
| C | $-3K_{B1}$ | $K_{a1}$ | $J_{B1}$ | $K_{b1}$ | $-3K_{B2}$ | | $K_{B2}$ | | | | | | | | | | | | | | | | | | | O |
| D | $-3K_{C1}$ | | $2K_{b1}$ | $J_{C1}$ | $-3K_{C2}$ | | | $K_{C2}$ | | | | | | | | | | | | | | | | | | O |
| E | | $2K_{A2}$ | $2K_{B2}$ | $K_{C2}$ | $-N_2$ | $2K_{A2}$ | $2K_{B2}$ | $K_{C2}$ | | | | | | | | | | | | | | | | | | $-W_2 h_2 \div 6E$ |
| F | | $K_{A2}$ | | | $-3K_{A2}$ | $J_{A2}$ | $K_{a2}$ | | $-3K_{A3}$ | $K_{A3}$ | | | | | | | | | | | | | | | | O |
| G | | | $K_{B2}$ | | $-3K_{B2}$ | $K_{a2}$ | $J_{B2}$ | $K_{b2}$ | $-3K_{B3}$ | | $K_{B3}$ | | | | | | | | | | | | | | | O |
| H | | | | $K_{C2}$ | $-3K_{C2}$ | | $2K_{b2}$ | $J_{C2}$ | $-3K_{C3}$ | | | $K_{C3}$ | | | | | | | | | | | | | | O |
| | Similar equations for intervening stories | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | | | | | | | | | | | | | | $2K_{AY}$ | $2K_{BY}$ | $K_{CY}$ | $-N_Y$ | $2K_{AY}$ | $2K_{BY}$ | $K_{CY}$ | | | | | $-W_Y h_Y \div 6E$ |
| T | | | | | | | | | | | | | | | $K_{AY}$ | | | $-3K_{AY}$ | $J_{AY}$ | $K_{aY}$ | | $-3K_{AZ}$ | $K_{AZ}$ | | | O |
| U | | | | | | | | | | | | | | | | $K_{BY}$ | | $-3K_{BY}$ | $K_{aY}$ | $J_{BY}$ | $K_{bY}$ | $-3K_{BZ}$ | | $K_{BZ}$ | | O |
| V | | | | | | | | | | | | | | | | | $K_{CY}$ | $-3K_{CY}$ | | $2K_{bY}$ | $J_{CY}$ | $-3K_{CZ}$ | | | $K_{CZ}$ | O |
| W | | | | | | | | | | | | | | | | | | | $2K_{AZ}$ | $2K_{BZ}$ | $K_{CZ}$ | $-N_Z$ | $2K_{ZV}$ | $2K_{BZ}$ | $K_{CZ}$ | $-W_Z h_Z \div 6E$ |
| X | | | | | | | | | | | | | | | | | | | $K_{AZ}$ | | | $-3K_{AZ}$ | $J_{AZ}$ | $K_{aZ}$ | | O |
| Y | | | | | | | | | | | | | | | | | | | | $K_{BZ}$ | | $-3K_{BZ}$ | $K_{aZ}$ | $J_{BZ}$ | $K_{bZ}$ | O |
| Z | | | | | | | | | | | | | | | | | | | | | $K_{CZ}$ | $-3K_{CZ}$ | | $2K_{bZ}$ | $J_{CZ}$ | O |

# TABLE 5.

## GENERAL EQUATIONS FOR A SYMMETRICAL FIVE-SPAN BENT ANY NUMBER OF STORIES HIGH.

| Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | | | Story No. 2 | | | | Story No. 3 | | | | Intervening Stories | Story No. X (Second from Top) | | | | Story No. Y (Next to Top) | | | | Story No. Z (Top) | | | | |
| | $R_1$ | $\theta_{A1}$ | $\theta_{B1}$ | $\theta_{C1}$ | $R_2$ | $\theta_{A2}$ | $\theta_{B2}$ | $\theta_{C2}$ | $R_3$ | $\theta_{A3}$ | $\theta_{B3}$ | $\theta_{C3}$ | | $R_X$ | $\theta_{AX}$ | $\theta_{BX}$ | $\theta_{CX}$ | $R_Y$ | $\theta_{AY}$ | $\theta_{BY}$ | $\theta_{CY}$ | $R_Z$ | $\theta_{AZ}$ | $\theta_{BZ}$ | $\theta_{CZ}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | $-N_1$ | $2K_{A1}$ | $2K_{B1}$ | $2K_{C1}$ | | | | | | | | | | | | | | | | | | | | | | $-W_1 h_1 \div 6E$ |
| B | $-3K_{A1}$ | $J_{A1}$ | $K_{a1}$ | | $-3K_{A2}$ | $K_{A2}$ | | | | | | | | | | | | | | | | | | | | O |
| C | $-3K_{B1}$ | $K_{a1}$ | $J_{B1}$ | $K_{b1}$ | $-3K_{B2}$ | | $K_{B2}$ | | | | | | | | | | | | | | | | | | | O |
| D | $-3K_{C1}$ | | $K_{b1}$ | $J_{C1}+K_{c1}$ | $-3K_{C2}$ | | | $K_{C2}$ | | | | | | | | | | | | | | | | | | O |
| E | | $2K_{A2}$ | $2K_{B2}$ | $2K_{C2}$ | $-N_2$ | $2K_{A2}$ | $2K_{B2}$ | $2K_{C2}$ | | | | | | | | | | | | | | | | | | $-W_2 h_2 \div 6E$ |
| F | | $K_{A2}$ | | | $-3K_{A2}$ | $J_{A2}$ | $K_{a2}$ | | $-3K_{A3}$ | $K_{A3}$ | | | | | | | | | | | | | | | | O |
| G | | | $K_{B2}$ | | $-3K_{B2}$ | $K_{a2}$ | $J_{B2}$ | $K_{b2}$ | $-3K_{B3}$ | | $K_{B3}$ | | | | | | | | | | | | | | | O |
| H | | | | $K_{C2}$ | $-3K_{C2}$ | | $K_{b2}$ | $J_{C2}+K_{c2}$ | $-3K_{C3}$ | | | $K_{C3}$ | | | | | | | | | | | | | | O |
| | Similar equations for intervening stories | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | | | | | | | | | | | | | | $2K_{AY}$ | $2K_{BY}$ | $2K_{CY}$ | $-N_Y$ | $2K_{AY}$ | $2K_{BY}$ | $2K_{CY}$ | | | | | $-W_Y h_Y \div 6E$ |
| T | | | | | | | | | | | | | | | $K_{AY}$ | | | $-3K_{AY}$ | $J_{AY}$ | $K_{aY}$ | | $-3K_{AZ}$ | $K_{AZ}$ | | | O |
| U | | | | | | | | | | | | | | | | $K_{BY}$ | | $-3K_{BY}$ | $K_{aY}$ | $J_{BY}$ | $K_{bY}$ | $-3K_{BZ}$ | | $K_{BZ}$ | | O |
| V | | | | | | | | | | | | | | | | | $K_{CY}$ | $-3K_{CY}$ | | $K_{bY}$ | $J_{CY}+K_{cY}$ | $-3K_{CZ}$ | | | $K_{CZ}$ | O |
| W | | | | | | | | | | | | | | | | | | | $2K_{AZ}$ | $2K_{BZ}$ | $2K_{CZ}$ | $-N_Z$ | $2K_{AZ}$ | $2K_{BZ}$ | $2K_{CZ}$ | $-W_Z h_Z \div 6E$ |
| X | | | | | | | | | | | | | | | | | | | $K_{AZ}$ | | | $-3K_{AZ}$ | $J_{AZ}$ | $K_{aZ}$ | | O |
| Y | | | | | | | | | | | | | | | | | | | | $K_{BZ}$ | | $-3K_{BZ}$ | $K_{aZ}$ | $J_{BZ}$ | $K_{bZ}$ | O |
| Z | | | | | | | | | | | | | | | | | | | | | $K_{CZ}$ | $-3K_{CZ}$ | | $K_{bZ}$ | $J_{CZ}+K_{cZ}$ | O |

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## TABLE 6.

## GENERAL EQUATIONS FOR AN UNSYMMETRICAL SINGLE-SPAN BENT ANY NUMBER OF STORIES HIGH.

| Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | | Story No. 2 | | | Story No. 3 | | | Intervening Stories | Story No. X (Second from Top) | | | Story No. Y (Next to Top) | | | Story No. Z (Top) | | | |
| | $R_1$ | $\theta_{A1}$ | $\theta_{B1}$ | $R_2$ | $\theta_{A2}$ | $\theta_{B2}$ | $R_3$ | $\theta_{A3}$ | $\theta_{B3}$ | | $R_X$ | $\theta_{AX}$ | $\theta_{BX}$ | $R_Y$ | $\theta_{AY}$ | $\theta_{BY}$ | $R_Z$ | $\theta_{AZ}$ | $\theta_{BZ}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | | | | | |
| A | $-N_1$ | $K_{A1}$ | $K_{B1}$ | | | | | | | | | | | | | | | | | $-W_1h_1 \div 6E$ |
| B | $-3K_{A1}$ | $J_{A1}$ | $K_{a1}$ | $-3K_{A2}$ | $K_{A2}$ | | | | | | | | | | | | | | | 0 |
| C | $-3K_{B1}$ | $K_{a1}$ | $J_{B1}$ | $-3K_{B2}$ | | $K_{B2}$ | | | | | | | | | | | | | | 0 |
| D | | $K_{A2}$ | $K_{B2}$ | $-N_2$ | $K_{A2}$ | $K_{B2}$ | | | | | | | | | | | | | | $-W_2h_2 \div 6E$ |
| E | | $K_{A2}$ | | $-3K_{A2}$ | $J_{A2}$ | $K_{a2}$ | $-3K_{A3}$ | $K_{A3}$ | | | | | | | | | | | | 0 |
| F | | | $K_{B2}$ | $-3K_{B2}$ | $K_{a2}$ | $J_{B2}$ | $-3K_{B3}$ | | $K_{B3}$ | | | | | | | | | | | 0 |
| | Similar equations for intervening stories | | | | | | | | | | | | | | | | | | | |
| U | | | | | | | | | | | | $K_{AY}$ | $K_{BY}$ | $-N_Y$ | $K_{AY}$ | $K_{BY}$ | | | | $-W_Yh_Y \div 6E$ |
| V | | | | | | | | | | | | $K_{AY}$ | | $-3K_{AY}$ | $J_{AY}$ | $K_{aY}$ | $-3K_{AZ}$ | $K_{AZ}$ | | 0 |
| W | | | | | | | | | | | | | $K_{BY}$ | $-3K_{BY}$ | $K_{aY}$ | $J_{BY}$ | $-3K_{BZ}$ | | $K_{BZ}$ | 0 |
| X | | | | | | | | | | | | | | | $K_{AZ}$ | $K_{BZ}$ | $-N_Z$ | $K_{AZ}$ | $K_{BZ}$ | $-W_Zh_Z \div 6E$ |
| Y | | | | | | | | | | | | | | | $K_{AZ}$ | | $-3K_{AZ}$ | $J_{AZ}$ | $K_{aZ}$ | 0 |
| Z | | | | | | | | | | | | | | | | $K_{BZ}$ | $-3K_{BZ}$ | $K_{aZ}$ | $J_{BZ}$ | 0 |

# TABLE 7.

## GENERAL EQUATIONS FOR AN UNSYMMETRICAL TWO-SPAN BENT ANY NUMBER OF STORIES HIGH.

| Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | | | Story No. 2 | | | | Story No. 3 | | | | Intervening Stories | Story No. X (Second from Top) | | | | Story No. Y (Next to Top) | | | | Story No Z (Top) | | | | |
| | $R_1$ | $\theta_{A1}$ | $\theta_{B1}$ | $\theta_{C1}$ | $R_2$ | $\theta_{A2}$ | $\theta_{B2}$ | $\theta_{C2}$ | $R_3$ | $\theta_{A3}$ | $\theta_{B3}$ | $\theta_{C3}$ | | $R_X$ | $\theta_{AX}$ | $\theta_{BX}$ | $\theta_{CX}$ | $R_Y$ | $\theta_{AY}$ | $\theta_{BY}$ | $\theta_{CY}$ | $R_Z$ | $\theta_{AZ}$ | $\theta_{BZ}$ | $\theta_{CZ}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | $-N_1$ | $K_{A1}$ | $K_{B1}$ | $K_{C1}$ | | | | | | | | | | | | | | | | | | | | | | $-W_1 h_1 \div 6E$ |
| B | $-3K_{A1}$ | $J_{A1}$ | $K_{a1}$ | | $-3K_{A2}$ | $K_{A2}$ | | | | | | | | | | | | | | | | | | | | 0 |
| C | $-3K_{B1}$ | $K_{a1}$ | $J_{B1}$ | $K_{b1}$ | $-3K_{B2}$ | | $K_{B2}$ | | | | | | | | | | | | | | | | | | | 0 |
| D | $-3K_{C1}$ | | $K_{b1}$ | $J_{C1}$ | $-3K_{C2}$ | | | $K_{C2}$ | | | | | | | | | | | | | | | | | | 0 |
| E | | $K_{A2}$ | $K_{B2}$ | $K_{C2}$ | $-N_2$ | $K_{A2}$ | $K_{B2}$ | $K_{C2}$ | | | | | | | | | | | | | | | | | | $-W_2 h_2 \div 6E$ |
| F | | $K_{A2}$ | | | $-3K_{A2}$ | $J_{A2}$ | $K_{a2}$ | | $-3K_{A3}$ | $K_{A3}$ | | | | | | | | | | | | | | | | 0 |
| G | | | $K_{B2}$ | | $-3K_{B2}$ | $K_{a2}$ | $J_{B2}$ | $K_{b2}$ | $-3K_{B3}$ | | $K_{B3}$ | | | | | | | | | | | | | | | 0 |
| H | | | | $K_{C2}$ | $-3K_{C2}$ | | $K_{B2}$ | $J_{C2}$ | $-3K_{C3}$ | | | $K_{C3}$ | | | | | | | | | | | | | | 0 |
| | Similar equations for intervening stories | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | | | | | | | | | | | | | | $K_{AY}$ | $K_{BY}$ | $K_{CY}$ | $-N_Y$ | $K_{AY}$ | $K_{BY}$ | $K_{CY}$ | | | | | $-W_Y h_Y \div 6E$ |
| T | | | | | | | | | | | | | | | $K_{AY}$ | | | $-3K_{AY}$ | $J_{AY}$ | $K_{aY}$ | | $-3K_{AZ}$ | $K_{AZ}$ | | | 0 |
| U | | | | | | | | | | | | | | | | $K_{BY}$ | | $-3K_{BY}$ | $K_{aY}$ | $K_{BY}$ | $K_{bY}$ | $-3K_{BZ}$ | | $K_{BZ}$ | | 0 |
| V | | | | | | | | | | | | | | | | | $K_{CY}$ | $-3K_{CY}$ | | $K_{bY}$ | $J_{CY}$ | $-3K_{CZ}$ | | | $K_{CZ}$ | 0 |
| W | | | | | | | | | | | | | | | | | | | $K_{AZ}$ | $K_{BZ}$ | $K_{CZ}$ | $-N_Z$ | $K_{AZ}$ | $K_{BZ}$ | $K_{CZ}$ | $-W_Z h_Z \div 6E$ |
| X | | | | | | | | | | | | | | | | | | | $K_{AZ}$ | | | $-3K_{AZ}$ | $J_{AZ}$ | $K_{aZ}$ | | 0 |
| Y | | | | | | | | | | | | | | | | | | | | $K_{BZ}$ | | $-3K_{BZ}$ | $K_{aZ}$ | $J_{BZ}$ | $K_{bZ}$ | 0 |
| Z | | | | | | | | | | | | | | | | | | | | | $K_{CZ}$ | $-3K_{CZ}$ | | $K_{bZ}$ | $J_{CZ}$ | 0 |

# TABLE 8.

## GENERAL EQUATIONS FOR AN UNSYMMETRICAL THREE-SPAN BENT ANY NUMBER OF STORIES HIGH.

| Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | | | | Story No. 2 | | | | | Story No. 3 | | | | | Intervening Stories | Story No. X (Second from Top) | | | | | Story No. Y (Next to Top) | | | | | Story No. Z (Top) | | | | | |
| | $R_1$ | $\theta_{A1}$ | $\theta_{B1}$ | $\theta_{C1}$ | $\theta_{D1}$ | $R_2$ | $\theta_{A2}$ | $\theta_{B2}$ | $\theta_{C2}$ | $\theta_{D2}$ | $R_3$ | $\theta_{A3}$ | $\theta_{B3}$ | $\theta_{C3}$ | $\theta_{D3}$ | | $R_X$ | $\theta_{AX}$ | $\theta_{BX}$ | $\theta_{CX}$ | $\theta_{DX}$ | $R_Y$ | $\theta_{AY}$ | $\theta_{BY}$ | $\theta_{CY}$ | $\theta_{DY}$ | $R_Z$ | $\theta_{AZ}$ | $\theta_{BZ}$ | $\theta_{CZ}$ | $\theta_{DZ}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | $-N_1$ | $K_{A1}$ | $K_{B1}$ | $K_{C1}$ | $K_{D1}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | $-W_1 h_1 \div 6E$ |
| B | $-3K_{A1}$ | $J_{A1}$ | $K_{a1}$ | | | $-3K_{A2}$ | $K_{A2}$ | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| C | $-3K_{B1}$ | $K_{a1}$ | $J_{B1}$ | $K_{b1}$ | | $-3K_{B2}$ | | $K_{B2}$ | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| D | $-3K_{C1}$ | | $K_{b1}$ | $J_{C1}$ | $K_{c1}$ | $-3K_{C2}$ | | | $K_{C2}$ | | | | | | | | | | | | | | | | | | | | | | | 0 |
| E | $-3K_{D1}$ | | | $K_{c1}$ | $J_{D1}$ | $-3K_{D2}$ | | | | $K_{D2}$ | | | | | | | | | | | | | | | | | | | | | | 0 |
| F | | $K_{A2}$ | $K_{B2}$ | $K_{C2}$ | $K_{D2}$ | $-N_2$ | $K_{A2}$ | $K_{B2}$ | $K_{C2}$ | $K_{D2}$ | | | | | | | | | | | | | | | | | | | | | | $-W_2 h_2 \div 6E$ |
| G | | $K_{A2}$ | | | | $-3K_{A2}$ | $J_{A2}$ | $K_{a2}$ | | | $-3K_{A3}$ | $K_{A3}$ | | | | | | | | | | | | | | | | | | | | 0 |
| H | | | $K_{B2}$ | | | $-3K_{B2}$ | $K_{a2}$ | $J_{B2}$ | $K_{b2}$ | | $-3K_{B3}$ | | $K_{B3}$ | | | | | | | | | | | | | | | | | | | 0 |
| I | | | | $K_{C2}$ | | $-3K_{C2}$ | | $K_{b2}$ | $J_{C2}$ | $K_{c2}$ | $-3K_{C3}$ | | | $K_{C3}$ | | | | | | | | | | | | | | | | | | 0 |
| J | | | | | $K_{D2}$ | $-3K_{D2}$ | | | $K_{c2}$ | $J_{D2}$ | $-3K_{D3}$ | | | | $K_{D3}$ | | | | | | | | | | | | | | | | | 0 |
| | Similar equations for intervening stories | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q | | | | | | | | | | | | | | | | | | $K_{AY}$ | $K_{BY}$ | $K_{CY}$ | $K_{DY}$ | $-N_Y$ | $K_{AY}$ | $K_{BY}$ | $K_{CY}$ | $K_{DY}$ | | | | | | $-W_Y h_Y \div 6E$ |
| R | | | | | | | | | | | | | | | | | | $K_{AY}$ | | | | $-3K_{AY}$ | $J_{AY}$ | $K_{aY}$ | | | $-3K_{AZ}$ | $K_{AZ}$ | | | | 0 |
| S | | | | | | | | | | | | | | | | | | | $K_{BY}$ | | | $-3K_{BY}$ | $K_{aY}$ | $J_{BY}$ | $K_{bY}$ | | $-3K_{BZ}$ | | $K_{BZ}$ | | | 0 |
| T | | | | | | | | | | | | | | | | | | | | $K_{CY}$ | | $-3K_{CY}$ | | $K_{bY}$ | $J_{CY}$ | $K_{cY}$ | $-3K_{CZ}$ | | | $K_{CZ}$ | | 0 |
| U | | | | | | | | | | | | | | | | | | | | | $K_{DY}$ | $-3K_{DY}$ | | | $K_{cY}$ | $J_{DY}$ | $-3K_{DZ}$ | | | | $K_{DZ}$ | 0 |
| V | | | | | | | | | | | | | | | | | | | | | | | $K_{AZ}$ | $K_{BZ}$ | $K_{CZ}$ | $K_{DZ}$ | $-N_Z$ | $K_{AZ}$ | $K_{BZ}$ | $K_{CZ}$ | $K_{DZ}$ | $-W_Z h_Z \div 6E$ |
| W | | | | | | | | | | | | | | | | | | | | | | | $K_{AZ}$ | | | | $-3K_{AZ}$ | $J_{AZ}$ | $K_{aZ}$ | | | 0 |
| X | | | | | | | | | | | | | | | | | | | | | | | | $K_{BZ}$ | | | $-3K_{BZ}$ | $K_{aZ}$ | $J_{BZ}$ | $K_{bZ}$ | | 0 |
| Y | | | | | | | | | | | | | | | | | | | | | | | | | $K_{CZ}$ | | $-3K_{CZ}$ | | $K_{bZ}$ | $J_{CZ}$ | $K_{cZ}$ | 0 |
| Z | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{DZ}$ | $-3K_{DZ}$ | | | $K_{cZ}$ | $J_{DZ}$ | 0 |

## TABLE 9.

## GENERAL EQUATIONS FOR AN UNSYMMETRICAL FOUR-SPAN BENT ANY NUMBER OF STORIES HIGH.

| Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | | | | | Story No. 2 | | | | | | Story No. 3 | | | | | | Intervening Stories | Story No. X (Second from Top) | | | | | | Story No. Y (Next to Top) | | | | | | Story No. Z (Top) | | | | | | |
| | $R_1$ | $\theta_{A1}$ | $\theta_{B1}$ | $\theta_{C1}$ | $\theta_{D1}$ | $\theta_{E1}$ | $R_2$ | $\theta_{A2}$ | $\theta_{B2}$ | $\theta_{C2}$ | $\theta_{D2}$ | $\theta_{E2}$ | $R_3$ | $\theta_{A3}$ | $\theta_{B3}$ | $\theta_{C3}$ | $\theta_{D3}$ | $\theta_{E3}$ | | $R_X$ | $\theta_{AX}$ | $\theta_{BX}$ | $\theta_{CX}$ | $\theta_{DX}$ | $\theta_{EX}$ | $R_Y$ | $\theta_{AY}$ | $\theta_{BY}$ | $\theta_{CY}$ | $\theta_{DY}$ | $\theta_{EY}$ | $R_Z$ | $\theta_{AZ}$ | $\theta_{BZ}$ | $\theta_{CZ}$ | $\theta_{DZ}$ | $\theta_{EZ}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | $-N_1$ | $K_{A1}$ | $K_{A1}$ | $K_{C1}$ | $K_{D1}$ | $K_{F1}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $-W_1h_1\div 6E$ |
| B | $-3K_{A1}$ | $J_{A1}$ | $K_{a1}$ | | | | $-3K_{A2}$ | $K_{A2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| C | $-3K_{B1}$ | $K_{a1}$ | $J_{B1}$ | $K_{b1}$ | | | $-3K_{B2}$ | | $K_{B2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| D | $-3K_{C1}$ | | $K_{b1}$ | $J_{C1}$ | $K_{c1}$ | | $-3K_{C2}$ | | | $K_{C2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| E | $-3K_{D1}$ | | | $K_{c1}$ | $J_{D1}$ | $K_{d1}$ | $-3K_{D2}$ | | | | $K_{D2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| F | $-3K_{E1}$ | | | | $K_{d1}$ | $J_{E1}$ | $-3K_{E2}$ | | | | | $K_{E2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| G | | $K_{A2}$ | $K_{B2}$ | $K_{C2}$ | $K_{D2}$ | $K_{F2}$ | $-N_2$ | $K_{A2}$ | $K_{B2}$ | $K_{C2}$ | $K_{D2}$ | $K_{E2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | $-W_2h_2\div 6E$ |
| H | | $K_{A2}$ | | | | | $-3K_{A2}$ | $J_{A2}$ | $K_{a2}$ | | | | $-3K_{A3}$ | $K_{A3}$ | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| I | | | $K_{B2}$ | | | | $-3K_{B2}$ | $K_{a2}$ | $J_{B2}$ | $K_{b2}$ | | | $-3K_{B3}$ | | $K_{B3}$ | | | | | | | | | | | | | | | | | | | | | | | 0 |
| J | | | | $K_{C2}$ | | | $-3K_{C2}$ | | $K_{b2}$ | $J_{C2}$ | $K_{c2}$ | | $-3K_{C3}$ | | | $K_{C3}$ | | | | | | | | | | | | | | | | | | | | | | 0 |
| K | | | | | $K_{D2}$ | | $-3K_{D2}$ | | | $K_{c2}$ | $J_{D2}$ | $K_{d2}$ | $-3K_{D3}$ | | | | $K_{D3}$ | | | | | | | | | | | | | | | | | | | | | 0 |
| L | | | | | | $K_{F2}$ | $-3K_{E2}$ | | | | $K_{d2}$ | $J_{E2}$ | $-3K_{E3}$ | | | | | $K_{E3}$ | | | | | | | | | | | | | | | | | | | | 0 |
| | Similar equations for intervening stories | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| O | | | | | | | | | | | | | | | | | | | | | $K_{AY}$ | $K_{BY}$ | $K_{CY}$ | $K_{DY}$ | $K_{EY}$ | $-N_Y$ | $K_{AY}$ | $K_{BY}$ | $K_{CY}$ | $K_{DY}$ | $K_{EY}$ | | | | | | | $-W_Yh_Y\div 6E$ |
| P | | | | | | | | | | | | | | | | | | | | | $K_{AY}$ | | | | | $-3K_{AY}$ | $J_{AY}$ | $K_{aY}$ | | | | $-3K_{AZ}$ | $K_{AZ}$ | | | | | 0 |
| Q | | | | | | | | | | | | | | | | | | | | | | $K_{BY}$ | | | | $-3K_{BY}$ | $K_{aY}$ | $J_{BY}$ | $K_{bY}$ | | | $-3K_{BZ}$ | | $K_{BZ}$ | | | | 0 |
| R | | | | | | | | | | | | | | | | | | | | | | | $K_{CY}$ | | | $-3K_{CY}$ | | $K_{bY}$ | $J_{CY}$ | $K_{cY}$ | | $-3K_{CZ}$ | | | $K_{CZ}$ | | | 0 |
| S | | | | | | | | | | | | | | | | | | | | | | | | $K_{DY}$ | | $-3K_{DY}$ | | | $K_{cY}$ | $J_{DY}$ | $K_{dY}$ | $-3K_{DZ}$ | | | | $K_{DZ}$ | | 0 |
| T | | | | | | | | | | | | | | | | | | | | | | | | | $K_{EY}$ | $-3K_{EY}$ | | | | $K_{dY}$ | $J_{EY}$ | $-3K_{EZ}$ | | | | | $K_{EZ}$ | 0 |
| U | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{AZ}$ | $K_{BZ}$ | $K_{CZ}$ | $K_{DZ}$ | $K_{EZ}$ | $-N_Z$ | $K_{AZ}$ | $K_{BZ}$ | $K_{CZ}$ | $K_{DZ}$ | $K_{EZ}$ | $-W_Zh_Z\div 6E$ |
| V | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{AZ}$ | | | | | $-3K_{AZ}$ | $J_{AZ}$ | $K_{aZ}$ | | | | 0 |
| W | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{BZ}$ | | | | $-3K_{BZ}$ | $K_{aZ}$ | $J_{BZ}$ | $K_{bZ}$ | | | 0 |
| X | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{CZ}$ | | | $-3K_{CZ}$ | | $K_{bZ}$ | $J_{CZ}$ | $K_{cZ}$ | | 0 |
| Y | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{DZ}$ | | $-3K_{DZ}$ | | | $K_{cZ}$ | $J_{DZ}$ | $K_{dZ}$ | 0 |
| Z | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{EZ}$ | $-3K_{EZ}$ | | | | $K_{dZ}$ | $J_{EZ}$ | 0 |

## TABLE 10.

## GENERAL EQUATIONS FOR AN UNSYMMETRICAL FIVE-SPAN BENT
## ANY NUMBER OF STORIES HIGH.

| Equation | Left Hand Member of Equation | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | | | | | | Story No. 2 | | | | | | | Story No. 3 | | | | | | | Intervening Stories | Story No. X (Second from Top) | | | | | | | Story No. Y (Next to Top) | | | | | | | Story No. Z (Top) | | | | | | | |
| | $R_1$ | $\theta_{A1}$ | $\theta_{B1}$ | $\theta_{C1}$ | $\theta_{D1}$ | $\theta_{E1}$ | $\theta_{F1}$ | $R_2$ | $\theta_{A2}$ | $\theta_{B2}$ | $\theta_{C2}$ | $\theta_{D2}$ | $\theta_{E2}$ | $\theta_{F2}$ | $R_3$ | $\theta_{A3}$ | $\theta_{B3}$ | $\theta_{C3}$ | $\theta_{D3}$ | $\theta_{E3}$ | $\theta_{F3}$ | | $R_X$ | $\theta_{AX}$ | $\theta_{BX}$ | $\theta_{CX}$ | $\theta_{DX}$ | $\theta_{EX}$ | $\theta_{FX}$ | $R_Y$ | $\theta_{AY}$ | $\theta_{BY}$ | $\theta_{CY}$ | $\theta_{DY}$ | $\theta_{EY}$ | $\theta_{FY}$ | $R_Z$ | $\theta_{AZ}$ | $\theta_{BZ}$ | $\theta_{CZ}$ | $\theta_{DZ}$ | $\theta_{EZ}$ | $\theta_{FZ}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | $-N_1$ | $K_{A1}$ | $K_{B1}$ | $K_{C1}$ | $K_{D1}$ | $K_{E1}$ | $K_{F1}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $-W_1 h_1 \div 6E$ |
| B | $-3K_{A1}$ | $J_{A1}$ | $K_{a1}$ | | | | | $-3K_{A2}$ | $K_{A2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| C | $-3K_{B1}$ | $K_{a1}$ | $J_{B1}$ | $K_{b1}$ | | | | $-3K_{B2}$ | | $K_{B2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| D | $-3K_{C1}$ | | $K_{b1}$ | $J_{C1}$ | $K_{c1}$ | | | $-3K_{C2}$ | | | $K_{C2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| E | $-3K_{D1}$ | | | $K_{c1}$ | $J_{D1}$ | $K_{d1}$ | | $-3K_{D2}$ | | | | $K_{D2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| F | $-3K_{E1}$ | | | | $K_{d1}$ | $J_{E1}$ | $K_{e1}$ | $-3K_{E2}$ | | | | | $K_{E2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| G | $-3K_{F1}$ | | | | | $K_{e1}$ | $J_{F1}$ | $-3K_{F2}$ | | | | | | $K_{F2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| H | | $K_{A2}$ | $K_{B2}$ | $K_{C2}$ | $K_{D2}$ | $K_{E2}$ | $K_{F2}$ | $-N_2$ | $K_{A2}$ | $K_{B2}$ | $K_{C2}$ | $K_{D2}$ | $K_{E2}$ | $K_{F2}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $-W_2 h_2 \div 6E$ |
| I | | $K_{A2}$ | | | | | | $-3K_{A2}$ | $J_{A2}$ | $K_{a2}$ | | | | | $-3K_{A3}$ | $K_{A3}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| J | | | $K_{B2}$ | | | | | $-3K_{B2}$ | $K_{a2}$ | $J_{B2}$ | $K_{b2}$ | | | | $-3K_{B3}$ | | $K_{B3}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| K | | | | $K_{C2}$ | | | | $-3K_{C2}$ | | $K_{b2}$ | $J_{C2}$ | $K_{c2}$ | | | $-3K_{C3}$ | | | $K_{C3}$ | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| L | | | | | $K_{D2}$ | | | $-3K_{D2}$ | | | $K_{c2}$ | $J_{D2}$ | $K_{d2}$ | | $-3K_{D3}$ | | | | $K_{D3}$ | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| M | | | | | | $K_{E2}$ | | $-3K_{F2}$ | | | | $K_{d2}$ | $J_{E2}$ | $K_{e2}$ | $-3K_{E3}$ | | | | | $K_{E3}$ | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| N | | | | | | | $K_{F2}$ | $-3K_{F2}$ | | | | | $K_{e2}$ | $J_{F2}$ | $-3K_{F3}$ | | | | | | $K_{F3}$ | | | | | | | | | | | | | | | | | | | | | | | 0 |
| | Similar equations for intervening stories | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| M | | | | | | | | | | | | | | | | | | | | | | | | $K_{AY}$ | $K_{BY}$ | $K_{CY}$ | $K_{DY}$ | $K_{EY}$ | $K_{FY}$ | $-N_Y$ | $K_{AY}$ | $K_{BY}$ | $K_{CY}$ | $K_{DY}$ | $K_{EY}$ | $K_{FY}$ | | | | | | | | $-W_Y h_Y \div 6E$ |
| N | | | | | | | | | | | | | | | | | | | | | | | | $K_{AY}$ | | | | | | $-3K_{AY}$ | $J_{AY}$ | $K_{aY}$ | | | | | $-3K_{AZ}$ | $K_{AZ}$ | | | | | | 0 |
| O | | | | | | | | | | | | | | | | | | | | | | | | | $K_{BY}$ | | | | | $-3K_{BY}$ | $K_{aY}$ | $J_{BY}$ | $K_{bY}$ | | | | $-3K_{BZ}$ | | $K_{BZ}$ | | | | | 0 |
| P | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{CY}$ | | | | $-3K_{CY}$ | | $K_{bY}$ | $J_{CY}$ | $K_{cY}$ | | | $-3K_{CZ}$ | | | $K_{CZ}$ | | | | 0 |
| Q | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{DY}$ | | | $-3K_{DY}$ | | | $K_{cY}$ | $J_{DY}$ | $K_{dY}$ | | $-3K_{DZ}$ | | | | $K_{DZ}$ | | | 0 |
| R | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{EY}$ | | $-3K_{EY}$ | | | | $K_{dY}$ | $J_{EY}$ | $K_{eY}$ | $-3K_{EZ}$ | | | | | $K_{EZ}$ | | 0 |
| S | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{FY}$ | $-3K_{FY}$ | | | | | $K_{eY}$ | $J_{FY}$ | $-3K_{FZ}$ | | | | | | $K_{FZ}$ | 0 |
| T | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{AZ}$ | $K_{BZ}$ | $K_{CZ}$ | $K_{DZ}$ | $K_{EZ}$ | $K_{FZ}$ | $-N_Z$ | $K_{AZ}$ | $K_{BZ}$ | $K_{CZ}$ | $K_{DZ}$ | $K_{EZ}$ | $K_{FZ}$ | $-W_Z h_Z \div 6E$ |
| U | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{AZ}$ | | | | | | $-3K_{AZ}$ | $J_{AZ}$ | $K_{aZ}$ | | | | | 0 |
| V | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{BZ}$ | | | | | $-3K_{BZ}$ | $K_{aZ}$ | $J_{BZ}$ | $K_{bZ}$ | | | | 0 |
| W | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{CZ}$ | | | | $-3K_{CZ}$ | | $K_{bZ}$ | $J_{CZ}$ | $K_{cZ}$ | | | 0 |
| X | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{DZ}$ | | | $-3K_{DZ}$ | | | $K_{cZ}$ | $J_{DZ}$ | $K_{dZ}$ | | 0 |
| Y | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{EZ}$ | | $-3K_{EZ}$ | | | | $K_{dZ}$ | $J_{FZ}$ | $K_{eZ}$ | 0 |
| Z | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | $K_{FZ}$ | $-3K_{FZ}$ | | | | | $K_{eZ}$ | $J_{FZ}$ | 0 |

bent was tested and the measured deflections and changes in the slopes were compared with the same quantities calculated by the above equations. As the entire model was cut from a sheet of celluloid the joints were perfectly rigid. The results of the tests are given in Fig. 15. The fact that the measured and computed deflections and changes in the slopes agree very closely indicates that the above analysis is correct.

A solution of a numerical problem illustrating the use of the equations in Table 3 is given in Section VII.

## VII. NUMERICAL PROBLEM.

13. *Determination of the Stresses in a Symmetrical Three-Span Twenty-Story Bent.*—Fig. 5 shows a symmetrical three-span bent twenty stories high. The bent resists a horizontal wind load of 30 lb. per sq. ft. on a vertical strip one foot wide. It is required to find the moment and the shear in the columns and girders and the direct stress in the columns.

The properties of the girder and column sections are shown in Table 11, page 48. The equations of Table 3 are applicable. The numerical values of the constants in these equations are given in Table 12. Table 13 contains the equations of Table 3 with the substitution of the numerical values of the constants given in Table 12. The figures given in the right-hand column are coefficients of .0001 as indicated at the head of the column; that is, the right-hand members of the equations are equal to .0001 times the numbers in the right-hand column. This method of writing the right-hand members of the equations obviates the necessity of repeating a large number of ciphers.

The unknown quantities in the equations of Table 13 are eliminated in Table 14. The quantity $R_1$ appears in the first three equations of Table 13 only. Dividing each of these equations by the coefficient of $R_1$ gives the three equations A, B, and C of Table 14, in all of which the coefficient of $R_1$ is equal to plus unity. Combining these latter equations as indicated, that is, subtracting equation B from A to get equation $(A-B)$ and subtracting equation C from B to get equation $(B-C)$, gives two new equations from which $R_1$ has been eliminated. Reducing the coefficients of the left-hand terms of equations $(A-B)$ and $(B-C)$ to plus unity gives equations 1 and 2 of Table 14. Equations D and E of Table 13 have also been reduced so that the coefficients of the left-hand term of each are equal to plus unity. They are rewritten in their new form as equations D and E in Table 14. Combining the four equations 1, 2, D, and E, as indicated in Table 14, eliminates

| Wind Load due to a Pressure of 30 lb. per sq. ft acting on a Vertical Strip 1 ft. Wide |  | Story Number. |
|---|---|---|
| Increment on Story = w | Total on all Stories Above = W. |  |
| 360 | 360 | 20 |
| 360 | 720 | 19 |
| 360 | 1080 | 18 |
| 360 | 1440 | 17 |
| 360 | 1800 | 16 |
| 360 | 2160 | 15 |
| 360 | 2520 | 14 |
| 360 | 2880 | 13 |
| 360 | 3240 | 12 |
| 360 | 3600 | 11 |
| 360 | 3960 | 10 |
| 360 | 4320 | 9 |
| 360 | 4680 | 8 |
| 360 | 5040 | 7 |
| 390 | 5430 | 6 |
| 420 | 5850 | 5 |
| 420 | 6270 | 4 |
| 420 | 6690 | 3 |
| 450 | 7140 | 2 |
| 570 | 7710 | 1 |

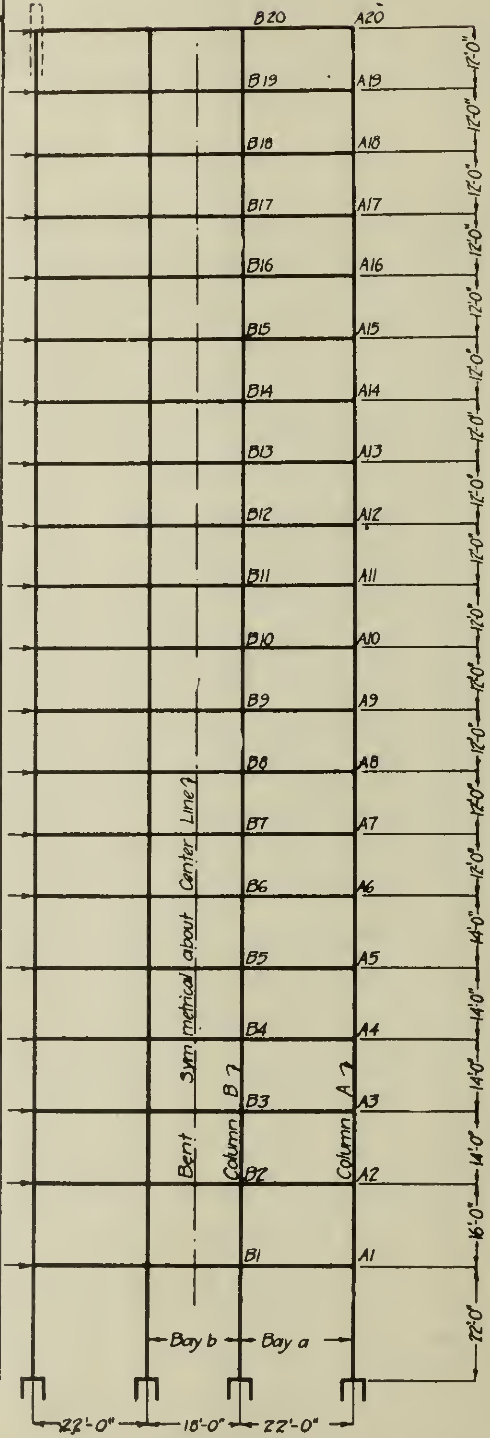

FIG. 5. SYMMETRICAL THREE-SPAN BENT, TWENTY STORIES HIGH.

the left-hand term, $\theta_{A1}$. By continuing this process all the unknown quantities are eliminated except the last, or $\theta_{B20}$. Its value thus becomes known and is found to be .0338 × .0001 radians. With the value of $\theta_{B20}$ known, the value of $\theta_{A20}$ can be determined from equation 153. With $\theta_{B20}$ and $\theta_{A20}$ both known, the value of $R_{20}$ can be determined from equation 150. The other values of $\theta$ and $R$ can be determined in a similar manner.

The process of determining the values of $\theta$ and $R$ is given in Table 15. The equations used are taken from Table 14.

As in Tables 13 and 14, the right-hand member is equal to the number in the column at the right multiplied by .0001. To illustrate, equation 150 is

$$R_{20} - .2702\ \theta_{A20} - .2993\ \theta_{B20} = .0735 \times .0001.$$

The left-hand term of the left-hand member of each equation is the unknown which is to be determined. The first line in each group is the algebraic form of the equation; and the successive lines are the numerical values of the corresponding terms. For example, equation 155 is

$$\theta_{B20} = .0338 \times .0001 \text{ or } .0000338.$$

Again, equation 153 is:

$$\theta_{A20} - .0443\ \theta_{B20} = .0502 \times .0001.$$

From the preceding equation

$$.0443\ \theta_{B20} = .0443 \times .0000338 = .0015 \times .0001.$$

The sign of this quantity as a term of the left-hand member was negative, and the term will, therefore, be positive after it has been transferred.

Equation 153 now becomes

$$\theta_{A20} = .0502 \times .0001 + .0015 \times .0001 = .0517 \times .0001,$$

as indicated in the last line of the group containing equation 153. Equation 150 contains $R_{20}$, $\theta_{A20}$, and $\theta_{B20}$; but $\theta_{A20}$ and $\theta_{B20}$ have been determined by equations 153 and 155, respectively, therefore $R_{20}$ can be determined by equation 150. Similarly, equation 147, containing $\theta_{B19}$ and the three known quantities, $R_{20}$, $\theta_{A20}$, and $\theta_{B20}$ can be used to determine $\theta_{B19}$. Equation 147 differs from the preceding equations in that it contains both positive and negative terms in the left-hand member. The second term of the left-hand member, being negative, when transferred, is added to the right-hand member. The sum of the two quantities is .0890 × .0001. Since the third and fourth terms of the left-hand member are positive, their sum, when transferred, is subtracted from the quantity above. The final form of the equation is: $\theta_{B_{19}} = .0835 \times .0001$. In a similar manner the other slopes, $\theta$, and the ratios of the deflection to the story height, $R$, can be determined. $\theta$ is expressed in radians, and $R$ is an abstract quantity.

The tabulation of the calculations as shown in Tables 14 and 15 facilitates the solution of a large number of equations. There are however a number of practical considerations which should be borne in mind when solving a problem by this method.

An error at any point affects all of the calculations which follow. It is therefore desirable to have two computers carry on the work simultaneously and compare results at frequent intervals in order to avoid the loss of time due to errors.

A number of combinations can be made from each group of equations. It is desirable to combine the equations so as to make the coefficient of the left-hand term of the left-hand member of the resulting equation as large as possible compared with the coefficients of the other terms in the equation. To illustrate this point, consider equations 25, 26, and M of Table 14. If equation M had been subtracted from equation 26, the coefficient of $\theta_{B4}$ in the resulting equation would have been .1463 and the coefficient of $R_5$ would have been 2.9027. The latter is about twenty times as great as the former. Any small error in the actual value of the former coefficient would be a large percentage of error. This same percentage of error would be introduced into the latter coefficient and produce a large actual error. With the combination of equations 25 and M as used, the coefficient of $\theta_{B4}$ is 1.0695 and of $R_5$, 3.4313. The latter coefficient is only about three times as large as the former and hence the effect of an inaccuracy in the value of the former upon the latter is correspondingly smaller than in the first case.

It is possible in some cases to combine the equations in such a way as to give equations which are algebraically independent, but which are numerically nearly identical. Combining such equations gives results which are likely to be inaccurate. Any tendency of the equations to become identities can be avoided by changing the order in which they are combined.

In getting the values of the remaining unknown quantities after one has been determined, each unknown is expressed in terms of the known quantities. By referring to Table 14 it is seen that in the first equation of a group of equations, any one of which could be used to get the value of an unknown, the coefficient of the quantity which is to be determined is larger than the coefficients of the known quantities. If this equation is used, any inaccuracies in the known quantities will affect the accuracy of the results less than they would if an equation had been used in which the coefficient of the unknown is less than the coefficient of the known quantities. The various equations available should be examined and the one used which causes the least accumulation of inaccuracies.

In order to determine the extent to which inaccuracies in the calculations accumulate, two independent sets of calculations were made; one set was made on a 20-inch slide rule and the other on a Fuller's cylindrical slide rule. The two sets of calculations were compared at frequent intervals, and mistakes were corrected; but no adjustment of inaccuracies was made. The maximum variation in the slopes and deflections as determined in the two sets of calculations was very small. This indicates that the calculations can be made with a slide rule without greater inaccuracies than are permissible.

The moments at the ends of the columns and girders can be determined by substituting the values of the deflections and the slopes, given in Table 15, in equations A and B of Section V. To facilitate this work, the quantities which occur in these equations are given in Tables 16 and 17. The values of $R$ and $\theta$ taken from Table 15 are given in the second, third, and fourth columns of Table 16; and the functions of the values of $R$ and $\theta$ which occur in the equations are given in the remaining columns of the same table. The values of $K$ taken from Table 12 are given in the second, third, fourth, and fifth columns of Table 17; and the functions of the values of $K$ which occur in the equations are given in the remaining columns of the table. The moments at the ends of the columns and girders given by the fundamental equation, $M_{AB} = 2EK\,(2\theta_A + \theta_B - 3R)$, are the products of two factors, one of which is given in Table 16 and the other in Table 17. The values of the moments are given in Table 18.

The shear in any member is equal to the algebraic sum of the moments at the two ends of the member divided by its length. The direct stress at any section in column $A$ is equal to the algebraic sum of the shears in the girders in bay $a$ above the section. The direct stress at any section in column $B$ is equal to the algebraic sum of the shears in the girders in bay $a$ plus the algebraic sum of the shears in the girders in bay $b$ above the section. The shears in the columns and girders and the direct stresses in the columns are given in Table 19.

The sum of all of the moments at the top and the bottom of all the columns in a story is equal to the total shear on the story multiplied by the story height. The algebraic sum of the moments in the girders on the two sides of a column is equal to the algebraic sum of the moments in the columns on the two sides of the girder.

The total shear on a story multiplied by the story height and the algebraic sum of all of the moments at the top and the bottom of all columns of a story, as given in Table 18, are given in the second and third columns of Table 20. The algebraic sum of the moments in column *A* at sections immediately above and below the girders and the moments in the girders in bay *a* at sections adjacent to column *A*, are given in the fourth and fifth columns of Table 20. The algebraic sum of the moments in column *B* at sections immediately above and below the girders and the algebraic sum of the moments in the girders in bays *a* and *b* at sections adjacent to column *B*, are given in the sixth and seventh columns of Table 20. The accuracy of the computations can be checked by comparing the two values for the same quantity as given in adjacent columns of Table 20. To illustrate, for the sixth story, the total shear in the story multiplied by the story height is 913,000 inch-pounds, and the sum of the moments at the tops and bottoms of all columns in the story is 914,400 inch-pounds. For a perfect check, the two quantities would be equal. An inspection of the table shows that the values check very closely.

The moments given in Table 18 are due to a horizontal wind load of 30 lbs. per sq. ft. acting on a vertical strip one foot wide. To obtain the moments due to the wind load on any portion of the building, multiply the moments in Table 18 by the width in feet of the portion of the building on which the wind load acts.

## VIII. Approximate Methods.

14. *Nomenclature of Methods.*—The method used in Section VII to determine the stresses in a symmetrical three-span twenty-story bent can be used in the actual design of a building, but a shorter method is desirable. The writers propose an approximate method which is much shorter than the method used in Section VII and which they believe is sufficiently accurate to be used in the actual design of buildings. For the sake of convenience in reference, this approximate method will be designated as the "Proposed Approximate Method" as distinguished from the method used in Section VII, which will be designated as the "Slope-Deflection Method." The slope-deflection method can be modified to advantage under certain special conditions. Such modifications

will be designated as "Modifications of the Slope-Deflection Method."

15. *Proposed Approximate Method.*—The proposed approximate method is based upon the following assumptions in addition to those used in the slope-deflection method.

1. The changes in the slope at the top of a column in the story above and in the story below the one in which the stresses are to be determined, are equal to the change in the slope at the top of the corresponding column in the latter story.

2. The ratio of the deflection to the length of the columns in the story above the one in which the stresses are to be determined, is equal to the ratio of the deflection to the length of the columns in the latter story.

In other words, in determining the stresses in the second story, $\theta_{A_1}$ and $\theta_{A_3}$ are assumed to be equal to $\theta_{A_2}$; $\theta_{B_1}$ and $\theta_{B_3}$ are assumed to be equal to $\theta_{B_2}$; and $R_3$ is assumed to be equal to $R_2$. Also, in figuring the stresses in the third story $\theta_{A_2}$ and $\theta_{A_4}$ are assumed to be equal to $\theta_{A_3}$; $\theta_{B_2}$ and $\theta_{B_4}$ are assumed to be equal to $\theta_{B_3}$; and $R_4$ is assumed to be equal to $R_3$. This does not mean, however, that the values of $\theta_{A_2}$, $\theta_{B_2}$, and $R_2$ used in determining the stresses in the second story are equal to the values of $\theta_{A_3}$, $\theta_{B_3}$, and $R_3$ respectively, used in determining the stresses in the third story.

An examination of the equations in Table 3 shows that, if assumptions 1 and 2 were true, three equations containing only three unknown quantities could be written for each story of the bent. To illustrate, consider the equations for the second story. In accordance with assumptions 1 and 2,

$$\theta_{A_1} = \theta_{A_2} = \theta_{A_3}$$
$$\theta_{B_1} = \theta_{B_2} = \theta_{B_3}$$
$$R_2 = R_3$$

Substituting $\theta_{A_2}$ for $\theta_{A_1}$ and $\theta_{A_3}$, $\theta_{B_2}$ for $\theta_{B_1}$ and $\theta_{B_3}$, and $R_2$ for $R_3$ in equations D, E, and F of Table 3 gives:

$$-N_2R_2 + 4K_{A_2}\,\theta_{A_2} + 4K_{B_2}\theta_{B_2} = -\frac{W_2h_2}{6E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$-3\,(K_{A_2} + K_{A_3})\,R_2 + (K_{A_2} + J_{A_2} + K_{A_3})\,\theta_{A_2} + K_{a_2}\theta_{B_2} = 0 \ldots\ldots\ldots (2)$$

$$-3\,(K_{B_2} + K_{B_3})\,R_2 + K_{a_2}\theta_{A_2} + (K_{B_2} + J_{B_2} + K_{b_2} + K_{B_3})\,\theta_{B_2} = 0 \ldots (3)$$

These equations have been written for the second story. Similar equations can be written for any other story by making the proper changes in the subscripts.

Equations 1, 2, and 3, obtained from the equations in Table 3, can be used in determining the stresses in a symmetrical three-span bent any number of stories high. If similar changes are made in the equations of Tables 1 to 10 inclusive, groups of equations can be obtained, one for each bent, which can be used to determine the stresses in any story of *symmetrical* bents of from one to five spans and also of *unsymmetrical* bents of from one to five spans.

The sum of the moments at the tops and bottoms of all columns of a story is equal to the total shear in the story multiplied by the story height. The distribution of this moment to the ends of the columns depends upon: first, the ratio of the $K$ of column $A$ to the $K$ of column $B$; second, the ratio of the $K$ of column $A$ to the $K$ of girder $a$; and third, the ratio of the $K$ of girder $a$ to the $K$ of girder $b$.

The distribution of the moment was determined in a number of bents for which the ratios of the $K$ of column $A$ to the $K$ of column $B$, the $K$ of column $A$ to the $K$ of girder $a$, and the $K$ of girder $a$ to the $K$ of girder $b$ had different values. Diagrams showing the distribution of the moment in these bents also give the distribution of the moment in other bents. The curves in Figs. 6, 7, and 8 show the distribution of the moment in symmetrical three-span bents. In Fig. 6, the curves in group I show the moment at the top and the bottom of column $A$ in bents in which the $K$ of column $A$ and the $K$ of column $B$ are equal. The abscissae represent the ratios of the $K$ of girder $a$ to the $K$ of girder $b$; and the ordinates represent the moment at the top and the bottom of column $A$ in per cent of $W \times h$. Beginning at the top and reading down, the curves are for bents for which the ratio of the $K$ of column $A$ to the $K$ of girder $a$ equals 0.5, 1, 2, and 4 respectively. The moment in column $A$ of a bent for which the ratio of the $K$ of column $A$ to the $K$ of girder $a$ has any intermediate value can be determined by interpolation.

The curves in groups II, III, IV, and V, of Fig. 6, when used with the curves in group I, give the moment at the top and the bottom of column $A$ in bents in which the $K$ of column $A$ and the $K$ of column $B$ are not equal. The curves in groups II, III, IV, and V are for bents for which the ratio of the $K$ for column $A$ to the $K$ for girder $a$ equals 4, 2, 1, and 0.5, respectively. Beginning at the left of each group and reading to the right, the curves are for bents for which the ratio of the $K$ for column $A$ to the $K$ for column $B$ equals 0.5, 1, and 2, respectively.

The moment at the top and the bottom of column $A$ can be obtained from the diagram in Fig. 6 in the following manner:

First consider the moment in a bent in which the $K$ of column $A$ is

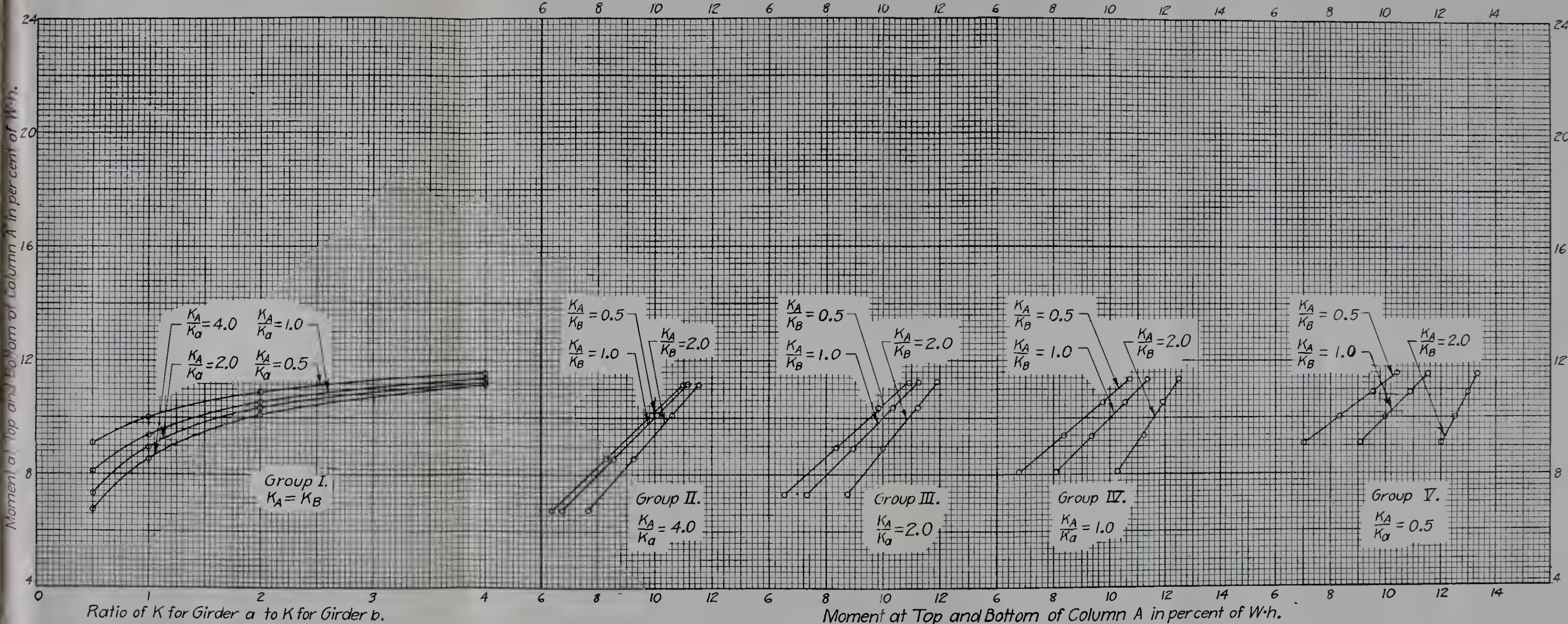

Fig. 6. Approximate Moment at the Top and the Bottom of Column *A*.

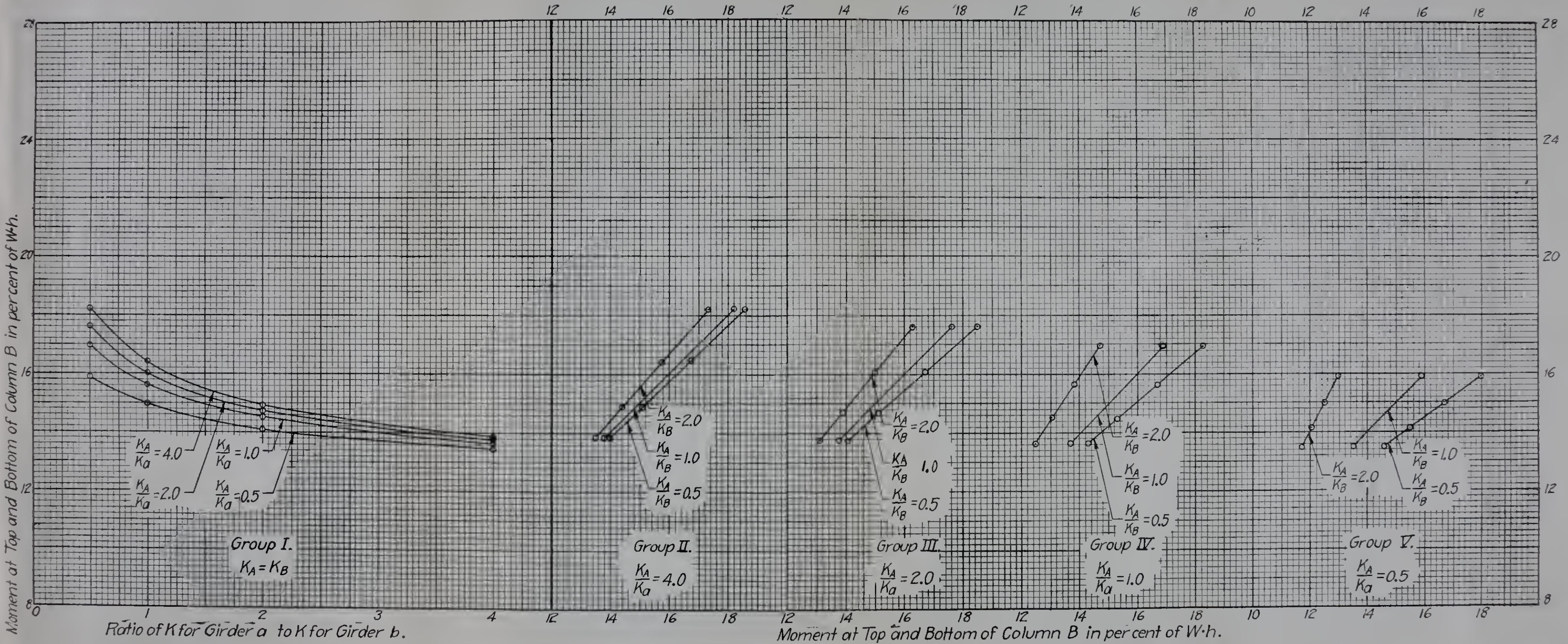

Fig. 7. Approximate Moment at the Top and the Bottom of Column *B*.

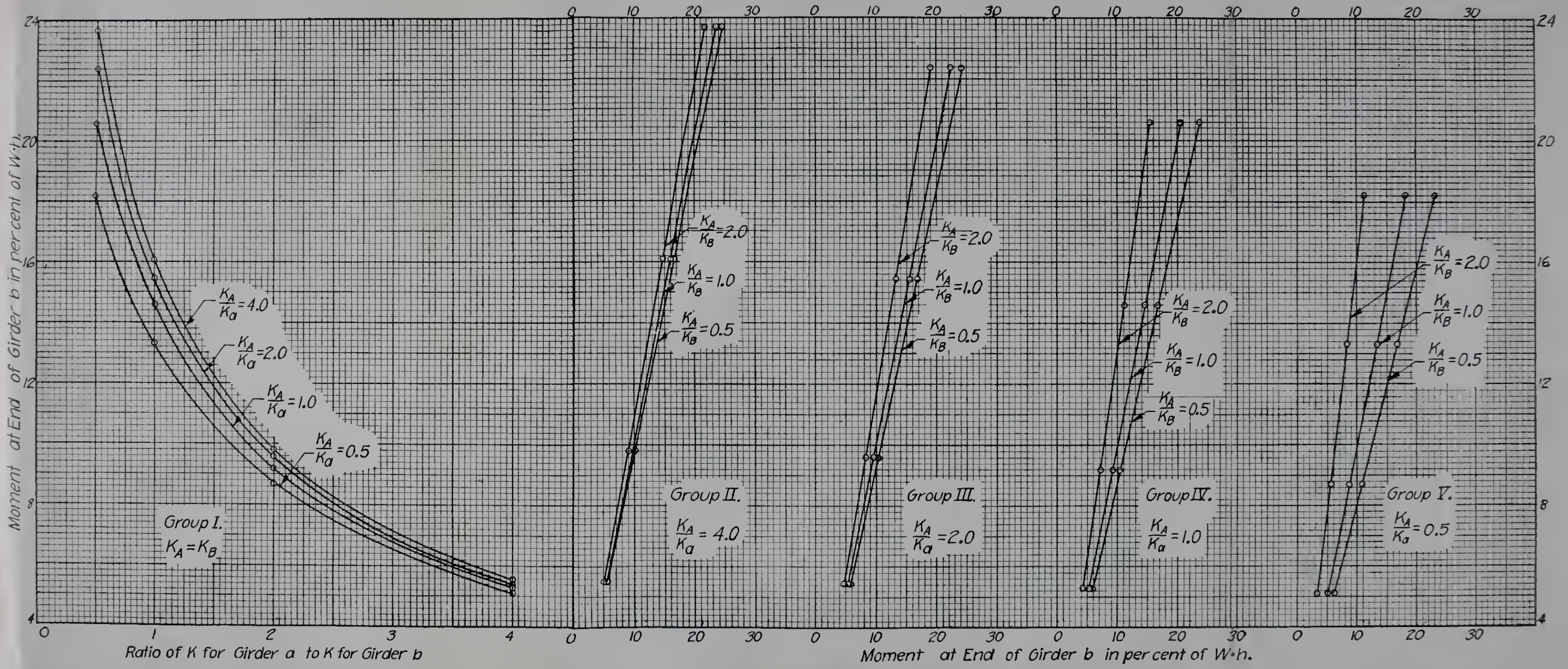

Fig. 8. Approximate Moment at the Ends of Girder *b*.

equal to the $K$ of column $B$. Determine the value of $\frac{K_a}{K_b}$ and of $\frac{K_A}{K_a}$. Use the curves in group I. Trace a vertical line whose abscissa equals $\frac{K_a}{K_b}$ until it intersects the curve corresponding to the value of $\frac{K_A}{K_a}$. Project the point of intersection horizontally to the left and read the moment in per cent of $W \times h$ from the vertical scale. As $W$ and $h$ are known quantities the moment is determined.

Next consider the moment in a bent in which the $K$ of column $A$ and the $K$ of column $B$ are not equal. Determine the value of $\frac{K_a}{K_b}$, $\frac{K_A}{K_a}$, and $\frac{K_A}{K_B}$. First use the curves of group I. Trace a vertical line whose abscissa equals $\frac{K_a}{K_b}$ until it intersects the curve corresponding to the value of $\frac{K_A}{K_a}$. Project the point of intersection horizontally to the right to the group of curves corresponding to the value of $\frac{K_A}{K_a}$ until it intersects the particular curve of this group corresponding to the value of $\frac{K_A}{K_B}$. Project this intersection point vertically downward and read the moment in per cent of $W \times h$ from the horizontal scale.

Similarly, the moments at the top and the bottom of column $B$ can be obtained from Fig. 7, and the moment at the end of girder $b$ can be obtained from Fig. 8. It should be noted, however, that the moment in girder $b$ depends equally upon the $W \times h$ in the story above and in the story below the girder. This being true, in getting the moment in girder $b$, the average of the $W \times h$ for the two stories should be used.

The moment at the right end of girder $a$ balances the sum of the moments in column $A$ just above and below the girder and is equal to their algebraic sum. The moment at the left end of girder $a$ balances the sum of the moments at the right end of girder $b$ together with the moments in column $B$ just above and below girder $a$ and is equal to their algebraic sum. It is therefore possible to obtain the moment at the ends of all members in a bent from the curves in Figs. 6, 7, and 8, subject, of course, to the error due to the use of assumptions 1 and 2 of this Section.

The curves in Figs. 6, 7, and 8 show that a large change in the ratio of the $K$ of one member to the $K$ of another member causes a relatively small change in the distribution of the moments in the bent.

16. *Numerical Problem.*—To illustrate the use of these curves the solution of a problem is presented.

The seventh story of a symmetrical three-span bent is 20 ft. high, and is subjected to a shear of 3,000 lbs. The eighth story of the same bent is 20 ft. high, and is subjected to a shear of 2,500 lbs. The properties of the members in the seventh and eighth stories are as follows:

$K_A = 30$ in.$^3$; $K_B = 40$ in.$^3$; $K_a = 20$ in.$^3$; and $K_b = 16$ in.$^3$. It is required to find the moments at the ends of all members in the seventh story.

$W \times h = 3{,}000 \times 20 \times 12 = 720{,}000$ in. lb. in the seventh story.

$W \times h = 2{,}500 \times 20 \times 12 = 600{,}000$ in. lb. in the eighth story.

$$\frac{K_A}{K_B} = \frac{30}{40} = .75$$

$$\frac{K_A}{K_a} = \frac{30}{20} = 1.5$$

$$\frac{K_a}{K_b} = \frac{20}{16} = 1.25$$

To get the moment in column *A* use Fig. 6. At the left of the figure trace the ordinate whose abscissa is 1.25 to a point half way between the two middle curves ($\frac{K_a}{K_A} = 1.5$, which is half way between 1 and 2), and project this point horizontally to a point half way ($\frac{K_A}{K_B} =$ .75, which is half way between .5 and 1.0) between the two left-hand curves of Group II, and also to a point half way between the two left-hand curves of Group III. The abscissa of the former point is 9.35 per cent, and of the latter point 9.15 per cent. As $\frac{K_A}{K_a}$ is equal to 1.5 or the average of 1 and 2, the moment at the top and the bottom of column *A* in the seventh story, $M_{A7}$, is the average of 9.35 per cent and 9.15 per cent or 9.25 per cent of $W \times h$, that is, $M_{A7} = .0925 \times 720{,}000 = 66{,}700$ in. lb. The moment, $M_{A8}$, at the top and the bottom of column *A* in the eighth story is $.0925 \times 600{,}000$, or $M_{A8} = 55{,}500$ in. lb. Similarly, the moment at the top and bottom of column *B*, $M_{B7}$, is 15.75 per cent of $W \times h$, that is, $M_{B7} = .1575 \times 720{,}000 = 113{,}500$ in. lb. in the seventh story; and $M_{B8} = .1575 \times 600{,}000 = 94{,}400$ in. lb. in the eighth story. The moment $M_{b7}$, at the end of girder *b* at the top of the seventh story is 13.85 per cent of $W \times h$, that is,

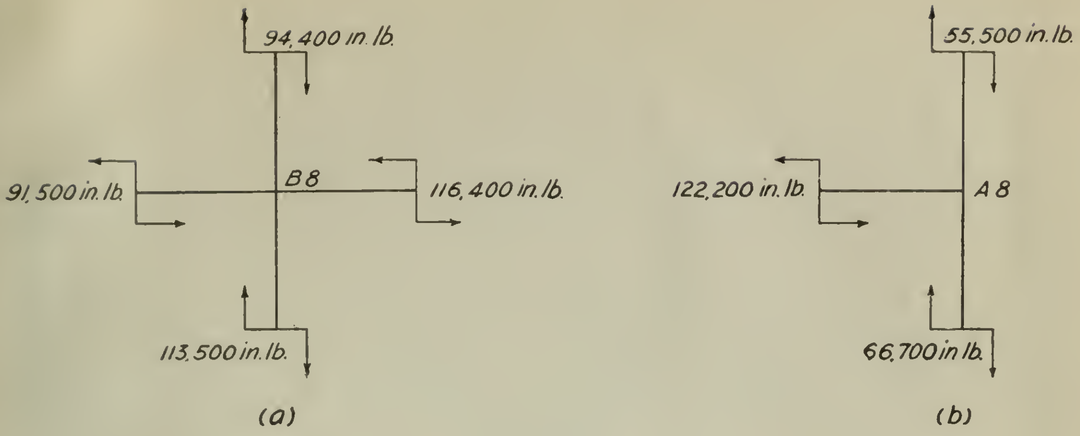

FIG. 9. MOMENTS ACTING AT POINTS *A8* AND *B8* OF A SYMMETRICAL THREE-SPAN BENT.

$$M_{b_7} = .1385 \left[ \frac{720{,}000 + 600{,}000}{2} \right] = 91{,}500 \text{ in. lb.}$$

The moment at the right end of girder *a* at the top of the seventh story is equal to the sum of 66,700 in. lb., the moment at the top of column *A* in the seventh story, and 55,500 in. lb., the moment at the bottom of column *A* in the eighth story, or 122,200 in. lb. (see Fig. 9b). The moment at the left end of girder *a* is equal to the sum of 113,500 in. lb., the moment at the top of column *B* in the seventh story, and 94,400 in. lb., the moment at the bottom of column *B* in the eighth story, less 91,500 in. lb., the moment in the end of girder *b*; that is, the moment at the left end of girder *a* is 116,400 in. lb. (see Fig. 9a).

A comparison of the moments in a bent as obtained by the proposed approximate method and by the slope-deflection method is given in Tables 23 to 26 inclusive.

17. *Modifications of the Slope-Deflection Method.*—The writers made a study to determine the effect of a change in the section of one member of a bent upon the moment in the other members. The moments were determined in a number of bents in which $K = 1$ for all members except the one whose effect upon the distribution of the moment was to be studied. The quantity $\frac{Wh}{6E}$ was taken equal to 1 for all stories and for all bents. The $K$ of column $B$ was given successively the values 0.5, 1, 2, and 4; and the corresponding moment at the ends of columns $A$ and $B$ and at the ends of girders $a$ and $b$ was calculated

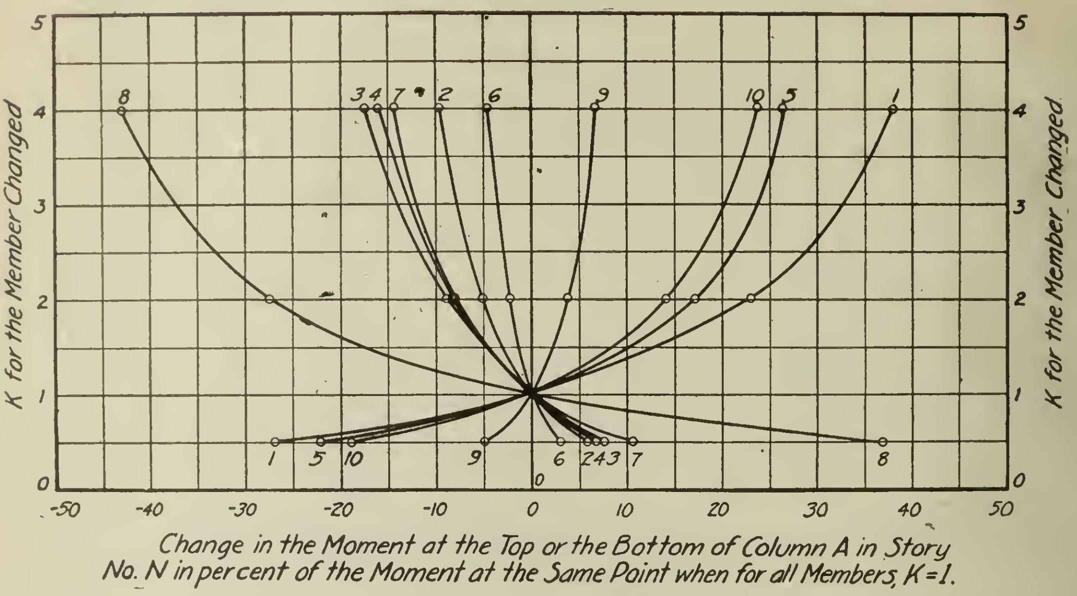

FIG. 10. DIAGRAM SHOWING CHANGE IN THE MOMENT AT THE TOP AND THE BOTTOM OF COLUMN *A* OF A SYMMETRICAL THREE-SPAN BENT DUE TO A CHANGE OF *K* IN THE OTHER MEMBERS.

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## KEY TO THE DIAGRAM IN FIG. 10.

| Member Changed | Designation of the Curve Showing the Change in the Moment at | |
|---|---|---|
| | Top of Column *A* | Bottom of Column *A* |
| Girder *a* in Story No. N | 1 | 2 |
| " " No. (N—1) | 2 | 1 |
| Girder *b* " " No. N | 3 | 4 |
| " " No. (N—1) | 4 | 3 |
| Column *A* " " No. N | 5 | 5 |
| " " No. (N—1) | 6 | 7 |
| " " No. (N+1) | 7 | 6 |
| Column *B* " " No. N | 8 | 8 |
| " " No. (N—1) | 9 | 10 |
| " " No. (N+1) | 10 | 9 |

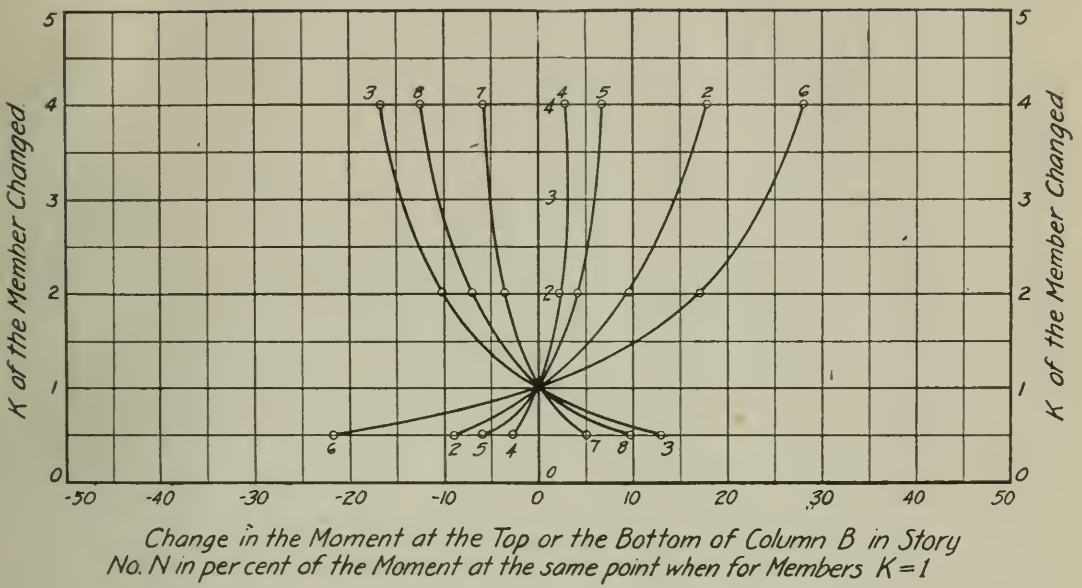

FIG. 11. DIAGRAM SHOWING CHANGE IN THE MOMENT AT THE TOP AND THE BOTTOM OF COLUMN *B* OF A SYMMETRICAL THREE-SPAN BENT DUE TO A CHANGE OF *K* IN THE OTHER MEMBERS.

---

## KEY TO THE DIAGRAM IN FIG. 11.

| Member Changed | Designation of the Curve Showing the Change in the Moment at | |
|---|---|---|
| | Top of Column *B* | Bottom of Column *B* |
| Girder *a* in Story No. N | — | 1 |
| " " No. (N—1) | 1 | — |
| Girder *b* " " No. N | 2 | — |
| " " No. (N—1) | — | 2 |
| Coulmn *A* " " No. N | 3 | 3 |
| " " No. (N—1) | 4 | 5 |
| " " No. (N+1) | 5 | 4 |
| Column *B* " " No. N | 6 | 6 |
| " " No. (N—1) | 7 | 8 |
| " " No. (N+1) | 8 | 7 |

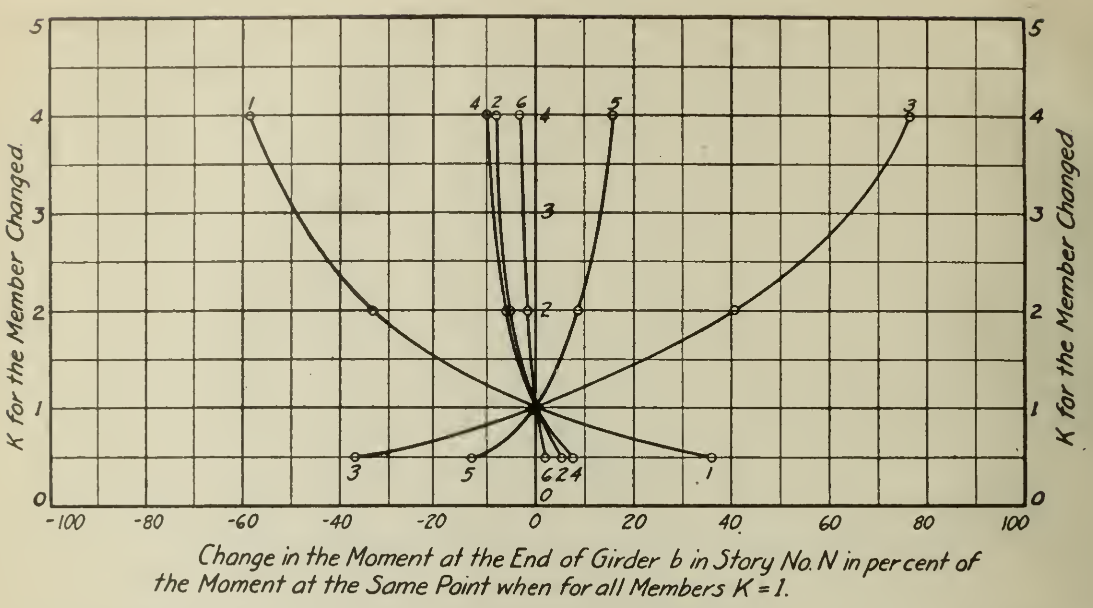

FIG. 12. DIAGRAM SHOWING CHANGE IN THE MOMENT AT THE END OF GIRDER *b* OF A SYMMETRICAL THREE-SPAN BENT DUE TO CHANGE OF *K* IN THE OTHER MEMBERS.

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## KEY TO THE DIAGRAM IN FIG. 12.

| Member Changed | Designation of the Curve Showing the Change in the Moment at the End of Girder *b* |
|---|---|
| Girder *a* in Story No. N | 1 |
| " " No. (N—1) | 2 |
| " " No. (N+1) | 2 |
| Girder *b* " " No. N | 3 |
| Column *A* " " No. N | 4 |
| " " No. (N+1) | 4 |
| Column *B* " " No. N | 5 |
| " " No. (N—1) | 6 |
| " " No. (N+1) | 5 |

in the story in which the $K$ of column $B$ was changed, and also in the stories immediately above and below that story. Similarly, the $K$ of column $A$, the $K$ of girder $a$ and the $K$ of girder $b$ were given successively the values 0.5, 1, 2, and 4, and the corresponding values of the moment at the ends of the girders and the columns were determined in the particular story and also in the stories immediately above and below that story. The effect of the changes in the values of the $K$ of the members upon the moment is given in Figs. 10, 11, and 12.

Fig. 10 shows the changes in the moment at the top and the bottom of column $A$. The abscissae are changes in the moment expressed in per cent of the moment in a bent for which the $K$ of all members is equal to 1. Increases in the moment are laid off to the right of the origin, and decreases are laid off to the left. The ordinates represent the values of the $K$ in the member changed. The story in which the moment was calculated is designated as story No. $N$, the story above is designated as story No. $(N+1)$, and the story below No. $(N-1)$. Each curve shows the change in the moment at the top or the bottom of column $A$ due to a change in the value of the $K$ of some member. The number of the curve which shows the change in the moment in column $A$ due to a change in the $K$ of any particular member can be obtained from the key to the diagram in Fig. 10. For example, curve No. 3 shows the change in the moment at the top of column $A$ due to a change in the $K$ of girder $b$ in story No. $N$. If the $K$ of girder $b$ is made equal to 2, the moment at the top of column $A$ is 8.9 per cent less than when the $K$ of girder $b$ is equal to 1. Curve No. 3 also shows the change in the moment at the bottom of column $A$ due to a change in the $K$ of girder $b$ in story No. $(N-1)$. If the $K$ for girder $b$ in story No. $(N-1)$ is equal to 2, the moment at the bottom of column $A$ is 8.9 per cent less than when the $K$ for girder $b$ is equal to 1.

Similarly, the changes in the moment in column $B$ due to changes in the $K$ of the members of the bent are given in Fig. 11 and the changes in the moment in girder $b$ are given in Fig. 12.

Fig. 10 shows that the moment in column $A$ of any story not only depends upon the value of the $K$ of the members in the same story but depends also upon the value of the $K$ of the members in the adjacent stories. Figs. 11 and 12 show that the same statement is true relative to the moment in column $B$ and girder $b$. This being true, any approximate method which considers only the members in the story in which the stresses are to be determined can not give accurate results except in bents which have no very sudden changes in the sections of the columns and girders.

Where there are sudden changes in the column and girder sections, the proposed approximate method is not accurate. In such cases it is sometimes possible to use a modification of the slope-deflection method and obtain quite accurate results with a comparatively small expenditure of labor. This modification of the slope-deflection method may be made in either of two ways, as follows: (a), the bent may be divided by a horizontal plane between any two adjacent floors, and the lower part be treated independently of the upper part; or (b), the bent may be divided by two horizontal planes any number of stories apart, and the portion between these planes may be treated independently of the other portions of the building. These modifications will be considered in detail.

(a) The wind stresses in the top stories of a bent are comparatively small and their exact determination is not important. An examination of the equations of Table 15 shows that the coefficient of the quantity to be determined is greater than the coefficients of the quantities in which the unknown is expressed. This being true, errors in the slopes and the deflection of one story have but little effect upon the slopes and deflections of the succeeding stories. Further examination of the same table shows that there is a comparatively small difference between the change in the slope at the top of a column in one story and the change in the slope at the top of the same column in the story below. If the two changes in slope are assumed to be equal, the error introduced is small. Bearing these facts in mind, consider again the twenty-story bent of Section VII. If $\theta_{A_{12}}$ and $\theta_{B_{12}}$ are assumed equal to $\theta_{A_{11}}$ and $\theta_{B_{11}}$ respectively in equations f, g, and h of Table 13, the first 34 equations will contain 34 unknown quantities which can be determined by the method used in Tables 14 and 15. That is, assuming that the changes in the slopes of the columns in one story are equal to the changes in the slopes of the same columns in the story below, is equivalent to dividing the bent into two parts; and the equations for the lower part can be solved independently of those of the upper part. If the slopes which are assumed equal are not really equal, the calculated slopes and deflections in the top story of the lower part will not be exact, but the results of a number of calculations show that the error in the slopes and the deflection in the next to the top story is so small that it may be neglected. This being true, if, in the actual design of the twenty-story bent of Section VII, it is considered unnecessary to calculate the wind stresses in the top ten stories, the bottom eleven stories may be treated as a complete bent and the stresses determined by the method outlined above, thereby decreasing the work. The stresses in the bottom ten stories will be quite exact. The bent could have been divided at any other story without affecting the accuracy of the results.

If there are sudden changes in the members the bent can be divided above the story in which the change occurs, and the stresses in the lower part can be determined by the method outlined above.

Table 23 shows that the largest errors in the proposed approximate method are in the first story. The moments can be determined in the first story by the modification of the slope-deflection method as follows:

Assume that the slopes in the third story are equal to the corresponding slopes in the second story. The first seven equations of Table 13 will then contain only seven unknown quantities. The solution of these equations is given in Tables 21 and 22. The changes in the slopes and the deflection for the first story as given in Table 22 agree very closely with the values for the same quantities given in Table 15, and the moments, being functions of the slopes and deflection, will also agree very closely with the moments given in Table 18.

(b) The moments in any particular story may be determined if there is some story below the one in question in which it can be seen by inspection that the changes in the slopes at the tops and the bottoms of the columns are equal. Suppose the moments are to be determined in the tenth story of the bent shown in Fig. 5, and that it is apparent from inspection that $\theta_{A8} = \theta_{A9}$, and $\theta_{B8} = \theta_{B9}$. Then assume that $\theta_{A12} = \theta_{A11}$, and $\theta_{B12} = \theta_{B11}$. The ten equations, Y to h inclusive of Table 13, will contain ten unknown quantities. If the slopes in the eighth and ninth stories, which have been assumed to be equal, are equal then the slopes and the deflection, and therefore also the moments, in the tenth story will be quite accurate, but if the slopes *below* the story in question which are assumed to be equal, are not really equal, the required moments will not be exact. That is, a difference between the slopes *below* the story in question affects the results, whereas a difference between the slopes *above* the story does not materially affect the results.

18. *Application of the Proposed Approximate Method and Modification of the Slope-Deflection Method.*—To obtain the moment in a bent in which there are sudden changes in the sections, the following com-

bination of methods can sometimes be used to advantage. Use the proposed approximate method for obtaining the moment in the portion of the bent in which there are no sudden changes in the sections of the members. Use a modification of the slope-deflection method to get the moments in the bottom story and in any intermediate stories in which sudden changes in the members occur. The results thus obtained will be sufficiently accurate to be used in the actual designs of buildings, and the amount of work required will not be excessive.

## IX. Comparison of the Approximate Methods With the Slope-Deflection Method.

19. *Symmetrical Three-Span Bent with Short Middle Span.*—For comparison, the moments in the symmetrical three-span twenty-story bent shown in Fig. 5 were calculated by the five following methods:

1. The slope-deflection method.
2. The proposed approximate method.
3. The three methods described by Mr. Fleming and known as methods I, II, and III.

The moments as determined by these methods are given in Tables 23 and 24. For each story the moments in the upper line are in 1,000 in. lb., and those in the lower line are in per cent of the moments as determined by the slope-deflection method. Tables 23 and 24 show in the case of this bent: first, that the moments determined by methods II and III are very seriously in error; second, that the moment determined by method I and by the proposed approximate method agree very closely with the moment determined by the slope-deflection method except at points where there are sudden changes in the members of the bent; third, that the errors in the moment determined by the proposed approximate method are less for the girders than for the columns.

20. *Symmetrical Three-Span Bent with Long Middle Span.*—The distribution of the moment determined by the slope-deflection method is affected by the ratio of the $K$ of girder $a$ to the $K$ of girder $b$. As this ratio does not affect the distribution of the moment determined by methods I, II, and III, the accuracy of the latter methods will depend upon the relative values of $K$ for the two girders.

In the bent shown in Fig. 5, the $K$ is less for girder $a$ than for girder $b$, since the girders have substantially the same sections and girder $a$ is longer than girder $b$. In order to determine the effect of the relative values of the $K$ of girders $a$ and $b$ upon the accuracy of the approximate methods, the moment in the bottom four stories of the bent shown in Fig. 13, a bent like the one shown in Fig. 5 except that the long and short spans have been interchanged, was determined by each of the five methods mentioned in article 19. The results are given in Tables 25 and 26. The moments determined by the proposed approximate method are as accurate as those given in Tables 23 and 24; whereas the moments determined by method I are inaccurate, and those determined by methods II and III are very inaccurate.

21. *Effect of the Proportions of a Bent upon the Accuracy of Method I.*—In order to determine more fully the effect of the proportions of a bent upon the accuracy of method I, the moments were determined in a number of bents having different proportions, by the slope-deflection method and by method I. These moments are given in Table 27.

In bents 1 to 9 of this table all girders and columns have the same section. The ratio of the moment of inertia of the columns to the moment of inertia of the girders and also the ratio of the moment of inertia of the girder in bay *a* to the moment of inertia of the girder in bay *b*, affects the moment determined by the slope-deflection method but does not affect the moment as determined by method I. If the sections of the columns and girders are not the same, the difference between the moments determined by the two methods might vary even more than Table 27 indicates. Any errors in the moment due to sudden changes in the sections of the bent are in addition to the errors indicated in Table 27.

The difference between the two methods is due to the fact that in method I the direct unit stress in a column is proportional to the distance of the column from the neutral axis of the bent; whereas in the slope-deflection method the stress in a column depends upon the shears in the girders, and the shears in the girders depend upon the changes in the slopes at the ends of the girders and upon the moment of inertia of the girder sections.

22. *Accuracy of the Approximate Methods when the Moment of Inertia of the Girders is Proportional to the Bending Moment.*—In the comparison of the approximate methods with the slope-deflection method given in Tables 23 to 27, the sections of the girders were chosen without reference to the moment to which they were subjected, except the girders in the bent shown in Fig. 5. The girders in this bent were designed to resist the bending moment determined by method I. Another investigation was made to determine the accuracy of the approxi-

| Wind load due to a Pressure of 30 lb. per sq. ft. acting on a Vertical Strip 1 ft. Wide. | | Story Number |
|---|---|---|
| Increment on Story = w | Total on all Stories above = W | |
| 360 | 360 | 20 |
| 360 | 720 | 19 |
| 360 | 1080 | 18 |
| 360 | 1440 | 17 |
| 360 | 1800 | 16 |
| 360 | 2160 | 15 |
| 360 | 2520 | 14 |
| 360 | 2880 | 13 |
| 360 | 3240 | 12 |
| 360 | 3600 | 11 |
| 360 | 3960 | 10 |
| 360 | 4320 | 9 |
| 360 | 4680 | 8 |
| 360 | 5040 | 7 |
| 390 | 5430 | 6 |
| 420 | 5850 | 5 |
| 420 | 6270 | 4 |
| 420 | 6690 | 3 |
| 450 | 7140 | 2 |
| 570 | 7710 | 1 |

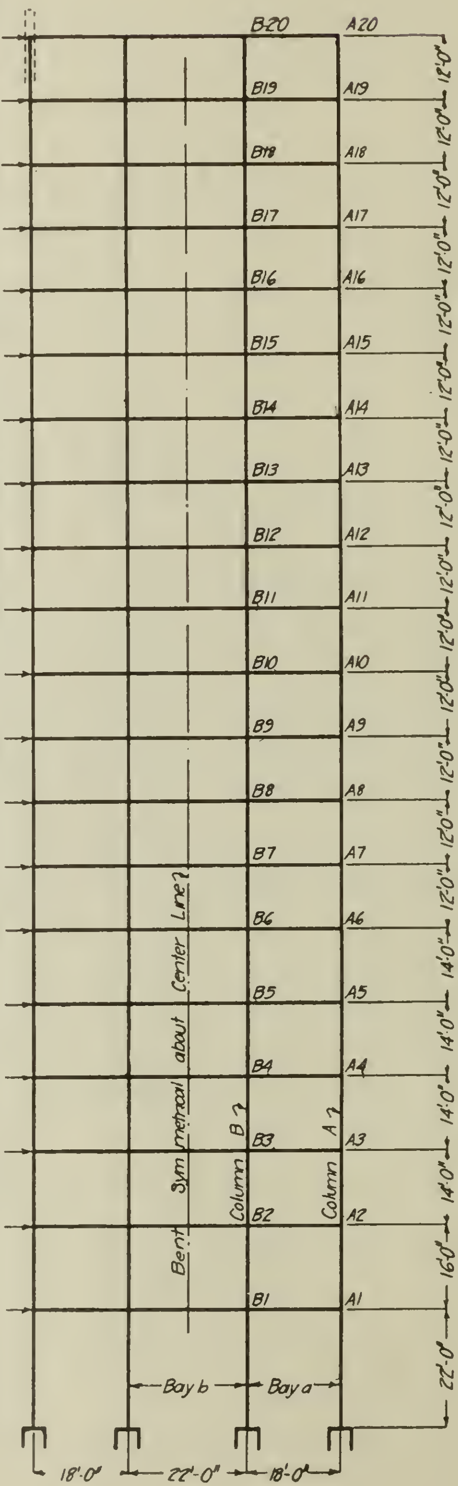


FIG. 13. SYMMETRICAL THREE-SPAN BENT TWENTY STORIES HIGH WITH LONG SPAN AT THE CENTER.

mate methods when applied to bents in which the girders were designed to resist the bending moment due to the wind load as determined by the method which is to be compared with the slope-deflection method. The results of this investigation are given in Tables 28 to 31. In Table 28 the moments of inertia of the sections of the girders are proportional to the bending moments determined by method I. The sections of the columns are equal, and the moment of inertia of the sections of the columns is equal to the moment of inertia of the section of girder *a*. Table 28 shows that method I gives the moment in the columns and girders quite accurately when the spans are equal and the story height does not exceed the length of the span. This statement is true, in general, only when the two column sections are equal and when the moments of inertia of the sections of column *A* and girder *a* are equal.

In considering the merits of the approximate methods, it should be noted that the moment which should be determined with the greatest accuracy is the moment to which the joint that connects the girder to the column is subjected. This moment is the moment at the ends of the girders. If the moments of inertia of the sections are not made proportional to the bending moment determined by method I, the method will not be as accurate as Table 28 indicates.

The moments of inertia of the girder sections of Table 29 are proportional to the bending moments in the respective girders as determined by method II. The moments of inertia of the girder sections of Table 30 are proportional to the bending moments in the respective girders determined by method III. It is apparent from Tables 29-30 and Tables 23-26 that methods II and III are so inaccurate that they should never be used.

The moments of inertia of the girder sections of Table 31 are proportional to the bending moments in the respective girders as determined by method IV. For bents having equal spans and equal column sections this method gives quite accurate results.

While methods I and IV may be quite accurate in some cases, their accuracy depends upon the proportions of the bent. For example, by comparing Tables 27 and 28 it is apparent that if the moments of inertia of the girder sections are not proportional to the bending moments determined by method I, that method will not be as accurate as when the girder sections are so proportioned. Again, in Tables 28 and 31 the sections of the columns are equal and the moments of inertia of the sections of column *A* and girder *a* are equal; but if these relations do not exist, the methods will not in general be as accurate as Tables 28 and 31 seem to indicate.

Any inaccuracy in methods I and IV due to sudden changes in the members of the bent, are in addition to the inaccuracies shown in Tables 28 and 31.

For the bents shown in Tables 28 to 31, the moments determined by the proposed approximate method are exactly the same as the moments determined by the slope-deflection method.

## X. TEST OF A CELLULOID MODEL OF A BENT.

23. *Description of Tests.*—In order to check the deflections and the changes in the slopes calculated by the slope-deflection method, a celluloid model of a bent was subjected to known shears, and the resulting deflections and changes in slopes were measured and compared with the calculated values. The model was made of celluloid 1/8 inch thick, and had the general dimensions shown in Fig. 14. A cord passing over a pulley and attached to a weight at one end and to the top of the model at the other, produced a uniform shear in all stories. Paper arms were fastened to the model at points where the members intersect. The movement of these arms indicated the changes in the slopes. One of these arms, $A_2D$, is shown in Fig. 14. The external force caused the point $A_2$ to move in approximately a horizontal line and at the same time to turn through a small angle, $\theta$. The paper arm $A_2D$ has the same motion as the point $A_2$; thus the vertical displacement of $D$ measures the angle $\theta$. The horizontal deflection of the model was obtained by measuring the displacement of a point at the middle of a girder. Paper arms were attached to all four columns at the tops of all stories simultaneously, and the changes in the slopes at all intersections were measured for each application of the load. Similarly, the horizontal deflection was measured at the middle of each of the three girders at the top of each story.

In the original model, known as model No. 1, members No. 1, 2, 3, and 4 were 1/2 inch wide; all other members were 1/4 inch wide. After this model had been tested, member No. 1 was reduced to 1/4 inch; and then the resulting model, now known as No. 2, was tested. Members No. 2, 3, and 4 were successively reduced to 1/4 inch; and the resulting models, now known as models No. 3, 4, and 5, were also tested.

In testing model No. 1, observations were made when loads of 2½ lb., 5 lb., 7½ lb., and 10 lb. were applied. The readings for the two sides of the model agreed very closely in the bottom three stories;

but in the fourth story they did not agree. Thinking that this discrepancy might be due to excessive stresses, when testing model No. 2, a maximum load of only 7½ lb. was used; but there was still the same discrepancy in the readings. Thinking that the apparatus might be out of order, the loads were removed and then applied a second time but the second readings agreed very closely with the first. In testing models No. 3, 4, and 5, loads of 1½ lb., 3 lb., and 4½ lb. were applied. The loads were then removed and applied in the opposite direction. The readings for the two sides of the model continued to agree very closely in the bottom three stories; and in the fourth story they continued to disagree. It is therefore probable that either the material was not homogeneous or that there were internal stresses or local weaknesses in the upper part of the model.

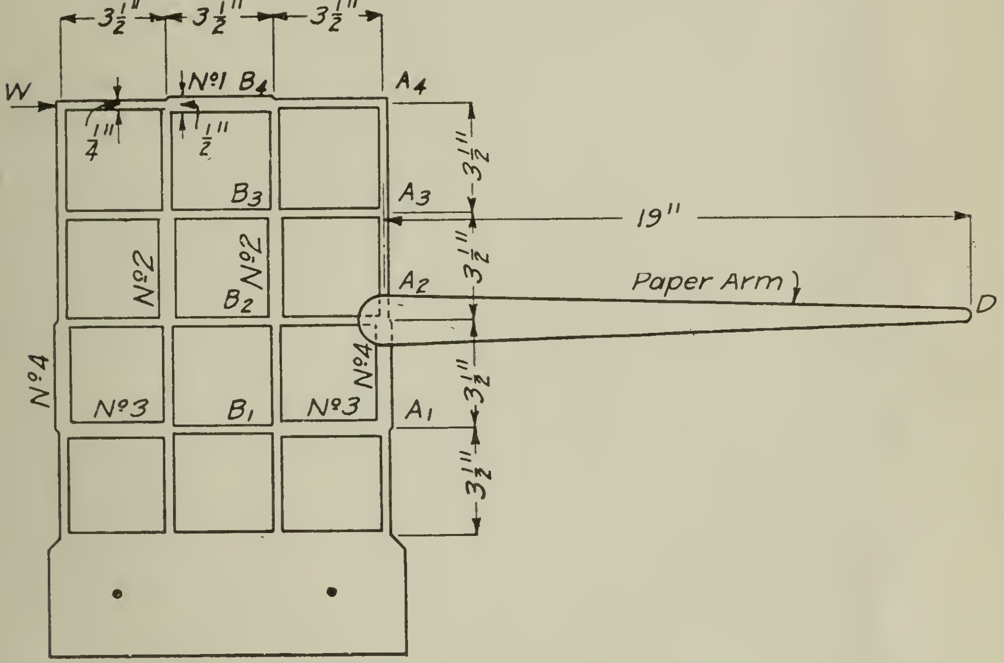


Fig. 14. Celluloid Model.

The observations made in the test of model No. 4 are given in Table 32.

24. *Results of Tests.*—The results of the tests of the models are given in Fig. 15. The models are shown by the sketches at the left of the figure. In these sketches members represented by heavy lines are ½ inch wide, and all other members are ¼ inch wide. The upper diagram of Fig. 15 shows the changes in the slopes and the lower diagram shows the deflections.

In the upper diagram the first group of lines at the left represents the changes in the slope at the point $A_1$, the second group represents the changes at $A_2$, and similarly the third and fourth groups represent the changes at $A_3$ and $A_4$. The fifth group of lines from the left represents the changes in the slope at the point $B_1$, the sixth group at $B_2$, and similarly for the other groups. In the left-hand group of lines the change in the slope at the point $A_1$ of model No. 1 is laid off from the origin on a horizontal line opposite the sketch of model No. 1; the change in the slope at $A_1$ of model No. 2 is laid off from the origin on a horizontal line opposite model No. 2; the change in the slope at $A_1$ of model No. 3 is laid off from the origin on the horizontal line opposite model No. 3; and the change in the slope at $A_1$ of model No. 4 is laid off from the origin on the horizontal line opposite model No. 4. The full lines connect points which represent the quantities as measured; and the dotted lines connect points which represent the same quantities as computed by the slope-deflection method. The changes in the slopes at $A_2$, $A_3$, $A_4$, $B_1$, $B_2$, $B_3$, and $B_4$ are shown in a similar manner.

The lower diagram shows the ratios of the deflection to the story height in the different stories of the models. The method of representing the ratios of deflection to story height is similar to the method used to represent the changes in the slopes in the upper diagram.

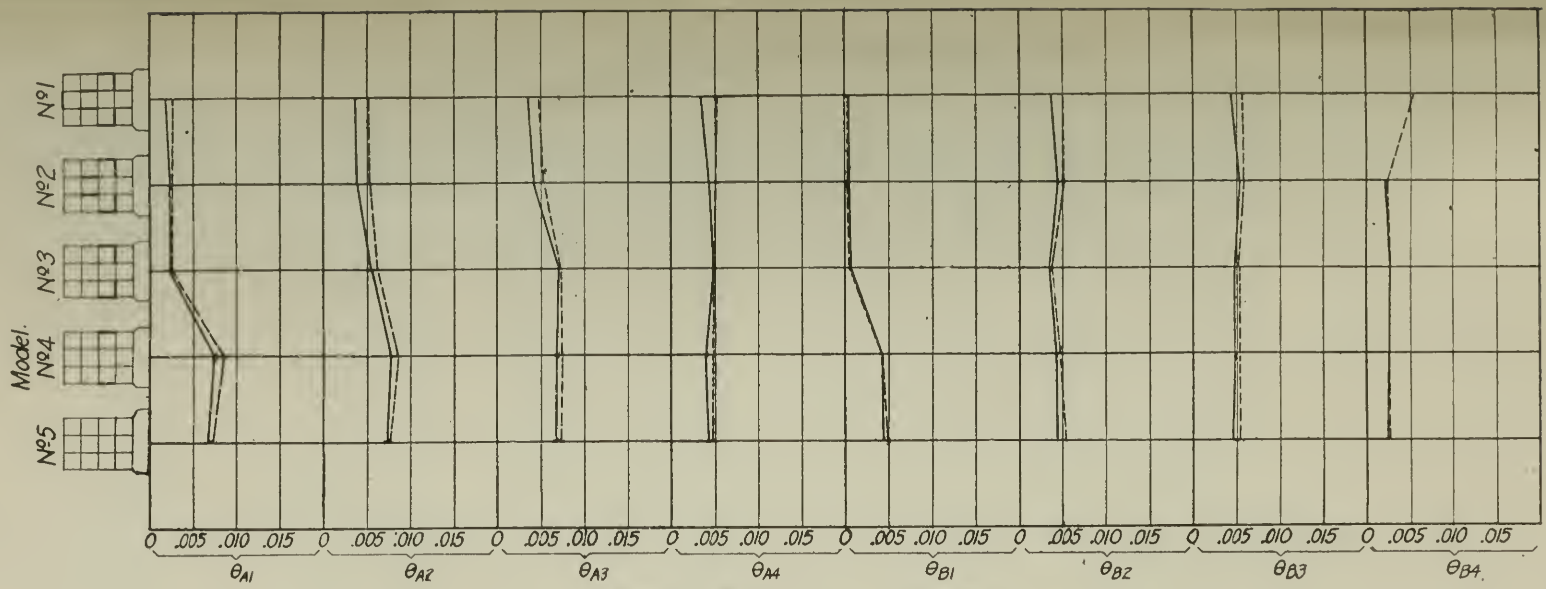
It will be noticed that the calculated quantities are in general greater than the observed quantities. The reason for this is: In the computations, the length of a member was taken equal to the distance between center lines, whereas the length that is actually free to bend is the distance from outside to outside of the members. This accounts for the difference between the observed and the computed values.

Fig. 15 shows that the changes in the slopes and the ratios of the deflection to the story height as observed and as computed agree closely. In other words, the tests support the theory upon which the slope-deflection method is based.

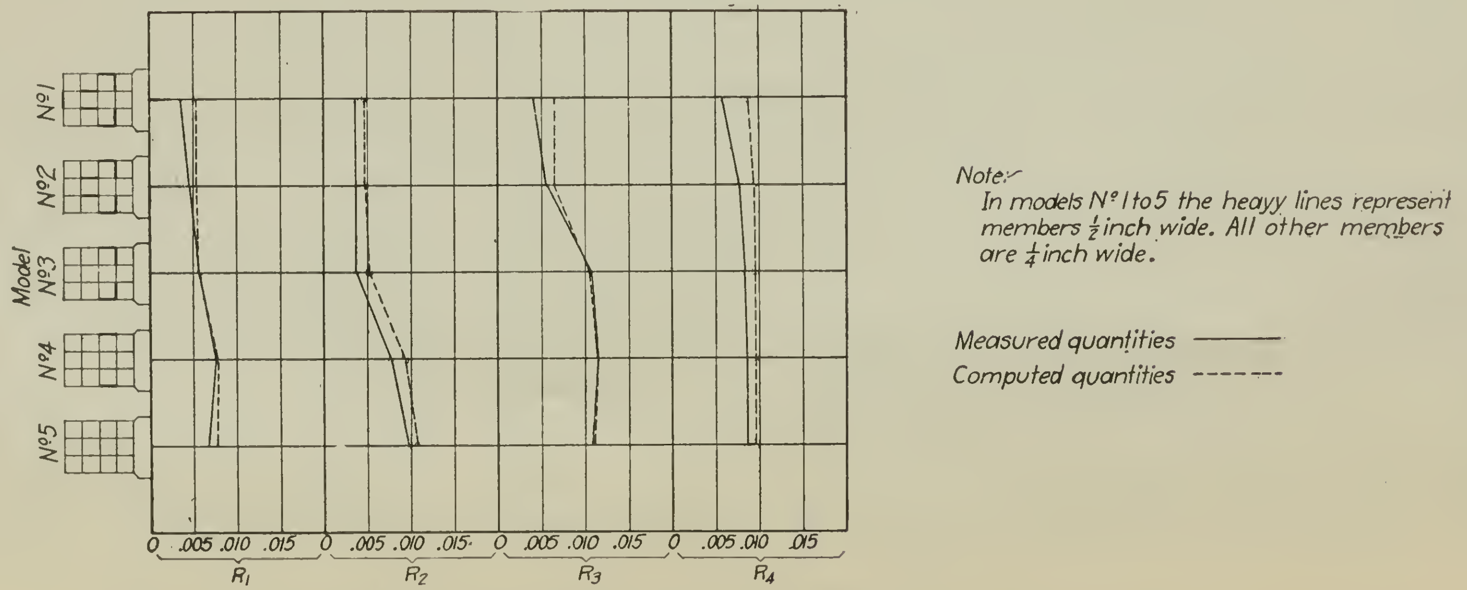
## XI. Discussion of the Assumptions.

25. *Preliminary.*—In making the analysis of the stresses the writers made certain assumptions in regard to the action of the frame when stressed. The effect of inaccuracies in these assumptions will now be considered.

If all of the columns of a story are taken together as a free body, the algebraic sum of the moments at the tops and bottoms of all the columns is equal to the shear on the story multiplied by the story height. As the moments in the columns are balanced by the moments in the



Changes in Slope—in Radians.



Note:–
In models No 1 to 5 the heavy lines represent members $\frac{1}{2}$ inch wide. All other members are $\frac{1}{4}$ inch wide.

Measured quantities ————
Computed quantities - - - - - -

Ratios of Deflection to Story Height—Abstract Numbers.

FIG. 15. DIAGRAM OF RESULTS OF THE TESTS OF CELLULOID MODEL.

girders, the algebraic sum of the moments at the ends of the girders is dependent upon the product of the shear in the story and the story height. These facts should be borne in mind in discussing the effect of inaccuracies in the assumptions upon the moments in a bent.

26. *Assumption of Perfect Rigidity.*—According to assumption No. 1, the connections between the columns and girders are perfectly rigid. The truth of this assumption can be determined only by tests. As far as the writers are aware no such tests have been made. While it is undesirable to make a mathematical analysis which is based upon an unverified assumption, some assumption relative to the rigidity of the connections must be made before the stresses in a frame can be determined. The distribution of the stresses depends more upon the relative stiffness than upon the actual stiffness of the connections. In view of these facts, it seems that the assumption of perfect rigidity of the joints is the most logical one that can be made.

27. *Assumption of the Unchanged Length.*—According to assumption No. 2, the change in the length of a member due to the direct stress is equal to zero.

If columns $A$ and $B$ change in length, the moment at the right-hand end of girder $a$ will be given by the equation

$$M_{AB} = 2EK_a\,(2\theta_A + \theta_B - 3R_a),$$

in which $R_a$ is equal to the difference between the changes in the lengths of columns $A$ and $B$ divided by the length of girder $a$. In the derivation of the general equations in Section VI, $R_a$ was assumed to be equal to zero. If the changes in the lengths of the columns do not alter the values of $\theta_A$ and $\theta_B$, the change in the moment $\Delta M_{AB}$, at the right-hand end of girder $a$ due to the changes in the lengths of the columns is:

$$\Delta M_{AB} = 2EK_a\,(-3R_a).$$

The change in the moment at the left-hand end of girder $a$ is equal to the change at the right-hand end. The difference between the changes in length of columns $A$ and $B$, or the deflection of one end of girder $a$ relative to the other end, is given by the equation:

$$d = \frac{Pl}{AE}, \text{ in which}$$

$d$ = the deflection,

$P$ = the difference between the stress in column $A$ and the stress in column $B$,

$l$ = the length of the columns,

$A$ = the area of the column section,

$E$ = the modulus of elasticity of steel.

From Table 19, page 75, the compressive stresses in the first story of columns $A$ and $B$ are 14,464 lb. and 4,587 lb. respectively.

Therefore $P = 14{,}464 - 4{,}587 = 9{,}877$ lb. From Table 11, page 48, $l = 264$ in., $K_a = 30.5$ in.$^3$, and $A = 112.8$ sq. in. $E = 29{,}000{,}000$ lb. per sq. in. Substituting these values in the above equation for $d$, gives:

$$d = .000798 \text{ in.}$$

By definition, $R_a = \frac{d}{l}$, therefore

$$R_a = \frac{.000798}{264} = .00000303, \text{ and}$$

$$\Delta M_{AB} = 2 \times 29{,}000{,}000 \times 30.5\ (-3 \times .00000303) = -16{,}050 \text{ in. lb.}$$

This change in the moment is 5.6 per cent of the moment at the right-hand end of girder $a$ as given in Table 18, page 74. Likewise the moment at the left-hand end of girder $a$ is decreased by 16,050 in. lb., which is 6.6 per cent of the moment as given in Table 18.

The moment at the end of girder $b$ is affected by the changes in the lengths of two columns, $B$, one on each side of the center line of the bent. One column is subjected to a tensile stress of 4,587 lb., and the other to a compressive stress of 4,587 lb. The deflection of girder $b$ is given by the equation

$$d = \frac{2 \times 4{,}587 \times 264}{112.8 \times 29{,}000{,}000} = .00074 \text{ inches.}$$

$$R_b = \frac{.00074}{216} = .00000343.$$

$$\Delta M_{BB} = 2 \times 29{,}000{,}000 \times 37.3 \times 3 \times .00000343 = 22{,}300 \text{ in. lb.,}$$

which is 9.0 per cent of the moment at the end of girder $b$ as given in Table 18.

The change in the length of a column is a function of the unit direct stress and also a function of the story height. The first story of the building considered is much higher (22 ft.) than the other stories, and the unit direct stress in the columns in the first story is greater than in the stories above; hence the change in the length of the columns is very much less in the other stories than in the first story. Therefore the changes in the moments for the other stories will be less than those in the first story computed above.

The changes in the moments in the columns have been determined and it remains to consider the effect of the direct stresses in the girders. The direct stress in a girder is very small in comparison with the direct stress in a column and therefore may be neglected.

If the changes in the slopes were not affected by the changes in the lengths of the columns, the moments at the ends of girders *a* and *b* would be decreased. As the sum of these moments is determined by the product of the shear in the story and the story height, they can not all be decreased simultaneously. Therefore all of the changes in the slopes must be increased until the sum of the moments at the ends of the girders will balance the moments in the columns. This will make the moments at the ends of the girders practically the same as they would have been if the columns had not changed in length. If the moments at the ends of the girders are not materially affected, the moments at the ends of the columns will not be materially affected. Therefore although the direct stresses in the columns do change the lengths of the columns, they do not affect the stresses in the frame to any great extent.

28. *Assumption as to Length of Members.*—In accordance with assumption No. 3, the length of a member was taken as the distance between the center lines of the members which it intersects. This makes the changes in the slopes and deflections, as calculated, greater than the actual values. The effect of the inaccuracy of this assumption upon the distribution of the moments is, however, not so apparent.

The curves in Fig. 6, 7, and 8 show that the distribution of the moment in a story depends upon the relative values of $K$ of the members, but that it takes a comparatively large change in the $K$ of a member to appreciably affect the distribution of the moments. The fact that the length of a member has been taken equal to the distance between center lines, has but little effect upon the relative values of $K$ for the members; and therefore does not materially affect the distribution of the moments.

29. *Assumption as to Deflection Due to Shear.*—According to assumption No. 4, the internal shearing stresses in a member produce no deflection. The distribution of the stresses is dependent, not upon the actual deflection of the columns due to shear, but upon the differences between the deflections in the different columns. The shears on the columns are small and the differences between the shears are still smaller; and therefore the assumption that the deflection due to shear is equal to zero will not cause any appreciable error.

30. *Assumption as to Load.*—According to assumption No. 5, the

entire wind load is resisted by the steel frame. The walls of a building no doubt help to resist the wind loads, but the resistance which they exert is uncertain. If a portion of the load is considered as being resisted by the walls the stresses in the steel frame are correspondingly reduced; but the method of determining the stresses is not affected.

## XII. Conclusions.

As a result of the investigation described in this bulletin the following conclusions can be made relative to the methods used to determine the wind stresses in the steel frames of office buildings.

a. Methods II and III of Section II are so inaccurate that they should never be used; I and IV are quite accurate in some cases, but they may give results which are seriously in error.

b. The method presented in Sections VI and VII, and designated as the slope-deflection method, contains no approximations except those in the assumptions. It has been shown that the inaccuracies in the assumptions do not materially affect the results. Therefore the method is very accurate.

c. While the slope-deflection method is long, it could be used in the actual design of a building; but it has its greatest value as a standard by means of which the accuracy of the approximate methods may be determined.

d. The proposed approximate method is short; and, except at points where there are very large changes in the size of the members, gives results which are accurate enough to be used in the actual design of a building.

## TABLE 11.

## PROPERTIES OF THE COLUMNS AND GIRDERS IN THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| | Story Number | Section of Member | | | Area of Section Sq. In. | Moment of Inertia—(Inches)$^4$ | | | | | Length of Columns $=h$ of Girders $=l$ Inches | $K$ for Columns $=\frac{I}{h}$ for Girders $=\frac{I}{l}$ (Inches)$^3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 4 Angles | | | | | |
| | | Web Plate | 4 Angles | Cover Plate | | Web | Primary | Secondary | Cover Plate | Total | | |
| Column $A$ | 1 | 17x$\frac{7}{8}$ | 8x8x$\frac{7}{8}$ | 2–18x1$\frac{1}{4}$ | 112.80 | 358 | 318 | 2190 | 3950 | 6816 | 264 | 25.8 |
| | 2 | 17x$\frac{7}{8}$ | 8x8x$\frac{7}{8}$ | 2–18x1$\frac{1}{4}$ | 112.80 | 358 | 318 | 2190 | 3950 | 6816 | 192 | 35.6 |
| | 3 and 4 | 17x$\frac{7}{8}$ | 8x8x$\frac{7}{8}$ | 2–18x1 | 103.80 | 358 | 318 | 2190 | 3080 | 5946 | 168 | 35.4 |
| | 5 and 6 | 17x$\frac{7}{8}$ | 8x8x$\frac{7}{8}$ | 2–18x$\frac{11}{16}$ | 92.55 | 358 | 318 | 2190 | 2060 | 4926 | 168 | 29.4 |
| | 7 and 8 | 17x$\frac{7}{8}$ | 8x8x$\frac{7}{8}$ | 2–18x$\frac{7}{16}$ | 83.55 | 358 | 318 | 2190 | 1266 | 4132 | 144 | 28.7 |
| | 9 and 10 | 17x$\frac{15}{16}$ | 8x8x$\frac{15}{16}$ | | 72.42 | 384 | 337 | 2315 | | 3036 | 144 | 21.1 |
| | 11 and 12 | 17x$\frac{7}{8}$ | 8x8x$\frac{13}{16}$ | | 64.24 | 358 | 299 | 2050 | | 2707 | 144 | 18.8 |
| | 13 and 14 | 17x$\frac{3}{4}$ | 8x6x$\frac{13}{16}$ | | 53.79 | 308 | 131 | 2195 | | 2634 | 144 | 18.3 |
| | 15 and 16 | 17x$\frac{1}{2}$ | 8x6x$\frac{5}{8}$ | | 42.26 | 205 | 105 | 1745 | | 2055 | 144 | 14.3 |
| | 17 and above | 17x$\frac{1}{2}$ | 8x6x$\frac{9}{16}$ | | 38.86 | 205 | 96 | 1590 | | 1891 | 144 | 13.1 |
| Column $B$ | 1 | 17x$\frac{7}{8}$ | 8x8x$\frac{7}{8}$ | 2–18x1$\frac{1}{4}$ | 112.80 | 358 | 318 | 2190 | 3950 | 6816 | 264 | 25.8 |
| | 2 | 17x$\frac{7}{8}$ | 8x8x$\frac{7}{8}$ | 2–18x1$\frac{1}{4}$ | 112.80 | 358 | 318 | 2190 | 3950 | 6816 | 192 | 35.6 |
| | 3 and 4 | 17x$\frac{7}{8}$ | 8x8x$\frac{7}{8}$ | 2–18x1 | 103.80 | 358 | 318 | 2190 | 3080 | 5946 | 168 | 35.5 |
| | 5 and 6 | 17x$\frac{7}{8}$ | 8x8x$\frac{7}{8}$ | 2–18x$\frac{3}{4}$ | 94.80 | 358 | 318 | 2190 | 2240 | 5106 | 168 | 30.4 |
| | 7 and 8 | 17x$\frac{7}{8}$ | 8x8x$\frac{7}{8}$ | 2–18x$\frac{1}{2}$ | 85.80 | 358 | 318 | 2190 | 1459 | 4325 | 144 | 30.0 |
| | 9 and 10 | 17x$\frac{7}{8}$ | 8x8x$\frac{7}{8}$ | 2–18x$\frac{5}{16}$ | 79.05 | 358 | 318 | 2190 | 892 | 3758 | 144 | 26.1 |
| | 11 and 12 | 17x$\frac{7}{8}$ | 8x8x$\frac{7}{8}$ | | 67.80 | 358 | 318 | 2190 | | 2866 | 144 | 19.9 |
| | 13 and 14 | 17x$\frac{7}{8}$ | 8x6x$\frac{13}{16}$ | | 57.92 | 358 | 131 | 2195 | | 2684 | 144 | 18.6 |
| | 15 and 16 | 17x$\frac{5}{8}$ | 8x6x$\frac{5}{8}$ | | 44.39 | 256 | 105 | 1745 | | 2106 | 144 | 14.6 |
| | 17 and above | 17x$\frac{5}{8}$ | 8x6x$\frac{9}{16}$ | | 41.00 | 256 | 96 | 1544 | | 1896 | 144 | 13.2 |
| Girders in Bay $a$ | 1 | 42x$\frac{3}{8}$ | 6x3$\frac{1}{2}$x$\frac{3}{8}$ | | 29.43 | 2315 | 13 | 5730 | | 8058 | 264 | 30.5 |
| | 2 and 3 | 36x$\frac{3}{8}$ | 6x3$\frac{1}{2}$x$\frac{3}{8}$ | | 27.18 | 1458 | 13 | 4170 | | 5641 | 264 | 21.4 |
| | 4 and 5 | 36x$\frac{3}{8}$ | 5x3$\frac{1}{2}$x$\frac{3}{8}$ | | 25.70 | 1458 | 13 | 3690 | | 5161 | 264 | 19.5 |
| | 6 | 30x$\frac{3}{8}$ | 6x3$\frac{1}{2}$x$\frac{3}{8}$ | | 24.93 | 844 | 13 | 2860 | | 3717 | 264 | 14.1 |
| | 7 | 30x$\frac{3}{8}$ | 5x3$\frac{1}{2}$x$\frac{3}{8}$ | | 23.45 | 844 | 13 | 2530 | | 3387 | 264 | 12.8 |
| | 8 and above | 24x$\frac{3}{8}$ | 5x3$\frac{1}{2}$x$\frac{3}{8}$ | | 21.20 | 432 | 13 | 1580 | | 2025 | 264 | 7.7 |
| Girders in Bay $b$ | 1 | 42x$\frac{3}{8}$ | 6x3$\frac{1}{2}$x$\frac{3}{8}$ | | 29.43 | 2315 | 13 | 5730 | | 8058 | 216 | 37.3 |
| | 2 | 36x$\frac{3}{8}$ | 6x3$\frac{1}{2}$x$\frac{7}{16}$ | | 29.38 | 1458 | 15 | 4830 | | 6303 | 216 | 29.2 |
| | 3 and 4 | 36x$\frac{3}{8}$ | 6x3$\frac{1}{2}$x$\frac{3}{8}$ | | 27.18 | 1458 | 13 | 4170 | | 5641 | 216 | 26.2 |
| | 5 | 36x$\frac{3}{8}$ | 5x3$\frac{1}{2}$x$\frac{3}{8}$ | | 25.70 | 1458 | 13 | 3690 | | 5161 | 216 | 23.8 |
| | 6 | 30x$\frac{3}{8}$ | 6x3$\frac{1}{2}$x$\frac{3}{8}$ | | 24.93 | 844 | 13 | 2860 | | 3717 | 216 | 17.2 |
| | 7 | 30x$\frac{3}{8}$ | 5x3$\frac{1}{2}$x$\frac{3}{8}$ | | 23.45 | 844 | 13 | 2530 | | 3387 | 216 | 15.7 |
| | 8 and above | 24x$\frac{3}{8}$ | 5x3$\frac{1}{2}$x$\frac{3}{8}$ | | 21.20 | 432 | 13 | 1580 | | 2025 | 216 | 9.4 |

## Table 12.

## Numerical Values of the Constants in the Equations of Table 3 for the Symmetrical Three-span Twenty-story Bent shown in Fig. 5.

All quantities are expressed in inches³

| Story No. | Values of $K\left(=\frac{I}{h}\right)$ for | | Values of $K\left(=\frac{I}{l}\right)$ for | | Values of $J\left[=2\Sigma\left(\frac{I}{h}+\frac{I}{l}\right)\right]$ for all members intersecting at the | | Values of $N\,(=2\sum K)$ for all Columns in a Story |
|---|---|---|---|---|---|---|---|
| | Column *A* | Column *B* | Girders in Bay *a* at Top of Story | Girders in Bay *b* at Top of Story | Top of Column *A* | Top of Column *B* | |
| 1 | 25.8 | 25.8 | 30.5 | 37.3 | 183.8 | 258.4 | 206.8 |
| 2 | 35.6 | 35.6 | 21.4 | 29.2 | 184.8 | 243.4 | 284.8 |
| 3 | 35.4 | 35.5 | 21.4 | 26.2 | 184.4 | 237.2 | 283.6 |
| 4 | 35.4 | 35.5 | 19.5 | 26.2 | 168.6 | 223.2 | 283.6 |
| 5 | 29.4 | 30.4 | 19.5 | 23.8 | 156.6 | 208.2 | 239.2 |
| 6 | 29.4 | 30.4 | 14.1 | 17.2 | 144.4 | 183.4 | 239.2 |
| 7 | 28.7 | 30.0 | 12.8 | 15.7 | 140.4 | 177.0 | 234.8 |
| 8 | 28.7 | 30.0 | 7.7 | 9.4 | 115.0 | 146.4 | 234.8 |
| 9 | 21.1 | 26.1 | 7.7 | 9.4 | 99.8 | 138.6 | 188.8 |
| 10 | 21.1 | 26.1 | 7.7 | 9.4 | 95.2 | 126.2 | 188.8 |
| 11 | 18.8 | 19.9 | 7.7 | 9.4 | 90.6 | 113.8 | 154.8 |
| 12 | 18.8 | 19.9 | 7.7 | 9.4 | 89.6 | 111.2 | 154.8 |
| 13 | 18.3 | 18.6 | 7.7 | 9.4 | 88.6 | 108.6 | 147.6 |
| 14 | 18.3 | 18.6 | 7.7 | 9.4 | 80.6 | 100.6 | 147.6 |
| 15 | 14.3 | 14.6 | 7.7 | 9.4 | 72.6 | 92.6 | 115.6 |
| 16 | 14.3 | 14.6 | 7.7 | 9.4 | 70.2 | 89.8 | 115.6 |
| 17 | 13.1 | 13.2 | 7.7 | 9.4 | 67.8 | 87.0 | 105.2 |
| 18 | 13.1 | 13.2 | 7.7 | 9.4 | 67.8 | 87.0 | 105.2 |
| 19 | 13.1 | 13.2 | 7.7 | 9.4 | 67.8 | 87.0 | 105.2 |
| 20 | 13.1 | 13.2 | 7.7 | 9.4 | 41.6 | 60.6 | 105.2 |

# TABLE 13.

## GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | | Story No. 2 | | | Story No. 3 | | | Story No. 4 | | | Story No. 5 | | | |
| | $R_1$ | $\theta_{A1}$ | $\theta_{B1}$ | $R_2$ | $\theta_{A2}$ | $\theta_{B2}$ | $R_3$ | $\theta_{A3}$ | $\theta_{B3}$ | $R_4$ | $\theta_{A4}$ | $\theta_{B4}$ | $R_5$ | $\theta_{A5}$ | $\theta_{B5}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | Co-efficient of .0001 |
| A | —206.8 | 51.6 | 51.6 | | | | | | | | | | | | | —117.2 |
| B | — 77.4 | 183.8 | 30.5 | —106.8 | 35.6 | | | | | | | | | | | 0.0 |
| C | — 77.4 | 30.5 | 295.7 | —106.8 | | 35.6 | | | | | | | | | | 0.0 |
| D | | 71.2 | 71.2 | —284.8 | 71.2 | 71.2 | | | | | | | | | | —78.9 |
| E | | 35.6 | | —106.8 | 184.8 | 21.4 | —106.2 | 35.4 | | | | | | | | 0.0 |
| F | | | 35.6 | —106.8 | 21.4 | 272.6 | —106.5 | | 35.5 | | | | | | | 0.0 |
| G | | | | | 70.8 | 71.0 | —283.6 | 70.8 | 71.0 | | | | | | | —64.5 |
| H | | | | | 35.4 | | —106.2 | 184.4 | 21.4 | —106.2 | 35.4 | | | | | 0.0 |
| I | | | | | | 35.5 | —106.5 | 21.4 | 263.4 | —106.5 | | 35.5 | | | | 0.0 |
| J | | | | | | | | 70.8 | 71.0 | —283.6 | 70.8 | 71.0 | | | | —60.5 |
| K | | | | | | | | 35.4 | | —106.2 | 168.6 | 19.5 | —88.2 | 29.4 | | 0.0 |
| L | | | | | | | | | 35.5 | —106.5 | 19.5 | 249.4 | —91.2 | | 30.4 | 0.0 |

TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 4 | | | Story No. 5 | | | Story No. 6 | | | Story No. 7 | | | Story No. 8 | | | |
| | $R_4$ | $\theta_{A4}$ | $\theta_{B4}$ | $R_5$ | $\theta_{A5}$ | $\theta_{B5}$ | $R_6$ | $\theta_{A6}$ | $\theta_{B6}$ | $R_7$ | $\theta_{A7}$ | $\theta_{B7}$ | $R_8$ | $\theta_{A8}$ | $\theta_{B8}$ | Co-efficient of .0001 |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | |
| M | | 58.8 | 60.8 | —239.2 | 58.8 | 60.8 | | | | | | | | | | —56.5 |
| N | | 29.4 | | — 88.2 | 156.6 | 19.5 | — 88.2 | 29.4 | | | | | | | | 0.0 |
| O | | | 30.4 | — 91.2 | 19.5 | 232.0 | — 91.2 | | —30.4 | | | | | | | 0.0 |
| P | | | | | 58.8 | 60.8 | —239.2 | 58.8 | 60.8 | | | | | | | —52.5 |
| Q | | | | | 29.4 | | — 88.2 | 144.4 | 14.1 | — 86.1 | 28.7 | | | | | 0.0 |
| R | | | | | | 30.4 | — 91.2 | 14.1 | 200.6 | — 90.0 | | 30.0 | | | | 0.0 |
| S | | | | | | | | 57.4 | 60.0 | —234.8 | 57.4 | 60.0 | | | | —41.8 |
| T | | | | | | | | 28.7 | | — 86.1 | 140.4 | 12.8 | —86.1 | 28.7 | | 0.0 |
| U | | | | | | | | | 30.0 | — 90.0 | 12.8 | 192.7 | —90.0 | | 30.0 | 0.0 |

## TABLE 13.—(Continued).

## GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 7 | | | Story No. 8 | | | Story No. 9 | | | Story No. 10 | | | Story No. 11 | | | |
| | $R_7$ | $\theta_{A7}$ | $\theta_{B7}$ | $R_8$ | $\theta_{A8}$ | $\theta_{B8}$ | $R_9$ | $\theta_{A9}$ | $\theta_{B9}$ | $R_{10}$ | $\theta_{A10}$ | $\theta_{B10}$ | $R_{11}$ | $\theta_{A11}$ | $\theta_{B11}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | Co-efficient of .0001 |
| V | | 57.4 | 60.0 | —234.8 | 57.4 | 60.0 | | | | | | | | | | —38.7 |
| W | | 28.7 | | — 86.1 | 115.0 | 7.7 | — 63.3 | 21.1 | | | | | | | | 0.0 |
| X | | | 30.0 | — 90.0 | 7.7 | 155.8 | — 78.3 | | 26.1 | | | | | | | 0.0 |
| Y | | | | | 42.2 | 52.2 | —188.8 | 42.2 | 52.2 | | | | | | | —35.8 |
| Z | | | | | 21.1 | | — 63.3 | 99.8 | 7.7 | — 63.3 | 21.1 | | | | | 0.0 |
| a | | | | | | 26.1 | — 78.3 | 7.7 | 148.0 | — 78.3 | | 26.1 | | | | 0.0 |
| b | | | | | | | | 42.2 | 52.2 | —188.8 | 42.2 | 52.2 | | | | —32.8 |
| c | | | | | | | | 21.1 | | — 63.3 | 95.2 | 7.7 | —56.4 | 18.8 | | 0.0 |
| d | | | | | | | | | 26.1 | — 78.3 | 7.7 | 135.6 | —59.7 | | 19.9 | 0.0 |

## TABLE 13.—(Continued).

## GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 10 | | | Story No. 11 | | | Story No. 12 | | | Story No. 13 | | | Story No. 14 | | | |
| | $R_{10}$ | $\theta_{A10}$ | $\theta_{B10}$ | $R_{11}$ | $\theta_{A11}$ | $\theta_{B11}$ | $R_{12}$ | $\theta_{A12}$ | $\theta_{B12}$ | $R_{13}$ | $\theta_{A13}$ | $\theta_{B13}$ | $R_{14}$ | $\theta_{A14}$ | $\theta_{B14}$ | |
| | Coefficient of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | Co-efficient of .0001 |
| e | | 37.6 | 39.8 | —154.8 | 37.6 | 39.8 | | | | | | | | | | —29.8 |
| f | | 18.8 | | — 56.4 | 90.6 | 7.7 | — 56.4 | 18.8 | | | | | | | | 0.0 |
| g | | | 19.9 | — 59.7 | 7.7 | 123.2 | — 59.7 | | 19.9 | | | | | | | 0.0 |
| h | | | | | 37.6 | 39.8 | —154.8 | 37.6 | 39.8 | | | | | | | —26.8 |
| i | | | | | 18.8 | | — 56.4 | 89.6 | 7.7 | — 54.9 | 18.3 | | | | | 0.0 |
| j | | | | | | 19.9 | — 59.7 | 7.7 | 120.6 | — 55.8 | | 18.6 | | | | 0.0 |
| k | | | | | | | | | | —147.6 | 36.6 | 37.2 | | | | —23.8 |
| l | | | | | | | | | | — 54.9 | 88.6 | 7.7 | —54.9 | 18.3 | | 0.0 |
| m | | | | | | | | | | — 55.8 | 7.7 | 118.0 | —55.8 | | 18.6 | 0.0 |

TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 13 | | | Story No. 14 | | | Story No. 15 | | | Story No. 16 | | | Story No. 17 | | | |
| | $R_{13}$ | $\theta_{A13}$ | $\theta_{B13}$ | $R_{14}$ | $\theta_{A14}$ | $\theta_{B14}$ | $R_{15}$ | $\theta_{A15}$ | $\theta_{B15}$ | $R_{16}$ | $\theta_{A16}$ | $\theta_{B16}$ | $R_{17}$ | $\theta_{A17}$ | $\theta_{B17}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | Co-efficient of .0001 |
| n | | 36.6 | 37.2 | —147.6 | 36.6 | 37.2 | | | | | | | | | | —20.9 |
| o | | 18.3 | | — 54.9 | 80.6 | 7.7 | — 42.9 | 14.3 | | | | | | | | 0.0 |
| p | | | 18.6 | — 55.8 | 7.7 | 110.0 | — 43.8 | | 14.6 | | | | | | | 0.0 |
| q | | | | | 28.6 | 29.2 | —115.6 | 28.6 | 29.2 | | | | | | | —17.9 |
| r | | | | | 14.3 | | — 42.9 | 72.6 | 7.7 | — 42.9 | 14.3 | | | | | 0.0 |
| s | | | | | | 14.6 | — 43.8 | 7.7 | 102.0 | — 43.8 | | 14.6 | | | | 0.0 |
| t | | | | | | | | 28.6 | 29.2 | —115.6 | 28.6 | 29.2 | | | | —14.9 |
| u | | | | | | | | 14.3 | | — 42.9 | 70.2 | 7.7 | —39.3 | 13.1 | | 0.0 |
| v | | | | | | | | | 14.6 | — 43.8 | 7.7 | 99.2 | —39.6 | | 13.2 | 0.0 |

## TABLE 13.—(Continued).

## GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 16 | | | Story No. 17 | | | Story No. 18 | | | Story No. 19 | | | Story No. 20 | | | |
| | $R_{16}$ | $\theta_{A16}$ | $\theta_{B16}$ | $R_{17}$ | $\theta_{A17}$ | $\theta_{B17}$ | $R_{18}$ | $\theta_{A18}$ | $\theta_{B18}$ | $R_{19}$ | $\theta_{A19}$ | $\theta_{B19}$ | $R_{20}$ | $\theta_{A20}$ | $\theta_{B20}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | | | | Co-efficient of .0001 |
| w | | 26.2 | 26.4 | —105.2 | 26.2 | 26.4 | | | | | | | | | | —11.9 |
| x | | 13.1 | | — 39.3 | 67.8 | 7.7 | — 39.3 | 13.1 | | | | | | | | 0.0 |
| y | | | 13.2 | — 39.6 | 7.7 | 96.4 | — 39.6 | | 13.2 | | | | | | | 0.0 |
| z | | | | | 26.2 | 26.4 | —105.2 | 26.2 | 26.4 | | | | | | | —8.94 |
| A′ | | | | | 13.1 | | — 39.3 | 67.8 | 7.7 | — 39.3 | 13.1 | | | | | 0.0 |
| B′ | | | | | | 13.2 | — 39.6 | 7.7 | 96.4 | — 39.6 | | 13.2 | | | | 0.0 |
| C′ | | | | | | | | 26.2 | 26.4 | —105.2 | 26.2 | 26.4 | | | | —5.96 |
| D′ | | | | | | | | 13.1 | | — 39.3 | 67.8 | 7.7 | — 39.3 | 13.1 | | 0.0 |
| E′ | | | | | | | | | 13.2 | — 39.6 | 7.7 | 96.4 | — 39.6 | | 13.2 | 0.0 |
| F′ | | | | | | | | | | | 26.2 | 26.4 | —105.2 | 26.2 | 26.4 | —2.98 |
| G′ | | | | | | | | | | | 13.1 | | — 39.3 | 41.6 | 7.7 | 0.0 |
| H′ | | | | | | | | | | | | 13.2 | — 39.6 | 7.7 | 70.0 | 0.0 |

## TABLE 14.

## ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | | Story No. 2 | | | Story No. 3 | | | Story No. 4 | | | |
| | $R_1$ | $\theta_{A1}$ | $\theta_{B1}$ | $R_2$ | $\theta_{A2}$ | $\theta_{B2}$ | $R_3$ | $\theta_{A3}$ | $\theta_{B3}$ | $R_4$ | $\theta_{A4}$ | $\theta_{B4}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | Co-efficient of .0001 |
| A | 1.0000 | —0.2495 | —0.2495 | | | | | | | | | | 0.5667 |
| B | 1.0000 | —2.3747 | —0.3940 | 1.3798 | —0.4599 | | | | | | | | 0.0000 |
| C | 1.0000 | —0.3940 | —3.8204 | 1.3798 | | —0.4599 | | | | | | | 0.0000 |
| A-B | 0 | 2.1252 | 0.1445 | —1.3798 | 0.4599 | | | | | | | | 0.5667 |
| B-C | 0 | —1.9806 | 3.4263 | 0 | —0.4599 | 0.4599 | | | | | | | 0.0000 |
| 1 | | 1.0000 | 0.0680 | —0.6494 | 0.2164 | | | | | | | | 0.2667 |
| 2 | | 1.0000 | —1.7300 | | 0.2322 | —0.2322 | | | | | | | 0.0000 |
| D | | 1.0000 | 1.0000 | —4.0000 | 1.0000 | 1.0000 | | | | | | | —1.1082 |
| E | | 1.0000 | | —3.0000 | 5.1912 | 0.6012 | —2.9834 | 0.9944 | | | | | 0.0000 |
| 1-2 | | 0 | 1.7980 | —0.6494 | —0.0158 | 0.2322 | | | | | | | 0.2667 |
| 2-D | | 0 | —2.7300 | 4.0000 | 0.7678 | —1.2322 | | | | | | | 1.1082 |
| D-E | | 0 | 1.0000 | —1.0000 | —4.1912 | 0.3988 | 2.9834 | —0.9944 | | | | | —1.1082 |
| 3 | | | 1 | —0.3611 | —0.0088 | 0.1292 | | | | | | | 0.1483 |
| 4 | | | 1 | —1.4652 | 0.2812 | 0.4513 | | | | | | | —0.4059 |
| 5 | | | 1 | —1.0000 | —4.1912 | 0.3988 | 2.9834 | —0.9941 | | | | | —1.1082 |
| F | | | 1 | —3.0000 | 0.6012 | 7.6575 | —2.9915 | | 0.9972 | | | | 0.0000 |
| 3-4 | | | 0 | 1.1040 | —0.2900 | —0.3222 | | | | | | | 0.5543 |
| 4-5 | | | 0 | —0.4652 | 4.4724 | 0.0525 | —2.9834 | 0.9944 | | | | | 0.7023 |
| 5-F | | | 0 | 2.0000 | —4.7924 | —7.2587 | 5.9749 | —0.9944 | —0.9972 | | | | —1.1082 |
| 6 | | | | 1 | —0.2627 | —0.2918 | | | | | | | 0.5021 |
| 7 | | | | 1 | —9.6140 | —0.1128 | 6.4127 | —2.1378 | | | | | —1.5095 |
| 8 | | | | 1 | —2.3962 | —3.6293 | 2.9874 | —0.4972 | —0.4986 | | | | —0.5541 |
| 6-7 | | | | 0 | 9.3513 | —0.1790 | —6.4127 | 2.1378 | | | | | 2.0116 |
| 7-8 | | | | 0 | —7.2178 | 3.5165 | 3.4252 | —1.6406 | 0.4986 | | | | —0.9554 |
| 9 | | | | | 1 | —0.0191 | —0.6857 | 0.2286 | | | | | 0.2151 |
| 10 | | | | | 1 | —0.4872 | —0.4746 | 0.2273 | —0.0691 | | | | 0.1324 |
| G | | | | | 1 | 1.0028 | —4.0055 | 1.0000 | 1.0028 | | | | —0.9110 |
| H | | | | | 1 | | —3.0000 | 5.2091 | 0.6045 | —3.0000 | 1.0000 | | 0.0000 |
| 9-10 | | | | | 0 | 0.4681 | —0.2111 | 0.0013 | 0.0691 | | | | 0.0827 |
| 10-G | | | | | 0 | —1.4900 | 3.5309 | —0.7727 | —1.0719 | | | | 1.0434 |
| G-H | | | | | 0 | 1.0028 | —1.0055 | —4.2091 | 0.3983 | 3.0000 | —1.0000 | | —0.9110 |
| 11 | | | | | | 1 | —0.4511 | 0.0027 | 0.1476 | | | | 0.1768 |
| 12 | | | | | | 1 | —2.3700 | 0.5186 | 0.7194 | | | | —0.6990 |
| 13 | | | | | | 1 | —1.0028 | —4.1971 | 0.3971 | 2.9915 | —0.9972 | | —0.9084 |
| I | | | | | | 1 | —3.0000 | 0.6028 | 7.4200 | —3.0000 | | 1.0000 | 0.0000 |
| 11-12 | | | | | | 0 | 1.0189 | —0.5159 | —0.5718 | | | | 0.8758 |
| 12-13 | | | | | | 0 | —1.3672 | 4.7157 | 0.3223 | —2.9915 | 0.9972 | | 0.2094 |
| 13-I | | | | | | 0 | 1.9972 | —4.7999 | —7.0230 | 5.9915 | —0.9972 | —1.0000 | —0.9084 |

## TABLE 14.—(Continued).

## ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 3 | | | Story No. 4 | | | Story No. 5 | | | Story No. 6 | | | |
| | $R_3$ | $\theta_{A3}$ | $\theta_{B3}$ | $R_4$ | $\theta_{A4}$ | $\theta_{B4}$ | $R_5$ | $\theta_{A5}$ | $\theta_{B5}$ | $R_6$ | $\theta_{A6}$ | $\theta_{B6}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | Co-efficient of .0001 |
| 14 | 1 | —0.2689 | —0.2980 | | | | | | | | | | 0.4564 |
| 15 | 1 | —3.4490 | —0.2357 | 2.1881 | —0.7294 | | | | | | | | —0.1532 |
| 16 | 1 | —2.4033 | —3.5179 | 2.9999 | —0.4993 | —0.5007 | | | | | | | —0.4548 |
| 14–15 | 0 | 3.1802 | —0.0623 | —2.1881 | 0.7294 | | | | | | | | 0.6096 |
| 14–16 | 0 | 2.1344 | 3.2199 | —2.9999 | 0.4993 | 0.5007 | | | | | | | 0.9112 |
| 17 | | 1 | —0.0196 | —0.6881 | 0.2294 | | | | | | | | 0.1917 |
| 18 | | 1 | 1.5086 | —1.4057 | 0.2339 | 0.2346 | | | | | | | 0.4270 |
| J | | 1 | 1.0028 | —4.0057 | 1.0000 | 1.0028 | | | | | | | —0.8546 |
| K | | 1 | | —3.0000 | 4.7628 | 0.5508 | —2.4914 | 0.8302 | | | | | 0.0000 |
| 17–18 | | 0 | —1.5282 | 0.7176 | —0.0046 | —0.2346 | | | | | | | —0.2353 |
| 18–K | | 0 | 1.5086 | 1.5943 | —4.5290 | —0.3162 | 2.4914 | —0.8302 | | | | | 0.4270 |
| J–K | | 0 | 1.0028 | —1.0057 | —3.7628 | 0.4520 | 2.4914 | —0.8302 | | | | | —0.8546 |
| 19 | | | 1 | —0.4696 | 0.0030 | 0.1535 | | | | | | | 0.1540 |
| 20 | | | 1 | 1.0568 | —3.0019 | —0.2096 | 1.6514 | —0.5503 | | | | | 0.2830 |
| 21 | | | 1 | —1.0028 | —3.7520 | 0.4507 | 2.4842 | —0.8279 | | | | | —0.8521 |
| L | | | 1 | —3.0000 | 0.5496 | 7.0255 | —2.5691 | | 0.8564 | | | | 0.0000 |
| 19–20 | | | 0 | —1.5264 | 3.0049 | 0.3631 | —1.6514 | 0.5503 | | | | | —0.1290 |
| 20–21 | | | 0 | 2.0596 | 0.7501 | —0.6603 | —0.8328 | 0.2776 | | | | | 1.1352 |
| 20–L | | | 0 | 4.0568 | —3.5515 | —7.2351 | 4.2205 | —0.5503 | —0.8564 | | | | 0.2830 |
| 22 | | | | 1 | —1.9687 | —0.2379 | 1.0819 | —0.3605 | | | | | 0.0846 |
| 23 | | | | 1 | 0.3642 | —0.3206 | —0.4044 | 0.1348 | | | | | 0.5512 |
| 24 | | | | 1 | —0.8754 | —1.7836 | 1.0404 | —0.1357 | —0.2111 | | | | 0.0697 |
| 22–23 | | | | 0 | —2.3329 | 0.0827 | 1.4863 | —0.4953 | | | | | —0.4666 |
| 23–24 | | | | 0 | 1.2397 | 1.4630 | —1.4448 | 0.2705 | 0.2111 | | | | 0.4815 |
| 25 | | | | | 1 | —0.0355 | —0.6371 | 0.2123 | | | | | 0.2000 |
| 26 | | | | | 1 | 1.1803 | —1.1657 | 0.2182 | 0.1703 | | | | 0.3884 |
| M | | | | | 1 | 1.0340 | —4.0684 | 1.0000 | 1.0340 | | | | —0.9609 |
| N | | | | | 1 | | —3.0000 | 5.3260 | 0.6632 | —3.0000 | 1.0000 | | 0.0000 |
| 25–26 | | | | | 0 | —1.2158 | 0.5286 | —0.0059 | —0.1703 | | | | —0.1884 |
| 25–M | | | | | 0 | —1.0695 | 3.4313 | —0.7877 | —1.0340 | | | | 1.1609 |
| M–N | | | | | 0 | 1.0340 | —1.0684 | —4.3260 | 0.3708 | 3.0000 | —1.0000 | | —0.9609 |
| 27 | | | | | | 1 | —0.4348 | 0.0048 | 0.1401 | | | | 0.1550 |
| 28 | | | | | | 1 | —3.2083 | 0.7365 | 0.9668 | | | | —1.0855 |
| 29 | | | | | | 1 | —1.0333 | —4.1842 | 0.3586 | 2.9016 | —0.9672 | | —0.9294 |
| O | | | | | | 1 | —3.0000 | —0.6415 | 7.6321 | —3.0000 | | 1.0000 | 0.0000 |
| 27–28 | | | | | | 0 | 2.7735 | —0.7316 | —0.8267 | | | | 1.2405 |
| 28–29 | | | | | | 0 | —2.1750 | 4.9207 | 0.6082 | —2.9016 | 0.9672 | | —0.1561 |
| 29–O | | | | | | 0 | 1.9667 | —4.8257 | —7.2735 | 5.9016 | —0.9672 | —1.0000 | —0.9294 |
| 30 | | | | | | | 1 | —0.2638 | —0.2981 | | | | 0.4473 |
| 31 | | | | | | | 1 | —2.2623 | —0.2796 | 1.3341 | —0.4447 | | 0.0718 |
| 32 | | | | | | | 1 | —2.4540 | —3.6986 | 3.0004 | —0.4918 | —0.5085 | —0.4726 |
| 30–31 | | | | | | | 0 | 1.9985 | —0.0185 | —1.3341 | 0.4447 | | 0.3755 |
| 30–32 | | | | | | | 0 | 2.1902 | 3.4005 | —3.0004 | 0.4918 | 0.5085 | 0.9199 |

## TABLE 14.—(Continued).

### ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation. | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 5 | | | Story No. 6 | | | Story No. 7 | | | Story No. 8 | | | |
| | $R_5$ | $\theta_{A5}$ | $\theta_{B5}$ | $R_6$ | $\theta_{A6}$ | $\theta_{B6}$ | $R_7$ | $\theta_{A7}$ | $\theta_{B7}$ | $R_8$ | $\theta_{A8}$ | $\theta_{B8}$ | Co-efficient of .0001 |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height. | | | | | | | | | | | | |
| 33 | | 1 | —0.0092 | —0.6676 | 0.2225 | | | | | | | | 0.1879 |
| 34 | | 1 | 1.5526 | —1.3698 | 0.2246 | 0.2322 | | | | | | | 0.4200 |
| P | | 1 | 1.0341 | —4.0682 | 1.0000 | 1.0341 | | | | | | | —0.8929 |
| Q | | 1 | | —3.0000 | 4.9120 | 0.4796 | —2.9285 | 0.9762 | | | | | 0.0000 |
| 33–34 | | 0 | —1.5618 | 0.7022 | —0.0020 | —0.2322 | | | | | | | —0.2321 |
| 33–P | | 0 | —1.0433 | 3.4006 | —0.7775 | —1.0341 | | | | | | | 1.0808 |
| P-Q | | 0 | 1.0341 | —1.0682 | —3.9120 | 0.5545 | 2.9285 | —0.9762 | | | | | —0.8929 |
| 35 | | | 1 | —0.4497 | 0.0013 | 0.1487 | | | | | | | 0.1486 |
| 36 | | | 1 | —3.2593 | 0.7452 | 0.9911 | | | | | | | —1.0359 |
| 37 | | | 1 | —1.0330 | —3.7818 | 0.5362 | 2.8320 | —0.9440 | | | | | —0.8634 |
| R | | | 1 | —3.0000 | 0.4638 | 6.5993 | —2.9606 | | 0.9870 | | | | 0.0000 |
| 35–36 | | | 0 | 2.8097 | —0.7439 | —0.8425 | | | | | | | 1.1845 |
| 36–37 | | | 0 | —2.2263 | 4.5270 | 0.4549 | —2.8320 | 0.9440 | | | | | —0.1724 |
| 37–R | | | 0 | 1.9670 | —4.2456 | —6.0631 | 5.7926 | —0.9440 | —0.9870 | | | | —0.8634 |
| 38 | | | | 1 | —0.2648 | —0.2998 | | | | | | | 0.4216 |
| 39 | | | | 1 | —2.0335 | —0.2044 | 1.2721 | —0.4240 | | | | | 0.0775 |
| 40 | | | | 1 | —2.1586 | —3.0827 | 2.9451 | —0.4799 | —0.5018 | | | | —0.4390 |
| 38–39 | | | | 0 | 1.7687 | —0.0955 | —1.2721 | 0.4240 | | | | | 0.3441 |
| 38–40 | | | | 0 | 1.8938 | 2.7828 | —2.9451 | 0.4799 | 0.5018 | | | | 0.8606 |
| 41 | | | | | 1 | —0.0540 | —0.7192 | 0.2397 | | | | | 0.1946 |
| 42 | | | | | 1 | 1.4695 | —1.5553 | 0.2534 | 0.2650 | | | | 0.4544 |
| S | | | | | 1 | 1.0453 | —4.0909 | 1.0000 | 1.0453 | | | | —0.7281 |
| T | | | | | 1 | | —3.0000 | 4.8920 | 0.4460 | —3.0000 | 1.0000 | | 0.0000 |
| 41–42 | | | | | 0 | —1.5235 | 0.8361 | —0.0137 | —0.2650 | | | | —0.2599 |
| 41–S | | | | | 0 | —1.0993 | 3.3717 | —0.7603 | —1.0453 | | | | 0.9227 |
| S–T | | | | | 0 | 1.0453 | —1.0909 | —3.8920 | 0.5993 | 3.0000 | —1.0000 | | —0.7281 |
| 43 | | | | | | 1 | —0.5489 | 0.0090 | 0.1739 | | | | 0.1706 |
| 44 | | | | | | 1 | —3.0672 | 0.6916 | 0.9510 | | | | —0.8394 |
| 45 | | | | | | 1 | —1.0436 | —3.7234 | 0.5733 | 2.8700 | —0.9567 | | 0.6964 |
| U | | | | | | 1 | —3.0000 | 0.4267 | 6.4233 | —3.0000 | | 1.0000 | 0.0000 |
| 43–44 | | | | | | 0 | 2.5183 | —0.6826 | —0.7781 | | | | 1.0100 |
| 44–45 | | | | | | 0 | —2.0236 | 4.4150 | 0.3777 | —2.8700 | 0.9567 | | —0.1430 |
| 45–U | | | | | | 0 | 1.9564 | —4.1501 | —5.8500 | 5.8700 | —0.9567 | —1.0000 | —0.6964 |
| 46 | | | | | | | 1 | —0.2711 | —0.3090 | | | | 0.4011 |
| 47 | | | | | | | 1 | —2.1818 | —0.1867 | 1.4184 | —0.4728 | | 0.0706 |
| 48 | | | | | | | 1 | —2.1213 | —2.9903 | 3.0006 | —0.4890 | —0.5111 | —0.3560 |
| 46–47 | | | | | | | 0 | 1.9108 | —0.1223 | —1.4184 | 0.4728 | | 0.3304 |
| 46–48 | | | | | | | 0 | 1.8502 | 2.6813 | —3.0006 | 0.4890 | 0.5111 | 0.7570 |

## TABLE 14.—(Continued).

## ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 7 | | | Story No. 8 | | | Story No. 9 | | | Story No. 10 | | | |
| | $R_7$ | $\theta_{A7}$ | $\theta_{B7}$ | $R_8$ | $\theta_{A8}$ | $\theta_{B8}$ | $R_9$ | $\theta_{A9}$ | $\theta_{B9}$ | $R_{10}$ | $\theta_{A10}$ | $\theta_{B10}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | Coefficient of .0001 |
| 49 | | 1 | —0.0640 | —0.7422 | 0.2474 | | | | | | | | 0.1729 |
| 50 | | 1 | 1.4491 | —1.6219 | 0.2643 | 0.2763 | | | | | | | 0.4093 |
| V | | 1 | 1.0453 | —4.0909 | 1.0000 | 1.0453 | | | | | | | —0.6741 |
| W | | 1 | | —3.0000 | 4.0073 | 0.2683 | —2.2056 | 0.7352 | | | | | 0.0000 |
| 49–50 | | 0 | —1.5131 | 0.8797 | —0.0169 | —0.2763 | | | | | | | —0.2363 |
| 49–V | | 0 | —1.1093 | 3.3487 | —0.7526 | —1.0453 | | | | | | | 0.8470 |
| V–W | | 0 | 1.0453 | —1.0909 | —3.0073 | —0.7770 | 2.2056 | —0.7352 | | | | | —0.6741 |
| 51 | | | 1 | —0.5814 | 0.0112 | 0.1826 | | | | | | | 0.1562 |
| 52 | | | 1 | —3.0186 | 0.6784 | 0.9423 | | | | | | | —0.7636 |
| 53 | | | 1 | —1.0436 | —2.8769 | 0.7433 | 2.1100 | —0.7036 | | | | | —0.6449 |
| X | | | 1 | —3.0000 | 0.2567 | 5.1933 | —2.6100 | | 0.8700 | | | | 0.0000 |
| 51–52 | | | 0 | 2.4372 | —0.6672 | —0.7598 | | | | | | | 0.9198 |
| 52–53 | | | 0 | —1.9750 | 3.5553 | 0.1990 | —2.1100 | 0.7036 | | | | | —0.1187 |
| 53–X | | | 0 | 1.9564 | —3.1336 | —4.4500 | 4.7200 | —0.7036 | —0.8700 | | | | —0.6449 |
| 54 | | | | 1 | —0.2738 | —0.3117 | | | | | | | 0.3774 |
| 55 | | | | 1 | —1.8002 | —0.1008 | 1.0685 | —0.3563 | | | | | 0.0601 |
| 56 | | | | 1 | —1 6019 | —2.2747 | 2.4127 | —0.3597 | —0.4447 | | | | —0.3296 |
| 54–55 | | | | 0 | 1.5265 | —0.2110 | —1.0685 | 0.3563 | | | | | 0.3173 |
| 54–56 | | | | 0 | 1.3281 | 1.9630 | —2.4127 | 0.3597 | 0.4447 | | | | 0.7070 |
| 57 | | | | | 1 | —0.1382 | —0.7000 | 0.2334 | | | | | 0.2079 |
| 58 | | | | | 1 | 1.4779 | —1.8165 | 0.2708 | 0.3348 | | | | 0.5323 |
| Y | | | | | 1 | 1.2371 | —4.4743 | 1.0000 | 1.2371 | | | | —0.8484 |
| Z | | | | | 1 | | —3.0000 | 4.7299 | 0.3649 | —3.0000 | 1.0000 | | 0.0000 |
| 57–58 | | | | | 0 | —1.6161 | 1.1165 | —0.0374 | —0.3348 | | | | —0.3245 |
| 57–Y | | | | | 0 | —1.3753 | 3.7743 | —0.7666 | —1.2371 | | | | 1.0562 |
| Y–Z | | | | | 0 | 1.2371 | —1.4743 | —3.7299 | 0.8722 | 3.0000 | —1.0000 | | —0.8484 |
| 59 | | | | | | 1 | —0.6908 | 0.0231 | 0.2072 | | | | 0.2008 |
| 60 | | | | | | 1 | —2.7443 | 0.5574 | 0.8995 | | | | —0.7681 |
| 61 | | | | | | 1 | —1.1918 | —3.0151 | 0.7051 | 2.4253 | —0.8084 | | —0.6859 |
| a | | | | | | 1 | —3.0000 | 0.2950 | 5.6705 | —3.0000 | | 1.0000 | 0.0000 |
| 59–60 | | | | | | 0 | 2.0535 | —0.5342 | —0.6924 | | | | 0.9689 |
| 60–61 | | | | | | 0 | —1.5525 | 3.5725 | 0.1945 | —2.4253 | 0.8084 | | —0.0822 |
| 61–a | | | | | | 0 | 1.8082 | —3.3102 | —4.9654 | 5.4253 | —0.8084 | —1.0000 | —0.6859 |
| 62 | | | | | | | 1 | —0.2602 | —0.3372 | | | | 0.4719 |
| 63 | | | | | | | 1 | —2.3013 | —0.1253 | 1.5624 | —0.5207 | | 0.0529 |
| 64 | | | | | | | 1 | —1.8308 | —2.7462 | 3.0000 | —0.4471 | —0.5531 | —0.3793 |
| 62–63 | | | | | | | 0 | 2.0411 | —0.2119 | —1.5624 | 0.5207 | | 0.4189 |
| 62–64 | | | | | | | 0 | 1.5706 | 2.4090 | —3.0000 | 0.4471 | 0.5531 | 0.8512 |

## TABLE 14.—(Continued).

## ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 9 | | | Story No. 10 | | | Story No. 11 | | | Story No. 12 | | | |
| | $R_9$ | $\theta_{A9}$ | $\theta_{B9}$ | $R_{10}$ | $\theta_{A10}$ | $\theta_{B10}$ | $R_{11}$ | $\theta_{A11}$ | $\theta_{B11}$ | $R_{12}$ | $\theta_{A12}$ | $\theta_{B12}$ | Co-efficient of .0001 |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | |
| 65 | | 1 | -0.1038 | -0.7655 | 0.2551 | | | | | | | | 0.2053 |
| 66 | | 1 | 1.5340 | -1.9103 | 0.2847 | 0.3522 | | | | | | | 0.5420 |
| b | | 1 | 1.2371 | -4.4743 | 1.0000 | 1.2371 | | | | | | | -0.7771 |
| c | | 1 | | -3.0000 | 4.5120 | 0.3649 | -2.6730 | 0.8911 | | | | | 0.0000 |
| 65-66 | | 0 | -1.6378 | 1.1448 | -0.0296 | -0.3522 | | | | | | | -0.3367 |
| 65-b | | 0 | -1.3409 | 3.7088 | -0.7449 | -1.2371 | | | | | | | 0.9824 |
| b-c | | 0 | 1.2371 | -1.4743 | -3.5120 | 0.8722 | 2.6730 | -0.8911 | | | | | -0.7771 |
| 67 | | | 1 | -0.6991 | 0.0180 | 0.2151 | | | | | | | 0.2056 |
| 68 | | | 1 | -2.7660 | 0.5555 | 0.9226 | | | | | | | -0.7327 |
| 69 | | | 1 | -1.1918 | -2.8392 | 0.7050 | 2.1610 | -0.7204 | | | | | -0.6283 |
| d | | | 1 | -3.0000 | 0.2950 | 5.1954 | -2.2873 | | 0.7624 | | | | 0.0000 |
| 67-68 | | | 0 | 2.0669 | -0.5375 | -0.7075 | | | | | | | 0.9383 |
| 68-69 | | | 0 | -1.5742 | 3.3947 | 0.2176 | -2.1610 | 0.7204 | | | | | -0.1044 |
| 69-d | | | 0 | 1.8082 | -3.1342 | -4.4904 | 4.4483 | -0.7204 | -0.7624 | | | | -0.6283 |
| 70 | | | | 1 | -0.2601 | -0.3423 | | | | | | | 0.4540 |
| 71 | | | | 1 | -2.1566 | -0.1382 | 1.3728 | -0.4576 | | | | | 0.0663 |
| 72 | | | | 1 | -1.7333 | -2.4833 | 2.4602 | -0.3984 | -0.4216 | | | | -0.3474 |
| 70-71 | | | | 0 | 1.8865 | -0.2041 | -1.3728 | 0.4576 | | | | | 0.3877 |
| 70-72 | | | | 0 | 1.4732 | 2.1310 | -2.4602 | 0.3984 | 0.4216 | | | | 0.8014 |
| 73 | | | | | 1 | -0.1082 | -0.7277 | 0.2426 | | | | | 0.2055 |
| 74 | | | | | 1 | 1.4464 | -1.6700 | 0.2705 | 0.2862 | | | | 0.5441 |
| e | | | | | 1 | 1.0585 | -4.1170 | 1.0000 | 1.0585 | | | | -0.7926 |
| f | | | | | 1 | | -3.0000 | 4.8195 | 0.4096 | -3.0000 | 1.0000 | | 0.0000 |
| 73-74 | | | | | 0 | -1.5547 | 0.9422 | -0.0278 | -0.2862 | | | | -0.3385 |
| 73-e | | | | | 0 | -1.1668 | 3.3893 | -0.7574 | -1.0585 | | | | 0.9981 |
| e-f | | | | | 0 | 1.0585 | -1.1170 | -3.8195 | 0.6499 | 3.0000 | -1.0000 | | -0.7926 |
| 75 | | | | | | 1 | -0.6061 | 0.0179 | 0.1841 | | | | 0.2178 |
| 76 | | | | | | 1 | -2.9051 | 0.6492 | 0.9076 | | | | -0.8555 |
| 77 | | | | | | 1 | -1.0553 | -3.6083 | 0.6140 | 2.8343 | -0.9449 | | -0.7488 |
| g | | | | | | 1 | -3.0000 | 0.3869 | 6.1913 | -3.0000 | | 1.0000 | 0.0000 |
| 75-76 | | | | | | 0 | 2.2990 | -0.6313 | -0.7235 | | | | 1.0733 |
| 76-77 | | | | | | 0 | -1.8498 | 4.2575 | 0.2936 | -2.8343 | 0.9449 | | -0.1067 |
| 77-g | | | | | | 0 | 1.9447 | -3.9952 | -5.5773 | 5.8343 | -0.9449 | -1.0000 | -0.7488 |
| 78 | | | | | | | 1 | -0.2746 | -0.3147 | | | | 0.4669 |
| 79 | | | | | | | 1 | -2.3018 | -0.1587 | 1.5323 | -0.5109 | | 0.0577 |
| 80 | | | | | | | 1 | -2.0547 | -2.8682 | 3.0003 | -0.4859 | -0.5143 | -0.3851 |
| 78-79 | | | | | | | 0 | 2.0272 | -0.1560 | -1.5323 | 0.5109 | | 0.4092 |
| 78-80 | | | | | | | 0 | 1.7801 | 2.5535 | -3.0000 | 0.4859 | 0.5143 | 0.8519 |

## TABLE 14.—(Continued).

### ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 11 | | | Story No. 12 | | | Story No. 13 | | | Story No. 14 | | | |
| | $R_{11}$ | $\theta_{A11}$ | $\theta_{B11}$ | $R_{12}$ | $\theta_{A12}$ | $\theta_{B12}$ | $R_{13}$ | $\theta_{A13}$ | $\theta_{B13}$ | $R_{14}$ | $\theta_{A14}$ | $\theta_{B14}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | Co-efficient of .0001 |
| 81 | | 1 | —0.0769 | —0.7559 | 0.2520 | | | | | | | | 0.2018 |
| 82 | | 1 | 1.4344 | —1.6854 | 0.2730 | 0.2889 | | | | | | | 0.4786 |
| h | | 1 | 1.0585 | —4.1170 | 1.0000 | 1.0585 | | | | | | | —0.7128 |
| i | | 1 | | —3.0000 | 4.7660 | 0.4096 | —2.9205 | 0.9734 | | | | | 0.0000 |
| 81-82 | | 0 | —1.5113 | 0.9294 | —0.0210 | —0.2889 | | | | | | | —0.2768 |
| 81-h | | 0 | —1.1355 | 3.3611 | —0.7480 | —1.0585 | | | | | | | 0.9147 |
| h-i | | 0 | 1.0585 | —1.1170 | —3.7660 | 0.6489 | 2.9205 | —0.9734 | | | | | —0.7128 |
| 83 | | | 1 | —0.6150 | 0.0139 | 0.1911 | | | | | | | 0.1831 |
| 84 | | | 1 | —2.9602 | 0.6587 | 0.9323 | | | | | | | —0.8055 |
| 85 | | | 1 | —1.0553 | —3.5583 | 0.6131 | 2.7592 | —0.9197 | | | | | —0.6734 |
| j | | | 1 | —3.0000 | 0.3869 | 6.0609 | —2.8040 | | 0.9347 | | | | 0.0000 |
| 83-84 | | | 0 | 2.3452 | —0.6449 | —0.7412 | | | | | | | 0.9886 |
| 84-85 | | | 0 | —1.9049 | 4.2170 | 0.3192 | —2.7592 | 0.9197 | | | | | —0.1321 |
| 85-j | | | 0 | 1.9447 | —3.9452 | —5.4478 | 5.5632 | —0.9197 | —0.9347 | | | | —0.6734 |
| 86 | | | | 1 | —0.2750 | —0.3160 | | | | | | | 0.4216 |
| 87 | | | | 1 | —2.2137 | —0.1676 | 1.4484 | —0.4828 | | | | | 0.0693 |
| 88 | | | | 1 | —2.0288 | —2.8015 | 2.8609 | —0.4730 | —0.4807 | | | | —0.3463 |
| 86-87 | | | | 0 | 1.9387 | —0.1485 | —1.4484 | 0.4828 | | | | | 0.3522 |
| 86-88 | | | | 0 | 1.7539 | 2.4855 | —2.8609 | 0.4730 | 0.4807 | | | | 0.7679 |
| 89 | | | | | 1 | —0.0766 | —0.7471 | 0.2490 | | | | | 0.1816 |
| 90 | | | | | 1 | 1.4173 | —1.6314 | 0.2697 | 0.2741 | | | | 0.4378 |
| k | | | | | 1 | 1.0164 | —4.0329 | 1.0000 | 1.0164 | | | | —0.6503 |
| l | | | | | 1 | | —3.0000 | 4.8417 | 0.4208 | —3.0000 | 1.0000 | | 0.0000 |
| 89-90 | | | | | 0 | —1.4939 | 0.8842 | —0.0207 | —0.2741 | | | | —0.2562 |
| 89-k | | | | | 0 | —1.0930 | 3.2857 | —0.7510 | —1.0164 | | | | 0.8320 |
| k-l | | | | | 0 | 1.0164 | —1.0329 | —3.8417 | 0.5956 | 3.0000 | —1.0000 | | —0.6503 |
| 91 | | | | | | 1 | —0.5920 | 0.0138 | 0.1835 | | | | 0.1715 |
| 92 | | | | | | 1 | —3.0063 | 0.6871 | 0.9299 | | | | —0.7612 |
| 93 | | | | | | 1 | —1.0163 | —3.7802 | 0.5861 | 2.9520 | —0.9840 | | —0.6399 |
| m | | | | | | 1 | —3.0000 | 0.4140 | 6.3443 | —3.0000 | | 1.0000 | 0.0000 |
| 91-92 | | | | | | 0 | 2.4143 | —0.6733 | —0.7464 | | | | 0.9327 |
| 92-93 | | | | | | 0 | —1.9900 | 4.4673 | 0.3438 | —2.9520 | 0.9840 | | —0.1213 |
| 93-m | | | | | | 0 | 1.9837 | —4.1942 | —5.7582 | 5.9520 | —0.9840 | —1.0000 | —0.6399 |
| 94 | | | | | | | 1 | —0.2789 | —0.3092 | | | | 0.3863 |
| 95 | | | | | | | 1 | —2.2413 | —0.1728 | 1.4834 | —0.4945 | | 0.0610 |
| 96 | | | | | | | 1 | —2.1143 | —2.9030 | 3.0000 | —0.4961 | —0.5042 | —0.3226 |
| 94-95 | | | | | | | 0 | 1.9624 | —0.1364 | —1.4834 | 0.4945 | | 0.3254 |
| 94-96 | | | | | | | 0 | 1.8354 | 2.5938 | —3.0000 | 0.4961 | 0.5042 | 0.7089 |

## TABLE 14.—(Continued).

### ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 13 | | | Story No. 14 | | | Story No. 15 | | | Story No. 16 | | | |
| | $R_{13}$ | $\theta_{A13}$ | $\theta_{B13}$ | $R_{14}$ | $\theta_{A14}$ | $\theta_{B14}$ | $R_{15}$ | $\theta_{A15}$ | $\theta_{B15}$ | $R_{16}$ | $\theta_{A16}$ | $\theta_{B16}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | Co-efficient of .0001 |
| 97 | | 1 | —0.0695 | —0.7559 | 0.2520 | | | | | | | | 0.1658 |
| 98 | | 1 | 1.4135 | —1.6349 | 0.2704 | 0.2747 | | | | | | | 0.3863 |
| n | | 1 | 1.0164 | —4.0329 | 1.0000 | 1.0164 | | | | | | | —0.5711 |
| o | | 1 | | —3.0000 | 4.4044 | 0.4208 | —2.3443 | 0.7814 | | | | | 0.0000 |
| 97-98 | | 0 | —1.4830 | 0.8790 | —0.0184 | —0.2747 | | | | | | | —0.2205 |
| 97-n | | 0 | —1.0859 | 3.2769 | —0.7480 | —1.0164 | | | | | | | 0.7369 |
| n-o | | 0 | 1.0164 | —1.0329 | —3.4044 | 0.5956 | 2.3443 | —0.7814 | | | | | —0.5711 |
| 99 | | | 1 | —0.5928 | 0.0124 | 0.1853 | | | | | | | 0.1487 |
| 100 | | | 1 | —3.0177 | 0.6888 | 0.9360 | | | | | | | —0.6786 |
| 101 | | | 1 | —1.0163 | —3.3497 | 0.5860 | 2.3067 | —0.7689 | | | | | —0.5619 |
| p | | | 1 | —3.0000 | 0.4140 | 5.9148 | —2.3551 | | 0.7850 | | | | 0.0000 |
| 99-100 | | | 0 | 2.4249 | —0.6765 | —0.7507 | | | | | | | 0.8274 |
| 100-101 | | | 0 | —2.0014 | 4.0385 | 0.3499 | —2.3067 | 0.7689 | | | | | —0.1167 |
| 101-p | | | 0 | 1.9837 | —3.7637 | —5.3287 | 4.6618 | —0.7689 | —0.7850 | | | | —0.5619 |
| 102 | | | | 1 | —0.2790 | —0.3096 | | | | | | | 0.3412 |
| 103 | | | | 1 | —2.0179 | —0.1748 | 1.1525 | —0.3842 | | | | | 0.0583 |
| 104 | | | | 1 | —1.8974 | —2.6867 | 2.3505 | —0.3877 | —0.3953 | | | | —0.2833 |
| 102-103 | | | | 0 | 1.7389 | —0.1348 | —1.1525 | 0.3842 | | | | | 0.2829 |
| 102-104 | | | | 0 | 1.6184 | 2.3771 | —2.3505 | 0.3877 | 0.3958 | | | | 0.6246 |
| 105 | | | | | 1 | —0.0775 | —0.6624 | 0.2209 | | | | | 0.1627 |
| 106 | | | | | 1 | 1.4690 | —1.4525 | 0.2396 | 0.2446 | | | | 0.3859 |
| q | | | | | 1 | 1.0210 | —4.0419 | 1.0000 | 1.0210 | | | | —0.6258 |
| r | | | | | 1 | | —3.0000 | 5.0771 | 0.5385 | —3.0000 | 1.0000 | | 0.0000 |
| 105-106 | | | | | 0 | —1.5465 | 0.7901 | —0.0186 | —0.2446 | | | | —0.2232 |
| 105-q | | | | | 0 | —1.0985 | 3.3795 | —0.7790 | —1.0210 | | | | 0.7886 |
| q-r | | | | | 0 | 1.0210 | —1.0419 | —4.0771 | 0.4825 | 3.0000 | —1.0000 | | —0.6258 |
| 107 | | | | | | 1 | —0.5109 | 0.0120 | 0.1581 | | | | 0.1444 |
| 108 | | | | | | 1 | —3.0762 | 0.7092 | 0.9294 | | | | —0.7178 |
| 109 | | | | | | 1 | —1.0206 | —3.9933 | 0.4726 | 2.9383 | —0.9795 | | —0.6130 |
| s | | | | | | 1 | —3.0000 | 0.5274 | 6.9832 | —3.0000 | | 1.0000 | 0.0000 |
| 107-108 | | | | | | 0 | 2.5653 | —0.6972 | —0.7713 | | | | 0.8622 |
| 108-109 | | | | | | 0 | —2.0556 | 4.7025 | 0.4568 | —2.9383 | 0.9795 | | —0.1048 |
| 109-s | | | | | | 0 | 1.9794 | —4.5207 | —6.5106 | 5.9383 | —0.9795 | —1.0000 | —0.6130 |
| 110 | | | | | | | 1 | —0.2718 | —0.3006 | | | | 0.3361 |
| 111 | | | | | | | 1 | —2.2875 | —0.2222 | 1.4294 | —0.4765 | | 0.0510 |
| 112 | | | | | | | 1 | —2.2836 | —3.2888 | 3.0000 | —0.4946 | —0.5052 | —0.3097 |
| 110-111 | | | | | | | 0 | 2.0157 | —0.0784 | —1.4294 | 0.4765 | | 0.2851 |
| 111-112 | | | | | | | 0 | 2.0118 | 2.9882 | —3.0000 | 0.4946 | 5052 | 0.6457 |

## Table 14.—(Continued).

## Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

| No .of Equation | Left-Hand Member of Equation | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 15 | | | Story No. 16 | | | Story No. 17 | | | Story No. 18 | | | |
| | $R_{15}$ | $\theta_{A15}$ | $\theta_{B15}$ | $R_{16}$ | $\theta_{A16}$ | $\theta_{B16}$ | $R_{17}$ | $\theta_{A17}$ | $\theta_{B17}$ | $R_{18}$ | $\theta_{A18}$ | $\theta_{B18}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | Co-efficient of .0001 |
| 113 | | 1 | —0.0389 | —0.7091 | 0.2364 | | | | | | | | 0.1414 |
| 114 | | 1 | 1.4854 | —1.4913 | 0.2459 | 0.2511 | | | | | | | 0.3210 |
| t | | 1 | 1.0210 | —4.0419 | 1.0000 | 1.0210 | | | | | | | —0.5209 |
| u | | 1 | | —3.0000 | 4.9092 | 0.5385 | —2.7480 | 0.9161 | | | | | 0.0000 |
| 113-114 | | 0 | —1.5243 | 0.7822 | —0.0095 | —0.2511 | | | | | | | —0.1796 |
| 113-t | | 0 | —1.0599 | 3.3328 | —0.7636 | —1.0210 | | | | | | | 0.6624 |
| t-u | | 0 | 1.0210 | —1.0419 | —3.9092 | 0.4825 | 2.7480 | —0.9161 | | | | | —0.5209 |
| 115 | | | 1 | —0.5132 | 0.0062 | 0.1647 | | | | | | | 0.1178 |
| 116 | | | 1 | —3.1444 | 0.7204 | 0.9633 | | | | | | | —0.6249 |
| 117 | | | 1 | —1.0206 | —3.8290 | 0.4726 | 2.6916 | —0.8973 | | | | | —0.5102 |
| v | | | 1 | —3.0000 | 0.5281 | 6.8055 | —2.7161 | | 0.9054 | | | | 0.0000 |
| 115-116 | | | 0 | 2.6312 | —0.7142 | —0.7986 | | | | | | | 0.7427 |
| 116-117 | | | 0 | —2.1238 | 4.5494 | 0.4907 | —2.6916 | 0.8973 | | | | | —0.1147 |
| 117-v | | | 0 | 1.9794 | —4.3571 | —6.3329 | 5.4077 | —0.8973 | —0.9054 | | | | —0.5102 |
| 118 | | | | 1 | —0.2714 | —0.3035 | | | | | | | 0.2823 |
| 119 | | | | 1 | —2.1422 | —0.2310 | 1.2673 | —0.4225 | | | | | 0.0540 |
| 120 | | | | 1 | —2.2013 | —3.1993 | 2.7318 | —0.4533 | —0.4574 | | | | —0.2579 |
| 118-119 | | | | 0 | 1.8708 | —0.0724 | —1.2673 | 0.4225 | | | | | 0.2282 |
| 118-120 | | | | 0 | 1.9299 | 2.8958 | —2.7318 | 0.4533 | 0.4574 | | | | 0.5399 |
| 121 | | | | | 1 | —0.0387 | —0.6774 | 0.2259 | | | | | 0.1220 |
| 122 | | | | | 1 | 1.5027 | —1.4177 | 0.2353 | 0.2374 | | | | 0.2802 |
| w | | | | | 1 | 1.0092 | —4.0214 | 1.0000 | 1.0092 | | | | —0.4542 |
| x | | | | | 1 | | —3.0000 | 5.1753 | 0.5878 | —3.0000 | 1.0000 | | 0.0000 |
| 121-122 | | | | | 0 | —1.5414 | 0.7403 | —0.0094 | —0.2374 | | | | —0.1582 |
| 121-w | | | | | 0 | —1.0479 | 3.3440 | —0.7741 | —1.0092 | | | | 0.5762 |
| w-x | | | | | 0 | 1.0092 | —1.0214 | —4.1755 | 0.4214 | 3.0000 | —1.0000 | | —0.4542 |
| 123 | | | | | | 1 | —0.4810 | 0.0061 | 0.1542 | | | | 0.1028 |
| 124 | | | | | | 1 | —3.1958 | 0.7398 | 0.9630 | | | | —0.5507 |
| 125 | | | | | | 1 | —1.0136 | —4.1437 | 0.4182 | 2.9773 | —0.9924 | | —0.4507 |
| y | | | | | | 1 | —3.0000 | 0.5833 | 7.3021 | —3.0000 | | 1.0000 | 0.0000 |
| 123-124 | | | | | | 0 | 2.7148 | —0.7337 | —0.8088 | | | | 0.6535 |
| 124-125 | | | | | | 0 | —2.1822 | 4.8835 | 0.5448 | —2.9773 | 0.9924 | | —0.0999 |
| 125-y | | | | | | 0 | 1.9864 | —4.7270 | —6.8839 | 5.9773 | —0.9924 | —1.0000 | —0.4507 |
| 126 | | | | | | | 1 | —0.2703 | —0.2979 | | | | 0.2407 |
| 127 | | | | | | | 1 | —2.2380 | —0.2496 | 1.3644 | —0.4548 | | 0.0458 |
| 128 | | | | | | | 1 | —2.3797 | —3.4655 | 3.0093 | —0.4996 | —0.5035 | —0.2269 |
| 126-127 | | | | | | | 0 | 1.9677 | —0.0483 | —1.3644 | 0.4548 | | 0.1949 |
| 126-128 | | | | | | | 0 | 2.1094 | 3.1676 | —3.0093 | 0.4996 | 0.5035 | 0.4677 |

## TABLE 14.—(Continued).

## ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Story No. 17 | | | Story No. 18 | | | Story No. 19 | | | Story No. 20 | | | |
| | $R_{17}$ | $\theta_{A17}$ | $\theta_{B17}$ | $R_{18}$ | $\theta_{A18}$ | $\theta_{B18}$ | $R_{19}$ | $\theta_{A19}$ | $\theta_{B19}$ | $R_{20}$ | $\theta_{A20}$ | $\theta_{B20}$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | | | | | | Co-efficient of .0001 |
| 129 | | 1 | —0.0245 | —0.6935 | 0.2311 | | | | | | | | 0.0991 |
| 130 | | 1 | 1.5017 | —1.4268 | 0.2369 | 0.2387 | | | | | | | 0.2217 |
| z | | 1 | 1.0092 | —4.0214 | 1.0000 | 1.0092 | | | | | | | —0.3412 |
| A′ | | 1 | | —3.0000 | 5.1755 | 0.5878 | —3.0000 | 1.0000 | | | | | 0.0000 |
| 129-130 | | 0 | —1.5262 | 0.7333 | —0.0057 | —0.2387 | | | | | | | —0.1226 |
| 129-z | | 0 | —1.0337 | 3.3279 | —0.7689 | —1.0092 | | | | | | | 0.4403 |
| z-A′ | | 0 | 1.0092 | —1.0214 | —4.1755 | 0.4214 | 3.0000 | —1.0000 | | | | | —0.3412 |
| 131 | | | 1 | —0.4805 | 0.0037 | 0.1564 | | | | | | | 0.0804 |
| 132 | | | 1 | —3.2197 | 0.7438 | 0.9764 | | | | | | | —0.4260 |
| 133 | | | 1 | —1.0120 | —4.1373 | 0.4176 | 2.9727 | —0.9909 | | | | | —0.3381 |
| B′ | | | 1 | —3.0000 | 0.5833 | 7.3021 | —3.0000 | | 1.0000 | | | | 0.0000 |
| 131-132 | | | 0 | 2.7392 | —0.7401 | —0.8200 | | | | | | | 0.5063 |
| 132-133 | | | 0 | —2.2077 | 4.8811 | 0.5588 | —2.9727 | 0.9909 | | | | | —0.0879 |
| 133-B′ | | | 0 | 1.9880 | —4.7206 | —6.8846 | 5.9727 | —0.9909 | —1.0000 | | | | —0.3381 |
| 134 | | | | 1 | —0.2702 | —0.2993 | | | | | | | 0.1849 |
| 135 | | | | 1 | —2.2110 | —0.2531 | 1.3465 | —0.4485 | | | | | 0.0398 |
| 136 | | | | 1 | —2.3745 | —3.4629 | 3.0042 | —0.4980 | —0.5030 | | | | —0.1701 |
| 134-135 | | | | 0 | 1.9408 | —0.0462 | —1.3465 | 0.4485 | | | | | 0.1451 |
| 134-136 | | | | 0 | 2.1043 | 3.1635 | —3.0042 | 0.4980 | 0.5030 | | | | 0.3549 |
| 137 | | | | | 1 | —0.0238 | —0.6938 | 0.2311 | | | | | 0.0747 |
| 138 | | | | | 1 | 1.5034 | —1.4277 | 0.2367 | 0.2390 | | | | 0.1687 |
| C′ | | | | | 1 | 1.0092 | —4.0214 | 1.0000 | 1.0092 | | | | —0.2275 |
| D′ | | | | | 1 | | —3.0000 | 5.1755 | 0.5878 | —3.0000 | 1.0000 | | 0.0000 |
| 137-138 | | | | | 0 | —1.5272 | 0.7339 | —0.0056 | —0.2390 | | | | —0.0939 |
| 137-C′ | | | | | 0 | —1.0330 | 3.3276 | —0.7689 | —1.0092 | | | | 0.3022 |
| C′-D′ | | | | | 0 | 1.0092 | —1.0214 | —4.1755 | 0.4214 | 3.0000 | —1.0000 | | —0.2275 |
| 139 | | | | | | 1 | —0.4806 | 0.0036 | 0.1565 | | | | 0.0615 |
| 140 | | | | | | 1 | —3.2214 | 0.7444 | 0.9769 | | | | —0.2926 |
| 141 | | | | | | 1 | —1.0121 | —4.1377 | 0.4176 | 2.9727 | —0.9909 | | —0.2254 |
| E′ | | | | | | 1 | —3.0000 | 0.5833 | 7.3021 | —3.0000 | | 1.0000 | 0.0000 |
| 139-140 | | | | | | 0 | 2.7408 | —0.7407 | —0.8204 | | | | 0.3541 |
| 140-141 | | | | | | 0 | —2.2093 | 4.8821 | 0.5594 | —2.9727 | 0.9909 | | —0.0672 |
| 141-E′ | | | | | | 0 | 1.9879 | —4.7210 | —6.8846 | 5.9727 | —0.9909 | —1.0000 | —0.2254 |
| 142 | | | | | | | 1 | —0 2703 | —0.2993 | | | | 0.1292 |
| 143 | | | | | | | 1 | —2.2099 | —0.2532 | 1.3455 | —0.4485 | | 0.0304 |
| 144 | | | | | | | 1 | —2.3749 | —3.4632 | 3.0046 | —0.4985 | —0.5031 | —0.1134 |
| 142-143 | | | | | | | 0 | 1.9396 | —0.0461 | —1.3455 | 0.4485 | | 0.0988 |
| 142-144 | | | | | | | 0 | 2.1047 | 3.1639 | —3.0046 | 0.4985 | 0.5031 | 0.2426 |
| 145 | | | | | | | | 1 | —0.0238 | —0.6938 | 0.2313 | | 0.0509 |
| 146 | | | | | | | | 1 | 1.5034 | —1.4278 | 0.2369 | 0.2390 | 0.1153 |
| F′ | | | | | | | | 1 | 1.0092 | —4.0214 | 1.0000 | 1.0092 | —0.1137 |
| G′ | | | | | | | | 1 | | —3.0000 | 3.1759 | 0.5878 | 0.0000 |
| 145-146 | | | | | | | | 0 | —1.5272 | 0.7340 | —0.0056 | —0.2390 | —0.0643 |
| 145-F′ | | | | | | | | 0 | —1.0330 | 3.3276 | —0.7687 | —1.0092 | 0.1647 |
| F′-G′ | | | | | | | | 0 | 1.0092 | —1.0214 | —2.1759 | 0.4214 | —0.1137 |
| 147 | | | | | | | | | 1 | —0.4806 | 0.0037 | 0.1565 | 0.0421 |
| 148 | | | | | | | | | 1 | —3.2217 | 0.7442 | 0.9770 | —0.1594 |
| 149 | | | | | | | | | 1 | —1.0122 | —2.1563 | 0.4176 | —0.1127 |
| H′ | | | | | | | | | 1 | —3.0000 | 0.5834 | 5.3033 | 0.0000 |

## TABLE 14.—(Continued).

## ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|
| | Story No. 19 | | | Story No. 20 | | | | |
| | $R_{19}$ | $\theta_{A19}$ | $\theta_{B19}$ | $R_{20}$ | $\theta_{A20}$ | $\theta_{B20}$ | | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | Coefficient of .0001 |
| 147–148 | | | 0 | 2.7411 | —0.7406 | —0.8205 | | 0.2015 |
| 148–149 | | | 0 | —2.2095 | 2.9005 | 0.5595 | | —0.0467 |
| 149–H′ | | | 0 | 1.9878 | —2.7397 | —4.8857 | | —0.1127 |
| 150 | | | | 1 | —0.2702 | —0.2993 | | 0.0735 |
| 151 | | | | 1 | —1.3129 | —0.2531 | | 0.0211 |
| 152 | | | | 1 | —1.3783 | —2.4580 | | —0.0567 |
| 150–151 | | | | 0 | 1.0427 | —0.0462 | | 0.0524 |
| 150–152 | | | | 0 | 1.1081 | 2.1587 | | 0.1302 |
| 153 | | | | | 1 | —0.0443 | | 0.0502 |
| 154 | | | | | 1 | 1.9480 | | 0.1175 |
| 153–154 | | | | | | —1.9923 | | —0.0673 |
| 155 | | | | | | 1.0000 | | 0.0338 |

## TABLE 15.

## DETERMINATION OF THE CHANGES IN THE SLOPES AND OF THE RATIOS OF DEFLECTION TO STORY HEIGHT IN THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

The equations are taken from Table 14.

| No. of Equation | Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms. | | Right-Hand Member of Equation Coefficient of .0001 | |
|---|---|---|---|---|
| 155 | $\theta_{B20}$ | = | | .0338 |
| 153 | $\theta_{A20} - .0443\ \theta_{B20}$ | = | .0502 | |
| | .0443 x .00000338 | = | .0015 | |
| | $\theta_{A20}$ | = | | .0517 |
| 150 | $R_{20} - .2702\ \theta_{A20} - .2993\ \theta_{B20}$ | = | .0735 | |
| | .2702 x .00000517 | = | .0140 | |
| | .2993 x .00000338 | = | .0101 | |
| | $R_{20}$ | = | | .0976 |
| 147 | $\theta_{B19} - .4806\ R_{20} + .0037\ \theta_{A20} + .1565\ \theta_{B20}$ | = | .0421 | |
| | .4806 x .00000976 | = | 0469 | .0890 |
| | .0037 x .00000517 | = | .0002 | |
| | .1565 x .00000338 | = | .0053 | —.0055 |
| | $\theta_{B19}$ | = | | .0835 |
| 145 | $\theta_{A19} - .0238\ \theta_{B19} - .6938\ R_{20} + .2313\ \theta_{A20}$ | = | .0509 | |
| | .0238 x .00000835 | = | .0020 | |
| | .6938 x .00000976 | = | .0676 | .1205 |
| | .2313 x .00000517 | = | | —.0120 |
| | $\theta_{A19}$ | = | | .1085 |
| 142 | $R_{19} - .2703\ \theta_{A19} - .2993\ \theta_{B19}$ | = | .1292 | |
| | .2703 x .00001085 | = | .0294 | |
| | .2993 x .00000835 | = | .0250 | |
| | $R_{19}$ | = | | .1836 |
| 139 | $\theta_{B18} - .4806\ R_{19} + .0037\ \theta_{A19} + .1566\ \theta_{B19}$ | = | .0615 | |
| | .4806 x .00001836 | = | .0881 | .1496 |
| | .0037 x .00001085 | = | .0004 | |
| | .1565 x .00000835 | = | .0131 | —.0135 |
| | $\theta_{B18}$ | = | | .1361 |
| 137 | $\theta_{A18} - .0238\ \theta_{B18} - .6938\ R_{19} + .2311\ \theta_{A19}$ | = | .0748 | |
| | .0238 x .00001361 | = | .0032 | |
| | .6938 x .00001836 | = | .1272 | .2052 |
| | .2311 x .00001085 | = | | —.0251 |
| | $\theta_{A18}$ | = | | .1801 |
| 134 | $R_{18} - 2702\ \theta_{A18} - .2994\ \theta_{B18}$ | = | .1849 | |
| | .2702 x .00001801 | = | .0486 | |
| | .2994 x .00001361 | = | 0407 | |
| | $R_{18}$ | = | | .2742 |

## TABLE 15.—(Continued).

DETERMINATION OF THE CHANGES IN THE SLOPES AND OF THE RATIOS OF DEFLECTION TO STORY HEIGHT IN THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation<br>The first line in each group is the algebraic form of the equation.<br>The successive lines are the numerical values of the terms. | | Right-Hand Member of Equation<br>Coefficient of .0001 | |
|---|---|---|---|---|
| 131 | $\theta_{B17} - .4805 R_{18} + .0038\,\theta_{A18} + .1564\,\theta_{B18}$ | = | .0804 | |
| | .4805 x .00002742 | = | .1319 | .2123 |
| | .0038 x .00001801 | = | .0007 | |
| | .1564 x .00001361 | = | .0213 | —.0220 |
| | $\theta_{B17}$ | = | | .1903 |
| 129 | $\theta_{A17} - .0246\,\theta_{B17} - .6935 R_{18} + .2311\,\theta_{A18}$ | = | .0991 | |
| | .0246 x .00001903 | = | .0047 | |
| | .6935 x .00002742 | = | .1905 | 2943 |
| | .2311 x .00001801 | = | | —.0416 |
| | $\theta_{A17}$ | = | | .2527 |
| 126 | $R_{17} - .2703\,\theta_{A17} - .2979\,\theta_{B17}$ | = | .2407 | |
| | .2703 x .00002527 | = | .0682 | |
| | .2979 x .00001903 | = | .0566 | |
| | $R_{17}$ | = | | .3655 |
| 123 | $\theta_{B16} - .4810 R_{17} + .0061\,\theta_{A17} + .1542\,\theta_{B17}$ | = | .1028 | |
| | .4810 x .00003655 | = | .1758 | .2786 |
| | .0061 x .00002527 | = | .0015 | |
| | .1542 x .00001903 | = | .0294 | —.0309 |
| | $\theta_{B16}$ | = | | .2477 |
| 121 | $\theta_{A16} - .0387\,\theta_{B16} - .6774 R_{17} + .2259\,\theta_{A17}$ | = | .1220 | |
| | .0387 x .00002477 | = | .0096 | |
| | .6774 x .00003655 | = | .2473 | .3789 |
| | .2259 x .00002527 | = | | —.0570 |
| | $\theta_{A16}$ | = | | .3219 |
| 118 | $R_{16} - .2714\,\theta_{A16} - .3035\,\theta_{B16}$ | = | .2823 | |
| | .2714 x .00003219 | = | .0873 | |
| | .3035 x .00002477 | = | .0752 | |
| | $R_{16}$ | = | | .4448 |
| 115 | $\theta_{B15} - .5132 R_{16} + .0062\,\theta_{A16} + .1647\,\theta_{B16}$ | = | .1178 | |
| | .5132 x .00004448 | = | .2280 | .3458 |
| | .0062 x .00003219 | = | .0020 | |
| | .1647 x .00002477 | = | .0408 | —.0428 |
| | $\theta_{B15}$ | = | | .3030 |
| 113 | $\theta_{A15} - .0389\,\theta_{B15} - .7091 R_{16} + .2364\,\theta_{A16}$ | = | .1414 | |
| | .0389 x .00003030 | = | .0118 | |
| | .7091 x .00004448 | = | .3180 | .4712 |
| | .2364 x .00003219 | = | | —.0762 |
| | $\theta_{A15}$ | = | | .3950 |
| 110 | $R_{15} - .2718\,\theta_{A15} - .3006\,\theta_{B15}$ | = | .3361 | |
| | .2718 x .00003950 | = | .1072 | |
| | .3006 x .00003030 | = | .0912 | |
| | $R_{15}$ | = | | .5345 |
| 107 | $\theta_{B14} - .5109 R_{15} + .0120\,\theta_{A15} + .1581\,\theta_{B15}$ | = | .1444 | |
| | .5109 x .00005345 | = | .2730 | .4174 |
| | .0120 x .00003950 | = | .0047 | |
| | .1581 x .00003030 | = | .0480 | —.0527 |
| | $\theta_{B14}$ | = | | .3647 |

## TABLE 15.—(Continued).

## DETERMINATION OF THE CHANGES IN THE SLOPES AND OF THE RATIOS OF DEFLECTION TO STORY HEIGHT IN THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation<br>The first line in each group is the algebraic form of the equation.<br>The successive lines are the numerical values of the terms. | | Right-Hand Member of Equation<br>Coefficient of .0001 | |
|---|---|---|---|---|
| 105 | $\theta_{A14} - .0775\ \theta_{B14} - .6624\ R_{15} + .2210\ \theta_{A15}$ | = | .1627 | |
| | .0775 x .00003647 | = | .0283 | |
| | .6624 x .00005345 | = | .3541 | .5451 |
| | .2210 x .00003950 | = | | —.0873 |
| | $\theta_{A14}$ | = | | .4578 |
| 102 | $R_{14} - .2790\ \theta_{A14} - .3096\ \theta_{B14}$ | = | .3412 | |
| | .2790 x .00004578 | = | .1278 | |
| | .3096 x .00003647 | = | .1160 | |
| | $R_{14}$ | = | | .5850 |
| 99 | $\theta_{B13} - .5928\ R_{14} + .0124\ \theta_{A14} + .1853\ \theta_{B14}$ | = | .1487 | |
| | .5928 x .00005850 | = | .3470 | .4957 |
| | .0124 x .00004578 | = | .0057 | |
| | .1853 x .00003647 | = | .0676 | —.0733 |
| | $\theta_{B13}$ | = | | .4224 |
| 97 | $\theta_{A13} - .0695\ \theta_{B13} - .7560\ R_{14} + .2520\ \theta_{A14}$ | = | .1658 | |
| | .0695 x .00004224 | = | .0294 | |
| | .7560 x .00005850 | = | .4420 | .6372 |
| | .2520 x .00004578 | = | | —.1154 |
| | $\theta_{A13}$ | = | | .5218 |
| 94 | $R_{13} - .2789\ \theta_{A13} - .3092\ \theta_{B13}$ | = | .3864 | |
| | .2789 x .00005218 | = | .1458 | |
| | .3092 x .00004224 | = | .1308 | |
| | $R_{13}$ | = | | .6630 |
| 91 | $\theta_{B12} - .5920\ R_{13} + .0138\ \theta_{A13} + .1835\ \theta_{B13}$ | = | .1715 | |
| | .5920 x .00006630 | = | .3921 | .5636 |
| | .0138 x .00005218 | = | .0072 | |
| | .1835 x .00004224 | = | .0775 | —.0847 |
| | $\theta_{B12}$ | = | | .4789 |
| 89 | $\theta_{A12} - .0766\ \theta_{B12} - .7472\ R_{13} + .2490\ \theta_{A13}$ | = | .1816 | |
| | .0766 x .00004789 | = | .0366 | |
| | .7472 x .00006630 | = | .4960 | .7142 |
| | .2490 x .00005218 | = | | —.1300 |
| | $\theta_{A12}$ | = | | .5842 |
| 86 | $R_{12} - .2750\ \theta_{A12} - .3160\ \theta_{B12}$ | = | .4216 | |
| | .2750 x .00005842 | = | .1603 | |
| | .3160 x .00004789 | = | .1511 | |
| | $R_{12}$ | = | | .7330 |
| 83 | $\theta_{B11} - .6150\ R_{12} + .0139\ \theta_{A12} + .1912\ \theta_{B12}$ | = | .1831 | |
| | .6150 x .00007330 | = | .4512 | .6343 |
| | .0139 x .00005842 | = | .0081 | |
| | .1912 x .00004789 | = | .0916 | —.0997 |
| | $\theta_{B11}$ | = | | .5346 |
| 81 | $\theta_{A11} - .0770\ \theta_{B11} - .7560\ R_{12} + .2520\ \theta_{A12}$ | = | .2018 | |
| | .0770 x .00005346 | = | .0411 | |
| | .7560 x .00007330 | = | .5550 | .7979 |
| | .2520 x .00005842 | = | | —.1471 |
| | $\theta_{A11}$ | = | | .6508 |
| 78 | $R_{11} - .2746\ \theta_{A11} - .3147\ \theta_{B11}$ | = | .4669 | |
| | .2746 x .00006508 | = | .1790 | |
| | .3147 x .00005346 | = | .1681 | |
| | $R_{11}$ | = | | .8140 |

## TABLE 15.—(Continued).

### DETERMINATION OF THE CHANGES IN THE SLOPES AND OF THE RATIOS OF DEFLECTION TO STORY HEIGHT IN THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation | Left-Hand Member of Equation<br>The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms. | | Right-Hand Member of Equation<br>Coefficient of .0001 | |
|---|---|---|---|---|
| 75 | $\theta_{B10} - .6061\, R_{11} + .0179\, \theta_{A11} + .1841\, \theta_{B11}$ | = | .2178 | |
| | .6061 x .00008140 | = | .4940 | .7118 |
| | .0179 x .00006508 | = | .0116 | |
| | .1841 x .00005346 | = | .0984 | —.1100 |
| | $\theta_{B10}$ | = | | .6018 |
| 73 | $\theta_{A10} - .1082\, \theta_{B10} - .7278\, R_{11} + .2426\, \theta_{A11}$ | = | .2055 | |
| | .1082 x .00006018 | = | .6651 | |
| | .7278 x .00008140 | = | .5925 | .8631 |
| | .2426 x .00006508 | = | | —.1580 |
| | $\theta_{A10}$ | = | | .7051 |
| 70 | $R_{10} - .2601\, \theta_{A10} - .3423\, \theta_{B10}$ | = | .4540 | |
| | .2601 x .00007051 | = | .1833 | |
| | .3423 x .00006018 | = | .2059 | |
| | $R_{10}$ | = | | .8432 |
| 67 | $\theta_{B9} - .6991\, R_{10} + .0181\, \theta_{A10} + .2151\, \theta_{B10}$ | = | .2056 | |
| | .6991 x .00008432 | = | .5890 | .7946 |
| | .0181 x .00007051 | = | .0127 | |
| | .2151 x .00006018 | = | .1293 | —.1420 |
| | $\theta_{B9}$ | = | | .6526 |
| 65 | $\theta_{A9} - .1038\, \theta_{B9} - .7655\, R_{10} + .2551\, \theta_{A10}$ | = | .2053 | |
| | .1038 x .00006526 | = | .0677 | |
| | .7655 x .00008432 | = | .6460 | .9190 |
| | .2551 x .00007051 | = | | —.1800 |
| | $\theta_{A9}$ | = | | .7390 |
| 62 | $R_9 - .2602\, \theta_{A9} - .3372\, \theta_{B9}$ | = | .4719 | |
| | .2602 x .00007390 | = | .1922 | |
| | .3372 x .00006526 | = | .2200 | |
| | $R_9$ | = | | .8841 |
| 59 | $\theta_{B8} - .6908\, R_9 + .0231\, \theta_{A9} + .2072\, \theta_{B9}$ | = | .2008 | |
| | .6908 x .00008841 | = | .6100 | .8108 |
| | .0231 x .00007390 | = | .0171 | |
| | .2072 x .00006526 | = | .1351 | —.1522 |
| | $\theta_{B8}$ | = | | .6586 |
| 57 | $\theta_{A8} - .1382\, \theta_{B8} - .7000\, R_9 + .2334\, \theta_{A9}$ | = | .2079 | |
| | .1382 x .00006586 | = | .0910 | |
| | .7000 x .00008841 | = | .6182 | .9171 |
| | .2334 x .00007390 | = | | —.1725 |
| | $\theta_{A8}$ | = | | .7446 |
| 54 | $R_8 - .2738\, \theta_{A8} - .3118\, \theta_{B8}$ | = | .3774 | |
| | .2738 x .00007446 | = | .2038 | |
| | .3118 x .00006586 | = | .2055 | |
| | $R_8$ | = | | .7867 |
| 51 | $\theta_{B7} - .5814\, R_8 + .0112\, \theta_{A8} + .1826\, \theta_{B8}$ | = | .1562 | |
| | .5814 x .00007867 | = | .4570 | .6132 |
| | .0112 x .00007446 | = | .0083 | |
| | .1826 x .00006586 | = | .1203 | —.1286 |
| | $\theta_{B7}$ | = | | .4846 |

## Table 15.—(Continued).

## Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

| No. of Equation | Left-Hand Member of Equation<br>The first line in each group is the algebraic form of the equation.<br>The successive lines are the numerical values of the terms. | | Right-Hand Member of Equation<br>Coefficient of .0001 | |
|---|---|---|---|---|
| 49 | $\theta_{A7} - .0640\ \theta_{B7} - .7422\ R_8 + .2474\ \theta_{A8}$ | = | .1729 | |
| | .0640 x .00004846 | = | .0310 | |
| | .7422 x .00007867 | = | .5845 | .7884 |
| | .2474 x .00007446 | = | | —.1840 |
| | $\theta_{A7}$ | = | | .6044 |
| 46 | $R_7 - .2711\ \theta_{A7} - .3090\ \theta_{B7}$ | = | .4011 | |
| | .2711 x .00006044 | = | .1639 | |
| | .3090 x .00004846 | = | .1497 | |
| | $R_7$ | = | | .7147 |
| 43 | $\theta_{B6} - .5489\ R_7 + .0090\ \theta_{A7} + .1739\ \theta_{B7}$ | = | .1706 | |
| | .5489 x .00007147 | = | .3920 | .5626 |
| | .0090 x .00006044 | = | .0054 | |
| | .1739 x .00004846 | = | .0842 | —.0896 |
| | $\theta_{B6}$ | = | | .4730 |
| 41 | $\theta_{A6} - .0540\ \theta_{B6} - .7192\ R_7 + .2397\ \theta_{A7}$ | = | .1946 | |
| | .0540 x .00004730 | = | .0256 | |
| | .7192 x .00007147 | = | .5140 | .7342 |
| | .2397 x .00006044 | = | | —.1448 |
| | $\theta_{A6}$ | = | | .5894 |
| 38 | $R_6 - .2648\ \theta_{A6} - .3000\ \theta_{B6}$ | = | .4216 | |
| | .2648 x .00005894 | = | .1561 | |
| | .3000 x .00004730 | = | .1420 | |
| | $R_6$ | = | | .7197 |
| 35 | $\theta_{B5} - .4496\ R_6 + .0013\ \theta_{A6} + .1487\ \theta_{B6}$ | = | .1486 | |
| | .4496 x .00007197 | = | .3240 | .4726 |
| | .0013 x .00005894 | = | .0008 | |
| | .1487 x .00004730 | = | .0704 | —.0712 |
| | $\theta_{B5}$ | = | | .4014 |
| 33 | $\theta_{A5} - .0092\ \theta_{B5} - .6676\ R_6 + .2225\ \theta_{A6}$ | = | .1879 | |
| | .0092 x .00004014 | = | .0037 | |
| | .6676 x .00007197 | = | .4802 | .6718 |
| | .2225 x .00005894 | = | | —.1310 |
| | $\theta_{A5}$ | = | | .5408 |
| 30 | $R_5 - .2638\ \theta_{A5} - .2981\ \theta_{B5}$ | = | .4473 | |
| | .2638 x .00005408 | = | .1424 | |
| | .2981 x .00004014 | = | .1196 | |
| | $R_5$ | = | | .7093 |
| 27 | $\theta_{B4} - .4348\ R_5 + .0048\ \theta_{A5} + .1401\ \theta_{B5}$ | = | .1550 | |
| | .4348 x .00007093 | = | .3080 | .4630 |
| | .0048 x .00005408 | = | .0026 | |
| | .1401 x .00004014 | = | .0563 | —.0589 |
| | $\theta_{B4}$ | = | | .4041 |
| 25 | $\theta_{A4} - .0355\ \theta_{B4} - .6371\ R_5 + .2123\ \theta_{A5}$ | = | .2000 | |
| | .0355 x .00004041 | = | .0143 | |
| | .6371 x .00007093 | = | .4515 | .6658 |
| | .2123 x .00005408 | = | | —.1150 |
| | $\theta_{A4}$ | = | | .5508 |

## TABLE 15.—(Continued).

### DETERMINATION OF THE CHANGES IN THE SLOPES AND OF THE RATIOS OF DEFLECTION TO STORY HEIGHT IN THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

| No. of Equation. | Left-Hand Member of Equation<br>The first line in each group is the algebraic form of the equation.<br>The successive lines are the numerical values of the terms. | | Right-Hand Member of Equation<br>Coefficient of .0001 | |
|---|---|---|---|---|
| 22 | $R_4 + .3642\ \theta_{A4} - .3206\ \theta_{B4} - .4044\ R_5 + .1348\ \theta_{A5}$ | = | .5512 | |
| | .3206 x .00004041 | = | .1295 | |
| | .4044 x .00007093 | = | .2862 | .9669 |
| | .3642 x .00005508 | = | .2008 | |
| | .1348 x .00005408 | = | .0729 | —.2737 |
| | $R_4$ | = | | .6932 |
| 19 | $\theta_{B3} - .4696\ R_4 + .0030\ \theta_{A4} + .1535\ \theta_{B4}$ | = | .1540 | |
| | .4696 x .00006932 | = | .3257 | .4797 |
| | .0030 x .00005508 | = | .0017 | |
| | .1535 x .00004041 | = | .0620 | —.0637 |
| | $\theta_{B3}$ | = | | .4160 |
| 17 | $\theta_{A3} - .0196\ \theta_{B3} - .6881\ R_4 + .2294\ \theta_{A4}$ | = | .1917 | |
| | .0196 x .00004160 | = | .0082 | |
| | .6881 x .00006932 | = | .4770 | .6769 |
| | .2294 x .00005508 | = | | —.1263 |
| | $\theta_{A3}$ | = | | .5506 |
| 14 | $R_3 - .2689\ \theta_{A3} - .2980\ \theta_{B3}$ | = | .4564 | |
| | .2689 x .00005506 | = | .1481 | |
| | .2980 x .00004160 | = | .1240 | |
| | $R_3$ | = | | .7285 |
| 11 | $\theta_{B2} - .4511\ R_3 + .0027\ \theta_{A3} + .1476\ \theta_{B3}$ | = | .1768 | |
| | .4511 x .0000 285 | = | .3285 | .5053 |
| | .0027 x .00005506 | = | .0015 | |
| | .1476 x .00004160 | = | .0614 | —.0629 |
| | $\theta_{B2}$ | = | | .4424 |
| 9 | $\theta_{A2} - .0191\ \theta_{B2} - .6857\ R_3 + .2286\ \theta_{A3}$ | = | .2151 | |
| | .0191 x .00004424 | = | .0085 | |
| | .6857 x .00007285 | = | .4990 | .7226 |
| | .2286 x .00005506 | = | | —.1259 |
| | $\theta_{A2}$ | = | | .5967 |
| 6 | $R_2 - .2627\ \theta_{A2} - .2918\ \theta_{B2}$ | = | .5021 | |
| | .2627 x .00005967 | = | .1565 | |
| | .2918 x .00004424 | = | .1291 | |
| | $R_2$ | = | | .7877 |
| 3 | $\theta_{B1} - .3612\ R_2 - .0088\ \theta_{A2} + .1292\ \theta_{B2}$ | = | .1483 | |
| | .3612 x .00007877 | = | .2845 | |
| | .0088 x .00005967 | = | .0053 | .4381 |
| | .1292 x .00004424 | = | | —.0572 |
| | $\theta_{B1}$ | = | | .3809 |
| 1 | $\theta_{A1} + .0680\ \theta_{B1} - .6494\ R_2 + .2164\ \theta_{A2}$ | = | .2667 | |
| | .6494 x .00007877 | = | .5110 | .7777 |
| | .0680 x .00003809 | = | .0259 | |
| | .2164 x .00005967 | = | .1291 | —.1550 |
| | $\theta_{A1}$ | = | | .6227 |
| A | $R_1 - .2495\ \theta_{A1} - .2495\ \theta_{B1}$ | = | .5667 | |
| | .2495 x .00006227 | = | .1553 | |
| | .2495 x .00003809 | = | .0951 | |
| | $R_1$ | = | | .8171 |

## TABLE 16.

### VALUES OF $R$ AND $\theta$ FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5, AND THE FUNCTIONS OF THESE VALUES THAT OCCUR IN THE EQUATIONS USED TO DETERMINE THE MOMENTS IN THE COLUMNS AND GIRDERS.

| Story No. | 10,000 x $R$ | 10,000 x $\theta_A$ | 10,000 x $\theta_B$ | 10,000 x $3R$ | 10,000 x $2\theta_A$ | 10,000 x $2\theta_B$ | 10,000 x $(2\theta_A+\theta_B)$ | 10,000 x $(2\theta_B+\theta_A)$ | 10,000 x $3\theta_B$ | For Column $A$* |  | For Column $B$ |  | Story No. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  |  |  |  |  |  |  |  |  | 10,000 x $(2\theta_{N-1}+\theta_N-3R)$ | 10,000 x $(2\theta_N+\theta_{N-1}-3R)$ | 10,000 x $(2\theta_{N-1}+\theta_N-3R)$ | 10,000 x $(2\theta_N+\theta_{N-1}-3R)$ |  |
| 1 | 8171 | .6227 | .3809 | 2.4413 | 1.2454 | .7618 | 1.6263 | 1.3845 | 1.1427 | 1.8186 | 1.1959 | 2.0604 | 1.6795 | 1 |
| 2 | .7877 | .5967 | .4424 | 2.3631 | 1.1934 | .8848 | 1.6358 | 1.4815 | 1.3372 | .5210 | .5470 | 1.1589 | 1.0974 | 2 |
| 3 | .7285 | .5506 | .4160 | 2.1855 | 1.1012 | 8320 | 1.5172 | 1.3826 | 1.2480 | .4415 | .4876 | .8847 | .9111 | 3 |
| 4 | .6932 | .5508 | .4041 | 2.0796 | 1.1016 | .8082 | 1.5057 | 1.3590 | 1.2123 | .4276 | .4274 | .8435 | .8554 | 4 |
| 5 | .7093 | .5408 | .4014 | 2.1279 | 1.0816 | .8028 | 1.4830 | 1.3436 | 1.2042 | .4855 | .4955 | .9183 | .9210 | 5 |
| 6 | .7197 | .5894 | .4730 | 2.1591 | 1.1788 | .9460 | 1.6518 | 1.5354 | 1.4190 | .4881 | .4395 | .8833 | .8117 | 6 |
| 7 | .7147 | .6044 | .4846 | 2.1441 | 1.2088 | .9692 | 1.6934 | 1.5736 | 1.4538 | .3609 | .3459 | .7135 | .7019 | 7 |
| 8 | .7867 | .7446 | .6586 | 2.3601 | 1.4892 | 1.3172 | 2.1478 | 2.0618 | 1.9758 | .4067 | .2665 | .7323 | .5583 | 8 |
| 9 | .8841 | .7390 | .6526 | 2.6523 | 1.4780 | 1.3052 | 2.1306 | 2.0442 | 1.9578 | .4241 | .4297 | .6825 | .6885 | 9 |
| 10 | .8432 | .7051 | .6018 | 2.5296 | 1.4102 | 1.2036 | 2.0120 | 1.9087 | 1.8054 | .3465 | .3804 | .6226 | .6734 | 10 |
| 11 | .8140 | .6508 | .5346 | 2.4420 | 1.3016 | 1.0692 | 1 8362 | 1 7200 | 1.6038 | .3810 | .4353 | .7038 | .7710 | 11 |
| 12 | .7330 | .5842 | .4789 | 2.1990 | 1.1684 | .9578 | 1.6473 | 1.5420 | 1.4367 | .3132 | .3798 | .6509 | .7066 | 12 |
| 13 | .6630 | .5218 | .4224 | 1.9890 | 1.0436 | .8448 | 1.4660 | 1.3666 | 1.2682 | .2988 | .3612 | .6088 | .6653 | 13 |
| 14 | .5850 | .4578 | .3647 | 1.7550 | .9156 | .7294 | 1.2803 | 1.1872 | 1.0941 | .2536 | .3176 | .5455 | .6032 | 14 |
| 15 | .5345 | .3950 | .3030 | 1.6035 | .7900 | .6060 | 1.0930 | 1.0010 | .9090 | .2929 | .3557 | .5711 | .6328 | 15 |
| 16 | .4448 | .3219 | .2477 | 1.3344 | .6438 | .4954 | .8915 | .8173 | .7431 | .2225 | .2956 | .4807 | .5360 | 16 |
| 17 | .3655 | .2527 | .1903 | 1.0965 | .5054 | .3806 | .6957 | .6333 | .5709 | .2000 | .2692 | .4108 | .4682 | 17 |
| 18 | .2742 | .1801 | .1361 | .8226 | .3602 | .2722 | .4963 | .4523 | .4083 | .1371 | .2097 | .3059 | .3601 | 18 |
| 19 | .1836 | .1085 | .0335 | .5503 | .2170 | .1670 | .3005 | .2755 | .2505 | .0821 | .1537 | .1951 | .2477 | 19 |
| 20 | .0976 | .0517 | .0338 | .2923 | .1034 | .0676 | .1372 | .1193 | .1014 | .0231 | .0809 | .0920 | .1417 | 20 |

* N represents the number of the story in question, and (N—1) the story below.

## TABLE 17.

VALUES OF $K$ FOR THE COLUMNS AND THE GIRDERS OF THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5, AND THE FUNCTIONS OF THESE VALUES THAT OCCUR IN THE EQUATIONS USED TO DETERMINE THE MOMENTS IN THE COLUMNS AND GIRDERS.

| Story No. | $K$ for Girder at Top of Story in | | $K$ for | | .0001 x $2EK$ for Girder at Top of Story in | | .0001 x $2EK$ for | |
|---|---|---|---|---|---|---|---|---|
| | Bay $a$ | Bay $b$ | Column $A$ | Column $B$ | Bay $a$ | Bay $b$ | Column $A$ | Column $B$ |
| 1 | 30.5 | 37.3 | 25.8 | 25.8 | 177 000 | 216 200 | 149 600 | 149 600 |
| 2 | 21.4 | 29.2 | 35.6 | 35.6 | 124 100 | 169 400 | 206 300 | 206 300 |
| 3 | 21.4 | 26.2 | 35.4 | 35.5 | 124 100 | 152 000 | 205 500 | 205 800 |
| 4 | 19.5 | 26.2 | 35.4 | 35.5 | 113 050 | 152 000 | 205 500 | 205 800 |
| 5 | 19.5 | 23.8 | 29.4 | 30.4 | 113 050 | 138 050 | 170 500 | 176 300 |
| 6 | 14.1 | 17.2 | 29.4 | 30.4 | 81 750 | 99 700 | 170 500 | 176 300 |
| 7 | 12.8 | 15.7 | 28.7 | 30.0 | 74 250 | 91 100 | 166 500 | 174 000 |
| 8 | 7.7 | 9.4 | 28.7 | 30.0 | 44 600 | 54 500 | 166 500 | 174 000 |
| 9 | 7.7 | 9.4 | 21.1 | 26.1 | 44 600 | 54 500 | 122 400 | 151 300 |
| 10 | 7.7 | 9.4 | 21.1 | 26.1 | 44 600 | 54 500 | 122 400 | 151 300 |
| 11 | 7.7 | 9.4 | 18.8 | 19.9 | 44 600 | 54 500 | 109 000 | 115 400 |
| 12 | 7.7 | 9.4 | 18.8 | 19.9 | 44 600 | 54 500 | 109 000 | 115 400 |
| 13 | 7.7 | 9.4 | 18.3 | 18.6 | 44 600 | 54 500 | 106 100 | 107 900 |
| 14 | 7.7 | 9.4 | 18.3 | 18.6 | 44 600 | 54 500 | 106 100 | 107 900 |
| 15 | 7.7 | 9.4 | 14.3 | 14.6 | 44 600 | 54 500 | 82 900 | 84 600 |
| 16 | 7.7 | 9.4 | 14 3 | 14.6 | 44 600 | 54 500 | 82 900 | 84 600 |
| 17 | 7.7 | 9.4 | 13.1 | 13.2 | 44 600 | 54 500 | 76 000 | 76 500 |
| 18 | 7.7 | 9.4 | 13.1 | 13.2 | 44 600 | 54 500 | 76 000 | 76 500 |
| 19 | 7.7 | 9.4 | 13.1 | 13.2 | 44 600 | 54 500 | 76 000 | 76 500 |
| 20 | 7.7 | 9.4 | 13.1 | 13.2 | 44 600 | 54 500 | 76 000 | 76 500 |

## TABLE 18.

## MOMENTS AT THE ENDS OF THE COLUMNS AND GIRDERS OF THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

Moments are expressed in inch-pounds.

| No. of Story | Moment in Column *A** | | Moment in Column *B* | | Moment in Girder at Top of Story in Bay *a* | | Moment in Girder at Top of Story in Bay *b* |
|---|---|---|---|---|---|---|---|
| | At Top = 2EK $(2\theta_N+\theta_{N-1}-3R)$ | At Bottom = 2EK $(2\theta_{N-1}+\theta_N-3R)$ | At Top = 2EK $(2\theta_N+\theta_{N-1}-3R)$ | At Bottom = 2EK $(2\theta_{N-1}+\theta_N-3R)$ | At Right-hand End = 2EK $(2\theta_A+\theta_B)$ | At Left-hand End = 2EK $(2\theta_B+\theta_A)$ | At Each End = 2EK $(3\theta_B)$ |
| 1 | 178 800 | 272 000 | 251 000 | 308 200 | 287 800 | 245 000 | 247 500 |
| 2 | 113 000 | 107 500 | 226 500 | 239 000 | 203 000 | 184 000 | 226 000 |
| 3 | 100 300 | 90 700 | 187 500 | 182 000 | 187 100 | 171 700 | 189 600 |
| 4 | 87 800 | 87 800 | 176 100 | 173 500 | 170 500 | 153 600 | 184 400 |
| 5 | 84 500 | 82 800 | 162 300 | 162 000 | 167 700 | 152 000 | 166 000 |
| 6 | 75 000 | 83 200 | 143 300 | 155 700 | 135 000 | 125 600 | 141 500 |
| 7 | 57 600 | 60 000 | 122 000 | 124 000 | 125 900 | 117 000 | 132 500 |
| 8 | 44 400 | 67 700 | 97 200 | 127 500 | 95 800 | 92 000 | 107 800 |
| 9 | 52 550 | 51 900 | 104 200 | 103 300 | 95 000 | 91 100 | 106 800 |
| 10 | 46 500 | 42 400 | 102 000 | 94 200 | 89 600 | 85 000 | 98 500 |
| 11 | 47 500 | 41 500 | 89 000 | 81 200 | 81 900 | 76 600 | 87 500 |
| 12 | 41 400 | 34 100 | 81 500 | 75 100 | 73 400 | 68 600 | 78 300 |
| 13 | 38 300 | 31 700 | 71 800 | 65 600 | 65 300 | 60 800 | 69 100 |
| 14 | 33 700 | 26 900 | 65 100 | 58 900 | 5 700 | 52 800 | 59 600 |
| 15 | 29 420 | 24 300 | 53 600 | 48 350 | 48 700 | 49 600 | 49 600 |
| 16 | 24 450 | 18 450 | 45 400 | 40 600 | 39 700 | 36 400 | 40 500 |
| 17 | 20 450 | 15 400 | 35 800 | 31 400 | 31 000 | 28 200 | 31 100 |
| 18 | 15 920 | 10 430 | 27 600 | 23 400 | 22 100 | 20 150 | 22 300 |
| 19 | 11 680 | 6 240 | 18 900 | 14 930 | 13 380 | 12 250 | 13 650 |
| 20 | 6 150 | 1 750 | 10 830 | 7 040 | 6 110 | 5 320 | 5 530 |

*N represents the number of the story in question, and (N—1) the story below.

## TABLE 19.

## DIRECT STRESSES IN THE COLUMNS, AND THE SHEARS IN THE COLUMNS AND GIRDERS OF THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

All quantities are in pounds.

| Story No. | Shear in | | Shear in Girder at Top of Story in | | Direct Stress in | |
|---|---|---|---|---|---|---|
| | Column *A* | Column *B* | Bay *a* | Bay *b* | Column *A* | Column *B* |
| 1 | 1 709 | 2 115 | 2 020 | 2 290 | 14 464 | 4 587 |
| 2 | 1 149 | 2 421 | 1 467 | 2 095 | 12 444 | 4 317 |
| 3 | 1 136 | 2 200 | 1 360 | 1 755 | 10 977 | 3 689 |
| 4 | 1 044 | 2 080 | 1 228 | 1 709 | 9 617 | 3 294 |
| 5 | 995 | 1 931 | 1 210 | 1 536 | 8 389 | 2 813 |
| 6 | 940 | 1 780 | 986 | 1 310 | 7 179 | 2 489 |
| 7 | 816 | 1 709 | 920 | 1 228 | 6 193 | 2 163 |
| 8 | 779 | 1 560 | 712 | 996 | 5 273 | 1 855 |
| 9 | 725 | 1 440 | 705 | 988 | 4 561 | 1 571 |
| 10 | 617 | 1 363 | 661 | 912 | 3 856 | 1 288 |
| 11 | 618 | 1 180 | 600 | 810 | 3 195 | 1 037 |
| 12 | 524 | 1 088 | 538 | 725 | 2 595 | 827 |
| 13 | 486 | 954 | 478 | 640 | 2 057 | 640 |
| 14 | 421 | 861 | 416 | 552 | 1 579 | 478 |
| 15 | 373 | 710 | 354 | 459 | 1 163 | 342 |
| 16 | 298 | 597 | 288 | 375 | 809 | 237 |
| 17 | 249 | 466 | 221 | 288 | 521 | 150 |
| 18 | 182 | 354 | 160 | 206 | 300 | 83 |
| 19 | 124 | 235 | 97 | 126 | 140 | 37 |
| 20 | 55 | 124 | 43 | 51 | 43 | 8 |

## TABLE 20.

## CHECK ON THE NUMERICAL VALUES OF THE MOMENTS AT THE ENDS OF THE COLUMNS AND GIRDERS OF THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

Quantities are in inch-pounds.

For a perfect check the corresponding quantities in the two columns of each group of columns should be identical.

| Story No. | Total Shear in a Story Multiplied by the Story Height | Sum of the Moments at the Tops and the Bottoms of all of the Columns in a Story | Sum of the Moments in Column A just above and below Girder *a* at the Top of a Story | Moment in Girder *a* at the Top of a Story, at the Section adjacent to Column A | Sum of the Moments in Column *B* just above and below Girders *a* and *b* at the Top of a Story | Sum of the Moments in Girders *a* and *b* at the Top of a Story at Sections adjacent to Column *B* |
|---|---|---|---|---|---|---|
| 1 | 2 035 000 | 2 020 000 | 286 300 | 287 800 | 490 000 | 492 500 |
| 2 | 1 370 000 | 1 372 000 | 203 700 | 203 000 | 408 500 | 410 000 |
| 3 | 1 123 000 | 1 121 000 | 187 100 | 188 100 | 361 000 | 361 300 |
| 4 | 1 053 000 | 1 050 400 | 170 500 | 170 600 | 338 100 | 338 000 |
| 5 | 983 000 | 983 200 | 167 700 | 167 700 | 318 000 | 318 000 |
| 6 | 913 000 | 914 400 | 135 000 | 135 000 | 267 300 | 267 100 |
| 7 | 725 000 | 727 200 | 125 900 | 125 300 | 249 500 | 249 500 |
| 8 | 673 000 | 673 600 | 95 800 | 96 300 | 200 500 | 199 800 |
| 9 | 622 000 | 624 000 | 95 000 | 94 950 | 198 400 | 197 900 |
| 10 | 570 000 | 570 200 | 89 600 | 88 000 | 183 200 | 183 500 |
| 11 | 518 000 | 518 400 | 81 900 | 81 600 | 164 100 | 164 100 |
| 12 | 466 000 | 464 200 | 73 400 | 72 100 | 147 100 | 146 900 |
| 13 | 414 000 | 414 800 | 65 300 | 65 200 | 130 700 | 129 900 |
| 14 | 363 000 | 369 200 | 57 000 | 58 000 | 113 450 | 112 400 |
| 15 | 311 000 | 311 400 | 48 700 | 47 870 | 94 200 | 94 200 |
| 16 | 259 000 | 259 800 | 39 700 | 39 850 | 76 800 | 76 900 |
| 17 | 207 500 | 206 200 | 31 000 | 30 880 | 59 200 | 59 300 |
| 18 | 155 300 | 154 800 | 22 100 | 22 160 | 42 530 | 42 450 |
| 19 | 103 600 | 103 500 | 13 380 | 13 430 | 25 940 | 25 900 |
| 20 | 51 800 | 51 540 | 6 110 | 6 150 | 10 830 | 10 850 |

## TABLE 21.

## ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS USED TO DETERMINE THE SLOPES AND THE DEFLECTIONS IN THE BOTTOM STORY OF THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5, BY A MODIFICATION OF THE SLOPE-DEFLECTION METHOD.

| No. of Equation | Left-Hand Member of Equation | | | | | | | Right-Hand Member of Equation |
|---|---|---|---|---|---|---|---|---|
| | Story No. 1 | | | Story No. 2 | | | Story No. 3 | |
| | $R_1$ | $\theta_{A1}$ | $\theta_{B1}$ | $R_2$ | $\theta_{A2}$ | $\theta_{B2}$ | $R_3$ | |
| | Coefficients of Unknown Slopes and Ratios of Deflection to Story Height | | | | | | | Co-efficient of .0001 |
| A | —206.8 | 51.6 | 51.6 | | | | | —117.2 |
| B | — 77.4 | 183.8 | 30.5 | —106.8 | 35.6 | | | 0.0 |
| C | — 77.4 | 30.5 | 295.7 | —106.8 | | 35.6 | | 0.0 |
| D | | 71.2 | 71.2 | —284.8 | 71.2 | 71.2 | | — 78.9 |
| E | | 35.6 | | —106.8 | 220.2 | 21.4 | —106.2 | 0.0 |
| F | | | 35.6 | —106.8 | 21.4 | 308.1 | —106.5 | 0.0 |
| G | | | | | 141.6 | 142.0 | —283.6 | — 64.5 |
| A | 1 | —0.2495 | —0.2495 | | | | | 0.5667 |
| B | 1 | —2.3 47 | —0.3941 | 1.3798 | —0.4599 | | | 0.0000 |
| C | 1 | —0.3941 | —3.8204 | 1.3798 | | —0.4599 | | 0.0000 |
| A–B | 0 | 2.1252 | 0.1445 | —1.3798 | 0.4599 | | | 0.5667 |
| B–C | 0 | —1.9807 | 3.4264 | | —0.4599 | 0.4599 | | 0.0000 |
| 1 | | 1 | 0.0680 | —0.6494 | 0.2164 | | | 0.2667 |
| 2 | | 1 | —1.7300 | | 0.2322 | —0.2322 | | 0.0000 |
| D | | 1 | 1.0000 | —4.0000 | 1.0000 | 1.0000 | | —1.1082 |
| E | | 1 | | —3.0000 | 6.1856 | 0.6012 | —2.9834 | 0.0000 |
| 1-2 | | 0 | 1.7980 | —0.6494 | —0.0158 | 0.2322 | | 0.2667 |
| 2–D | | 0 | —2.7300 | 4.0000 | —0.7678 | —1.2322 | | 1.1082 |
| D–E | | 0 | 1.0000 | —1.0000 | —5.1856 | 0.3988 | 2.9834 | —1.1082 |
| 3 | | | 1 | —0.3612 | —0.0088 | 0.1292 | | 0.1483 |
| 4 | | | 1 | —1.4652 | 0.2812 | 0.4513 | | —0.4059 |
| 5 | | | 1 | —1.0000 | —5.1856 | 0.3988 | 2.9834 | —1.1082 |
| F | | | 1 | —3.0000 | 0.6012 | 8.6547 | —2.9915 | 0.0000 |
| 3–4 | | | 0 | 1.1041 | —0.2900 | —0.3222 | | 0.5543 |
| 4–5 | | | 0 | —0.4652 | 5.4668 | 0.0525 | —2.9834 | 0.7023 |
| 5–F | | | 0 | 2.0000 | —5.7868 | 8.2559 | 5.9749 | —1.1082 |
| 6 | | | | 1 | —0.2627 | —0.2918 | | 0.5021 |
| 7 | | | | 1 | —11.7518 | —0.1128 | 6.4127 | —1.5095 |
| 8 | | | | 1 | —2.8934 | —4.1279 | 2.9875 | —0.5541 |
| 6–7 | | | | 0 | 11.4891 | —0.1790 | —6.4127 | 2.0116 |
| 7–8 | | | | 0 | —8.8584 | 4.0151 | 3.4252 | —0.9554 |
| 9 | | | | | 1 | —0.0156 | —0.5600 | 0.1750 |
| 10 | | | | | 1 | —0.4540 | —0.3862 | 0.1079 |
| G | | | | | 1 | 1.0030 | —2.0010 | —0.4555 |
| 9–10 | | | | | 0 | 0.4384 | —0.1738 | 0.0671 |
| 10–G | | | | | 0 | —1.4570 | 1.6148 | 0.5634 |
| 11 | | | | | | 1 | —0.3961 | 0.1530 |
| 12 | | | | | | 1 | —1.1090 | —0.3865 |
| 11–12 | | | | | | 0 | 0.7131 | 0.5395 |
| 13 | | | | | | | 1 | 0.7560 |

## TABLE 22.

### DETERMINATION OF THE CHANGES IN THE SLOPES AND THE RATIO OF THE DEFLECTION TO STORY HEIGHT IN THE BOTTOM STORY OF THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5, BY A MODIFICATION OF THE SLOPE-DEFLECTION METHOD.

| No. of Equation | Left-Hand Member of Equation<br>The first line in each group is the algebraic form of the equation.<br>The successive lines are the numerical values of the terms. | | | | Right-Hand Member of Equation<br>Coefficient of .0001 | |
|---|---|---|---|---|---|---|
| 13 | | | $R_3$ | = | .7560 | |
| 11 | $\theta_{B2} - .3961\ R_3$ | | | = | .1530 | |
| | | .3961 x .00007560 | | = | .2995 | |
| | | | $\theta_{B2}$ | = | | .4525 |
| 9 | $\theta_{A2} - .0156\ \theta_{B2} - .5600\ R_3$ | | | = | .1750 | |
| | | .0156 x .00004525 | | = | .0071 | |
| | | .5600 x .00007560 | | = | .4240 | |
| | | | $\theta_{A2}$ | = | | .6061 |
| 6 | $R_2 - .2627\ \theta_{A2} - .2918\ \theta_{B2}$ | | | = | .5021 | |
| | | .2627 x .00006061 | | = | .1593 | |
| | | .2918 x .00004525 | | = | .1320 | |
| | | | $R_2$ | = | | .7934 |
| 3 | $\theta_{B1} - .3612\ R_2 - .0088\ \theta_{A2} + .1292\ \theta_{B2}$ | | | = | .1483 | |
| | | .3612 x .00007934 | | = | .2865 | |
| | | .0088 x .00006061 | | = | .0053 | .4401 |
| | | .1292 x .00004525 | | = | | —.0585 |
| | | | $\theta_{B1}$ | = | | .3816 |
| 1 | $\theta_{A1} + .0680\ \theta_{B1} - .6494\ R_2 + .2164\ \theta_{A2}$ | | | = | .2667 | |
| | | .6494 x .00007934 | | = | .5150 | .7817 |
| | | .0680 x .00003816 | | = | .0260 | |
| | | .2164 x .00006061 | | = | .1313 | —.1573 |
| | | | $\theta_{A1}$ | = | | .6244 |
| A | $R_1 - .2495\ \theta_{A1} - .2495\ \theta_{B1}$ | | | = | .5667 | |
| | | .2495 x .00006244 | | = | .1556 | |
| | | .2495 x .00003816 | | = | .0951 | |
| | | | $R_1$ | = | | .8174 |

# TABLE 23.

## COMPARISON OF THE APPROXIMATE METHODS WITH THE SLOPE-DEFLECTION METHOD WHEN APPLIED TO THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

## MOMENTS IN THE COLUMNS.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

| Story No. | Moment at Top of Column *A* | | | | | Moment at Bottom of Column *A* | | | | | Moment at Top of Column *B* | | | | | Moment at Bottom of Column *B* | | | | | Story No. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | |
| | | | I | II | III | | | I | II | III | | | I | II | III | | | I | II | III | |
| 1 | 178.8<br>*100.0* | 191.5<br>*106.8* | 166.9<br>*93.7* | 254.5<br>*145.0* | 254.5<br>*145.0* | 272.0<br>*100.0* | 191.5<br>*70.3* | 166.9<br>*61.3* | 254.5<br>*93.6* | 254.5<br>*93.6* | 251.0<br>*100.0* | 319.0<br>*126.8* | 343.2<br>*136.9* | 254.5<br>*101.4* | 254.5<br>*101.4* | 308.2<br>*100.0* | 319.0<br>*103.5* | 343.2<br>*111.3* | 254.2<br>*82.5* | 254.5<br>*82.5* | 1 |
| 2 | 113.0<br>*100.0* | 113.8<br>*100.7* | 111.0<br>*98.3* | 171.4<br>*152.0* | 171.4<br>*152.0* | 107.5<br>*100.0* | 113.8<br>*106.0* | 111.0<br>*103 4* | 171.4<br>*159.4* | 171.4<br>*159.4* | 226.5<br>*100.0* | 227.5<br>*100.4* | 228.6<br>*101.0* | 171.4<br>*75.7* | 171.4<br>*75.7* | 239.0<br>*100.0* | 227.5<br>*95.3* | 228.6<br>*95.8* | 171.4<br>*71.7* | 171.4<br>*71.7* | 2 |
| 3 | 100.3<br>*100.0* | 96.8<br>*96.5* | 91.5<br>*91.2* | 140.5<br>*140.3* | 140.5<br>*140.3* | 90.7<br>*100.0* | 96.8<br>*106.8* | 91.5<br>*101.0* | 140.5<br>*155.0* | 140.5<br>*155.0* | 187.5<br>*100.0* | 184.1<br>*98.3* | 187.6<br>*100.0* | 140.5<br>*75.0* | 140.5<br>*75.0* | 182.0<br>*100.0* | 184.1<br>*101.2* | 187.6<br>*103.0* | 140.5<br>*77.2* | 140.5<br>*77.2* | 3 |
| 4 | 87.8<br>*100.0* | 87.1<br>*99.3* | 87.1<br>*99.2* | 131.7<br>*150.0* | 131.7<br>*150.0* | 87.8<br>*100.0* | 87.1<br>*99.3* | 87.1<br>*99.2* | 131.7<br>*150.0* | 131.7<br>*150.0* | 176.1<br>*100.0* | 175.0<br>*99.4* | 175.3<br>*99.5* | 131.7<br>*74.6* | 131.7<br>*74.6* | 173.5<br>*100.0* | 175.0<br>*101.0* | 175.3<br>*101.2* | 131.7<br>*75.9* | 131.7<br>*75.9* | 4 |
| 5 | 84.5<br>*100.0* | 85.6<br>*101.3* | 80.9<br>*95.6* | 120.4<br>*142.3* | 120.4<br>*142.3* | 82.8<br>*100.0* | 85.6<br>*103.3* | 80.9<br>*97.6* | 120.4<br>*145.3* | 120.4<br>*145.3* | 162.3<br>*100.0* | 160.0<br>*98.5* | 163.1<br>*100.6* | 125.3<br>*77.2* | 125.3<br>*77.2* | 162.0<br>*100.0* | 160.0<br>*98.8* | 163.1<br>*100.8* | 125.3<br>*77.5* | 125.3<br>*77.5* | 5 |
| 6 | 75.0<br>*100.0* | 76.8<br>*102.0* | 74.3<br>*99.0* | 111.7<br>*149.0* | 111.7<br>*149.0* | 83.2<br>*100.0* | 76.8<br>*92.2* | 74.3<br>*89.5* | 111.7<br>*134.2* | 111.7<br>*134.2* | 143.3<br>*100.0* | 151.8<br>*106.0* | 153.0<br>*106.8* | 116.3<br>*81.1* | 116.3<br>*81.1* | 155.7<br>*100.0* | 151.8<br>*97.5* | 153.0<br>*98.3* | 116.3<br>*74.8* | 116.3<br>*74.8* | 6 |
| 7 | 57.6<br>*100.0* | 60.3<br>*105.0* | 59.2<br>*102.8* | 88.6<br>*154.0* | 88.6<br>*154.0* | 60.0<br>*100.0* | 60.3<br>*100.5* | 59.2<br>*98.5* | 88.6<br>*147.6* | 88.6<br>*147.6* | 122.0<br>*100.0* | 120.7<br>*98.8* | 122.0<br>*100.0* | 92.8<br>*76.1* | 92.8<br>*76.1* | 124.0<br>*100.0* | 120.7<br>*97.3* | 122.0<br>*98.3* | 92.8<br>*74.8* | 92.8<br>*74.8* | 7 |
| 8 | 44.4<br>*100.0* | 54.7<br>*125.0* | 55.9<br>*126.0* | 82.3<br>*185.5* | 82.3<br>*185.5* | 67.7<br>*100.0* | 54.7<br>*80.7* | 55.9<br>*82.5* | 82.3<br>*121.8* | 82.3<br>*121.8* | 97.2<br>*100.0* | 114.0<br>*117.2* | 113.0<br>*116.2* | 86.2<br>*88.7* | 86.2<br>*88.7* | 127.5<br>*100.0* | 114.0<br>*89.5* | 113.0<br>*88.6* | 86.2<br>*67.6* | 86.2<br>*67.6* | 8 |
| 9 | 52.6<br>*100.0* | 49.8<br>*94.8* | 51.4<br>*97.7* | 69.5<br>*132.0* | 69.5<br>*132.0* | 51 9<br>*100.0* | 49.8<br>*96.1* | 51.4<br>*99.1* | 69.5<br>*134.0* | 69.5<br>*134.0* | 104 2<br>*100.0* | 105.5<br>*100.8* | 104.5<br>*100.3* | 86.0<br>*82.5* | 86.0<br>*82.5* | 103.3<br>*100.0* | 105.5<br>*102.1* | 104.5<br>*101.0* | 86.0<br>*83.2* | 86.0<br>*83.2* | 9 |
| 10 | 46.5<br>*100.0* | 45.6<br>*98.0* | 46.0<br>*99.0* | 63.7<br>*137.0* | 63.7<br>*137.0* | 42.4<br>*100.0* | 45.6<br>*107.8* | 46.0<br>*108.5* | 63.7<br>*150.4* | 63.7<br>*150.4* | 102.2<br>*100.0* | 96.3<br>*94.3* | 96.8<br>*94.9* | 78.8<br>*77.3* | 78.8<br>*77.3* | 94.2<br>*100.0* | 96.3<br>*102.1* | 96.8<br>*102.7* | 78.8<br>*83.6* | 78.8<br>*83.6* | 10 |
| 11 | 47.5<br>*100.0* | 43.0<br>*90.6* | 41.9<br>*88.1* | 62.9<br>*132.3* | 62.9<br>*132.3* | 41.5<br>*100.0* | 43.0<br>*103.8* | 41.9<br>*101.0* | 62.9<br>*152.8* | 62.9<br>*152.8* | 89.0<br>*100.0* | 86.4<br>*97.1* | 88.1<br>*99.0* | 66.7<br>*74.9* | 66.7<br>*74.9* | 81.2<br>*100.0* | 86.4<br>*106.5* | 88.1<br>*108.5* | 66.7<br>*82.0* | 66.7<br>*82.0* | 11 |
| 12 | 41.4<br>*100.0* | 38.7<br>*93.6* | 38.2<br>*92.4* | 56.7<br>*137.0* | 56.7<br>*137.0* | 34.1<br>*100.0* | 38.7<br>*113.7* | 38.2<br>*112.0* | 56.7<br>*166.4* | 56.7<br>*166.4* | 81.5<br>*100.0* | 77.9<br>*95.6* | 79.1<br>*97.0* | 60.0<br>*73.6* | 60.0<br>*73.6* | 75.1<br>*100.0* | 77.9<br>*103.5* | 79.1<br>*105.4* | 60.0<br>*79.9* | 60.0<br>*79.9* | 12 |

## TABLE 24.

## COMPARISON OF THE APPROXIMATE METHODS WITH THE SLOPE-DEFLECTION METHOD WHEN APPLIED TO THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

### MOMENTS IN THE GIRDERS.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

| Story No. | Moment at Right End of Girder *a* | | | | | Moment at Left End of Girder *a* | | | | | Moment at End of Girder *b* | | | | | Story No. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | |
| | | | I | II | III | | | I | II | III | | | I | II | III | |
| 1 | 287.8 | 305.3 | 279.3 | 425.9 | 425.9 | 245.0 | 274.5 | 279.3 | 143.7 | 130.9 | 247.5 | 272.0 | 295.0 | 282.2 | 295.0 | 1 |
| | *100.0* | *106.1* | *97.1* | *148.0* | *148.0* | *100.0* | *112.0* | *114.0* | *58.6* | *53.4* | *100.0* | *110.0* | *119.0* | *114.0* | *119.0* | |
| 2 | 203.0 | 210.6 | 199.8 | 311.9 | 311.9 | 184.0 | 185.6 | 199.8 | 104.4 | 100.7 | 226.0 | 226.0 | 211.2 | 207.5 | 211.2 | 2 |
| | *100.0* | *103.5* | *98.5* | *153.6* | *153.6* | *100.0* | *100.8* | *108.5* | *56.7* | *54.8* | *100.0* | *100.0* | *93.4* | *91.9* | *93.4* | |
| 3 | 187.1 | 183.9 | 178.2 | 272.2 | 272.2 | 171.7 | 173.1 | 178.2 | 90.7 | 84.3 | 189.6 | 186.0 | 187.9 | 181.5 | 187.9 | 3 |
| | *100.0* | *98.2* | *95.2* | *145.8* | *145.8* | *100.0* | *100.8* | *104.0* | *52.9* | *49.1* | *100.0* | *98.0* | *99.0* | *95.8* | *99.0* | |
| 4 | 170.5 | 172.7 | 169.2 | 252.1 | 252.1 | 153.6 | 152.0 | 169.2 | 87.3 | 84.9 | 184.4 | 183.0 | 172.1 | 169.7 | 172.1 | 4 |
| | *100.0* | *101.0* | *99.5* | *148.0* | *148.0* | *100.0* | *99.0* | *110.3* | *56.9* | *55.3* | *100.0* | *99.0* | *93.5* | *92.0* | *93.5* | |
| 5 | 167.7 | 162.4 | 154.8 | 232.1 | 232.1 | 152.0 | 151.8 | 154.8 | 83.7 | 77.3 | 166.0 | 160.0 | 164.4 | 157.9 | 164.4 | 5 |
| | *100.0* | *97.0* | *92.4* | *138.7* | *138.7* | *100.0* | *99.9* | *101.8* | *55.0* | *50.9* | *100.0* | *96.5* | *99.0* | *95.0* | *99.0* | |
| 6 | 135.0 | 137.1 | 133.2 | 200.3 | 200.3 | 125.6 | 128.5 | 133.2 | 73.0 | 68.3 | 141.5 | 144.0 | 140.8 | 136.1 | 140.8 | 6 |
| | *100.0* | *101.2* | *98.8* | *148.2* | *148.2* | *100.0* | *102.6* | *112.2* | *58.1* | *54.4* | *100.0* | *102.0* | *99.5* | *96.3* | *99.5* | |
| 7 | 125.9 | 115.0 | 114.5 | 170.9 | 170.9 | 117.0 | 110.9 | 114.5 | 62.4 | 57.4 | 132.5 | 123.8 | 121.6 | 116.6 | 121.6 | 7 |
| | *100.0* | *91.5* | *91.0* | *135.9* | *135.9* | *100.0* | *94.9* | *98.0* | *53.3* | *49.1* | *100.0* | *93.0* | *91.8* | *88.0* | *91.8* | |
| 8 | 95.8 | 104.5 | 108.7 | 151.8 | 151.8 | 92.0 | 103.5 | 108.7 | 64.2 | 56.0 | 107.8 | 116.0 | 106.2 | 108.0 | 106.2 | 8 |
| | *100.0* | *109.3* | *113.3* | *158.4* | *158.4* | *100.0* | *112.5* | *118.0* | *69.8* | *60.9* | *100.0* | *108.0* | *95.8* | *100.3* | *95.8* | |
| 9 | 95.0 | 95.4 | 96.8 | 133.2 | 133.2 | 91.1 | 95.8 | 96.8 | 65.5 | 60.5 | 106.8 | 103.0 | 104.3 | 99.3 | 104.3 | 9 |
| | *100.0* | *100.4* | *101.9* | *140.3* | *140.3* | *100.0* | *105.0* | *112.4* | *71.9* | *66.4* | *100.0* | *99.4* | *97.6* | *93.0* | *97.6* | |
| 10 | 89.6 | 88.6 | 87.1 | 126.6 | 126.6 | 85.0 | 85.7 | 87.1 | 54.8 | 48.0 | 98.5 | 97.0 | 97.5 | 90.7 | 97.5 | 10 |
| | *100.0* | *98.9* | *97.3* | *141.2* | *141.2* | *100.0* | *100.8* | *102.4* | *64.5* | *56.5* | *100.0* | *98.5* | *99.0* | *92.1* | *99.0* | |
| 11 | 81.9 | 81.7 | 80.3 | 119.6 | 119.6 | 76.6 | 77.0 | 80.3 | 44 6 | 40.8 | 87.5 | 87.3 | 85.9 | 82.1 | 85.9 | 11 |
| | *100.0* | *99.8* | *98.2* | *146.2* | *146.2* | *100.0* | *100.4* | *104.8* | *58.2* | *53.3* | *100.0* | *99.8* | *98.1* | *94.0* | *98.1* | |
| 12 | 73.4 | 73.1 | 72.3 | 108.0 | 108.0 | 68.6 | 68.5 | 72.3 | 38.9 | 36.4 | 78.3 | 78.1 | 75.9 | 73.4 | 75.9 | 12 |
| | *100.0* | *99.6* | *98.7* | *147.2* | *147 2* | *100 0* | *100 0* | *105 5* | *56.8* | *53.0* | *100.0* | *99.8* | *96.9* | *93.8* | *96.9* | |

## TABLE 25.

## COMPARISON OF THE APPROXIMATE METHODS WITH THE SLOPE-DEFLECTION METHOD WHEN APPLIED TO THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 13.

### MOMENTS IN THE COLUMNS.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

| Story No. | Moment at Top of Column *A* | | | | | Moment at Bottom of Column *A* | | | | | Moment at Top of Column *B* | | | | | Moment at Bottom of Column *B* | | | | | Story No. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | |
| | | | I | II | III | | | I | II | III | | | I | II | III | | | I | II | III | |
| 1 | 193.2 | 203.5 | 138.0 | 254.4 | 254.4 | 275.9 | 203.5 | 138.0 | 254.4 | 254.4 | 248.0 | 305.0 | 370.5 | 254.4 | 254.4 | 302.8 | 305.0 | 370.5 | 254.4 | 254.4 | 1 |
| | *100.0* | *105.5* | *71.4* | *131.5* | *131.5* | *100.0* | *73.8* | *50.0* | *92.0* | *92.0* | *100.0* | *122.8* | *149.5* | *102.2* | *102.2* | *100.0* | *100.5* | *122.0* | *84.0* | *80.4* | |
| 2 | 125.9 | 127.5 | 92.5 | 171.4 | 171.4 | 123.8 | 127.5 | 92.5 | 171.4 | 171.4 | 212.7 | 214.0 | 248.5 | 171.4 | 171.4 | 223.7 | 214.0 | 248.5 | 171.4 | 171.4 | 2 |
| | *100.0* | *101.3* | *73.5* | *136.0* | *136.0* | *100.0* | *102.8* | *75.0* | *148.2* | *148.2* | *100.0* | *100.5* | *117.0* | *80.5* | *80.5* | *100.0* | *95.8* | *111.3* | *76.8* | *76.8* | |
| 3 | 110.6 | 108.0 | 75.9 | 140.5 | 140.5 | 101.6 | 108.0 | 75.9 | 140.5 | 140.5 | 177.1 | 174.0 | 203.9 | 140.5 | 140.5 | 171.6 | 174.0 | 203.9 | 140.5 | 140.5 | 3 |
| | *100.0* | *97.5* | *68.7* | *127.0* | *127.0* | *100.0* | *106.5* | *74.7* | *138.2* | *138.2* | *100.0* | *98.2* | *114.9* | *79.3* | *79.3* | *100.0* | *101.5* | *118.6* | *82.0* | *82.0* | |
| 4 | 99.4 | 97.8 | 71.5 | 131.7 | 131.7 | 99.4 | 97.8 | 71.5 | 131.7 | 131.7 | 165.3 | 164.1 | 191.9 | 131.7 | 131.7 | 163.7 | 164.1 | 191.9 | 131.7 | 131.7 | 4 |
| | *100.0* | *98.3* | *72.0* | *132.2* | *132.2* | *100.0* | *98.3* | *72.0* | *132.0* | *132.0* | *100.0* | *99.5* | *116.0* | *79.6* | *79.6* | *100.0* | *100.3* | *117.5* | *80.5* | *80.5* | |

# TABLE 26

## COMPARISON OF THE APPROXIMATE METHODS WITH THE SLOPE-DEFLECTION METHOD WHEN APPLIED TO THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 13.

## MOMENTS IN THE GIRDERS.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

| Story No. | Moment at Right End of Girder *a* | | | | | Moment at Left End of Girder *a* | | | | | Moment at End of Girder *b* | | | | | Story No. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | Slope-Deflection Method | Proposed Approximate Method | Fleming's Method | | | |
| | | | I | II | III | | | I | II | III | | | I | II | III | |
| 1 | 316.9<br>*100.0* | 331.0<br>*104.5* | 230.9<br>*73.0* | 425.8<br>*135.2* | 425.8<br>*135.2* | 277.4<br>*100.0* | 309.0<br>*111.5* | 230.9<br>*83.3* | 138.5<br>*50.0* | 36.0<br>*13.0* | 194.5<br>*100.0* | 210.0<br>*108.0* | 389.2<br>*200.0* | 287.3<br>*148.0* | 389.8<br>*200.0* | 1 |
| 2 | 227.6<br>*100.0* | 235.5<br>*103.8* | 167.7<br>*73.5* | 311.9<br>*137.0* | 311.9<br>*137.0* | 209.4<br>*100.0* | 212.2<br>*101.9* | 167.7<br>*80.0* | 103.0<br>*49.4* | 23.6<br>*11.3* | 174.6<br>*100.0* | 175.8<br>*100.5* | 282.8<br>*162.0* | 208.8<br>*119.6* | 288.2<br>*165.0* | 2 |
| 3 | 210.1<br>*100.0* | 205.8<br>*98.0* | 147.7<br>*70.0* | 272.2<br>*129.5* | 272.2<br>*129.5* | 194.7<br>*100.0* | 194.1<br>*99.7* | 147.7<br>*75.8* | 90.7<br>*46.6* | 23.2<br>*11.9* | 146.4<br>*100.0* | 144.0<br>*98.2* | 249.1<br>*170.0* | 181.4<br>*124.0* | 249.0<br>*170.0* | 3 |
| 4 | 191.0<br>*100.0* | 192.3<br>*100.8* | 138.1<br>*72.3* | 254.5<br>*133.0* | 254.5<br>*133.0* | 175.6<br>*100.0* | 173.3<br>*98.8* | 138.1<br>*79.0* | 84.8<br>*48.3* | 21.7<br>*12.4* | 143.3<br>*100.0* | 143.0<br>*98.8* | 232.9<br>*162.2* | 169.7<br>*118.1* | 232.8<br>*162.2* | 4 |

# TABLE 27.

## EFFECT OF THE PROPORTIONS OF A BENT UPON THE ACCURACY OF METHOD I.

## ALL GIRDER SECTIONS ARE THE SAME.

All stories of a bent are identical, and the shears on all stories are equal. All column sections are equal, all girder sections are equal, and the column sections are equal to the girder sections.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

| Bent No. | Proportions of the Bent | | Moment at Top and Bottom of Column *A* | | Moment at Top and Bottom of Column *B* | | Moment at Right End of Girder *a* | | Moment at Left End of Girder *a* | | Moment at End of Girder *b* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Ratio of Story Height to Width of Bay *a* | Ratio of Width of Bay *a* to Width of Bay *b* | Slope-Deflection Method | Method I | Slope-Deflection Method | Method I | Slope-Deflection Method | Method I | Slope-Deflection Method | Method I | Slope-Deflection Method | Method I |
| 1 | 2 | 2 | 9.09<br>*100.0* | 9.62<br>*105.9* | 15.91<br>*100.0* | 15.38<br>*96.5* | 18.20<br>*100.0* | 19.24<br>*105.9* | 13.67<br>*100.0* | 19.24<br>*140.7* | 18.20<br>*100.0* | 11.52<br>*63.2* |
| 2 | 2 | 1 | 10.00<br>*100.0* | 7.50<br>*75.0* | 15.00<br>*100.0* | 17.50<br>*116.5* | 20.00<br>*100.0* | 15.00<br>*75.0* | 16.66<br>*100.0* | 15.0<br>*90.0* | 13.33<br>*100.0* | 20.00<br>*150.0* |
| 3 | 2 | 0.5 | 10.87<br>*100.0* | 5.00<br>*46.0* | 14.13<br>*100.0* | 20.00<br>*141.5* | 21.70<br>*100.0* | 10.00<br>46.1 | 19.54<br>*100.0* | 10.00<br>*51.2* | 8.70<br>*100.0* | 30.00<br>*345.0* |
| 4 | 1 | 2 | 8.09<br>*100.0* | 9.62<br>*119.00* | 16.91<br>*100.0* | 15.38<br>*90.9* | 16.18<br>*100.0* | 19.24<br>*119.3* | 13.23<br>*100.0* | 19.24<br>*145.5* | 20.60<br>*100.0* | 11.52<br>*55.9* |
| 5 | 1 | 1 | 9.38<br>*100.0* | 7.50<br>*79.9* | 15.62<br>*100.0* | 17.50<br>*111.9* | 18.75<br>*100.0* | 15.00<br>*80.0* | 16.66<br>*100.0* | 15.00<br>*90.0* | 14.60<br>*100.0* | 20.00<br>*137.0* |
| 6 | 1 | 0.5 | 10.53<br>*100.0* | 5.00<br>*47.5* | 14.47<br>*100.0* | 20.00<br>*138.0* | 21.04<br>*100.0* | 10.00<br>*47.5* | 19.74<br>*100.0* | 10.00<br>*50.6* | 9.20<br>*100.0* | 30.00<br>*326.0* |
| 7 | 0.5 | 2 | 7.32<br>*100.0* | 9.62<br>*131.3* | 17.67<br>*100.0* | 15.38<br>*87.0* | 14.66<br>*100.0* | 19.24<br>*131.0* | 12.93<br>*100.0* | 19.24<br>*149.0* | 22.40<br>*100.0* | 11.52<br>*51.3* |
| 8 | 0.5 | 1 | 8.93<br>*100.0* | 7.50<br>*83.9* | 16.06<br>*100.0* | 17.50<br>*108.7* | 17.84<br>*100.0* | 15.00<br>*83.9* | 16.66<br>*100.0* | 15.00<br>*90.00* | 15.45<br>*100.0* | 20.00<br>*129.5* |
| 9 | 0.5 | 0.5 | 10.33<br>*100.0* | 5.00<br>*48.4* | 14.68<br>*100.0* | 20.00<br>*136.1* | 20.61<br>*100.0* | 10.00<br>*48.5* | 19.76<br>*100.0* | 10.0<br>*50.6* | 9.60<br>*100.0* | 30.00<br>*312.0* |

## TABLE 28.

## EFFECT OF THE PROPORTIONS OF A BENT UPON THE ACCURACY OF METHOD I.

## ALL GIRDERS PROPORTIONAL TO THE BENDING MOMENTS.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column *A* and girder *a* are equal, and the ratio of the moment of inertia of girder *a* to the moment of inertia of girder *b* equals the ratio of the bending moment in girder *a* to the bending moment in girder b, as determined by method I.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

| Bent No. | Proportions of the Bent | | Moment at Top and Bottom of Column *A* | | Moment at Top and Bottom of Column *B* | | Moment at Right End of Girder *a* | | Moment at Left End of Girder *a* | | Moment at End of Girder *b* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Ratio of Story Height to Width of Bay *a* | Ratio of Width of Bay *a* to Width of Bay *b* | Slope-Deflection Methed | Method I | Slope-Deflection Method | Method I | Slope-Deflection Method | Method I | Slope-Deflection Method | Method I | Slope-Deflection Method | Method I |
| 1 | 2 | 2 | 9.75 | 9.62 | 15.25 | 15.38 | 19.50 | 19.24 | 16.05 | 19.24 | 14.45 | 11.52 |
| | | | *100.0* | *98.8* | *100.0* | *100.5* | *100.0* | *98.8* | *100.0* | *120.0* | *100.0* | *79.6* |
| 2 | 2 | 1 | 9.60 | 7.50 | 15.40 | 17.50 | 19.2 | 15.00 | 15.60 | 15.00 | 15.20 | 20.00 |
| | | | *100.0* | *78.1* | *100.0* | *113.6* | *100.0* | *78.1* | *100.0* | *96.1* | *100.0* | *131.5* |
| 3 | 2 | 0.5 | 9.45 | 5.00 | 15.55 | 20.00 | 18.9 | 10.00 | 15.10 | 10.00 | 16.00 | 30.00 |
| | | | *100.0* | *52.9* | *100.0* | *129.0* | *100.0* | *52.9* | *100.0* | *66.3* | *100.0* | *187.5* |
| 4 | 1 | 2 | 9.05 | 9.62 | 15.95 | 15.38 | 18.10 | 19.24 | 15.70 | 19.24 | 16.20 | 11.52 |
| | | | *100.0* | *106.2* | *100.0* | *96.2* | *100.0* | *106.2* | *100.0* | *122.5* | *100.0* | *92.6* |
| 5 | 1 | 1 | 8.85 | 7.50 | 16.15 | 17.50 | 17.70 | 15.00 | 15.20 | 15.00 | 17.10 | 20.00 |
| | | | *100.0* | *84.8* | *100.0* | *108.3* | *100.0* | *84.8* | *100.0* | *98.8* | *100.0* | *117.0* |
| 6 | 1 | 0.5 | 8.60 | 5.00 | 16.40 | 20.00 | 17.20 | 10.00 | 14.7 | 10.00 | 18.1 | 30.00 |
| | | | *100.0* | *58.0* | *100.0* | *122.0* | *100.0* | *58.1* | *100.0* | *68.0* | *100.0* | *165.8* |
| 7 | 0.5 | 2 | 8.55 | 9.62 | 16.45 | 15.38 | 17.10 | 19.24 | 15.80 | 19.24 | 17.1 | 11.52 |
| | | | *100.0* | *112.5* | *100.0* | *93.2* | *100.0* | *112.5* | *100.0* | *122.0* | *100.0* | *67.3* |
| 8 | 0.5 | 1 | 8.30 | 7.50 | 16.70 | 17.50 | 16.60 | 15.00 | 15.20 | 15.00 | 18.2 | 20.00 |
| | | | *100.0* | *90.3* | *100.0* | *105.0* | *100.0* | *90.4* | *100.0* | *98.8* | *100.0* | *110.0* |
| 9 | 0.5 | 0.5 | 8.00 | 5.00 | 17.00 | 20.00 | 16.00 | 10.00 | 14.60 | 10.00 | 19.4 | 30.00 |
| | | | *100.0* | *62.5* | *100.0* | *117.5* | *100.0* | *62.5* | *100.0* | *68.5* | *100.0* | *154.8* |

## TABLE 29.

## EFFECT OF THE PROPORTIONS OF A BENT UPON THE ACCURACY OF METHOD II.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column *A* and girder *a* are equal, and the ratio of the moment of inertia of girder *a* to the moment of inertia of girder *b* equals the ratio of the bending moment in girder *a* to the bending moment in girder *b*, as determined by method II.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

| Bent No. | Proportions of the Bent | | Moment at Top and Bottom of Column *A* | | Moment at Top and Bottom of Column *B* | | Moment at Right End of Girder *a* | | Moment at Left End of Girder *a* | | Moment at End of Girder *b* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Ratio of Story Height to Width of Bay *a* | Ratio of Width of Bay a to Width of Bay *b* | Slope-Deflection Method | Method II | Slope-Deflection Method | Method II | Slope-Deflection Method | Method II | Slope-Deflection Method | Method II | Slope-Deflection Method | Method II |
| 1 | 2 | 2 | 9.65<br>*100.0* | 12.50<br>*129.5* | 15.35<br>*100.0* | 12.50<br>*81.5* | 19.30<br>*100.0* | 25.00<br>*129.5* | 15.45<br>*100.0* | 8.33<br>*53.9* | 15.25<br>*100.0* | 16.66<br>*109.3* |
| 2 | 2 | 1 | 10.55<br>*100.0* | 12.50<br>*118.5* | 14.45<br>*100.0* | 12.50<br>*86.5* | 21.10<br>*100.0* | 25.00<br>*118.5* | 18.30<br>*100.0* | 8.33<br>*45.6* | 10.60<br>*100.0* | 16.66<br>*157.1* |
| 3 | 2 | 0.5 | 11.25<br>*100.0* | 12.50<br>*111.0* | 13.75<br>*100.0* | 12.50<br>*91.0* | 22.50<br>*100.0* | 25.00<br>*111.1* | 20.95<br>*100.0* | 8.33<br>*39.8* | 6.55<br>*100.0* | 16.66<br>*255.0* |
| 4 | 1 | 2 | 8.82<br>*100.0* | 12.50<br>*141.8* | 16.18<br>*100.0* | 12.50<br>*77.3* | 17.64<br>*100.0* | 25.00<br>*141.8* | 15.26<br>*100.0* | 8.33<br>*54.6* | 17.10<br>*100.0* | 16.66<br>*97.5* |
| 5 | 1 | 1 | 10.10<br>*100.0* | 12.50<br>*123.8* | 14.90<br>*100.0* | 12.50<br>*83.9* | 20.20<br>*100.0* | 25.00<br>*124.0* | 18.45<br>*100.0* | 8.33<br>*45.20* | 11.35<br>*100.0* | 16.66<br>*146.8* |
| 6 | 1 | 0.5 | 11.00<br>*100.0* | 12.50<br>*113.5* | 14.00<br>*100.0* | 12.50<br>*89.3* | 22.00<br>*100.0* | 25.00<br>*113.7* | 21.20<br>*100.0* | 8.33<br>*39.3* | 6.80<br>*100.0* | 16.66<br>*245.0* |
| 7 | 0.5 | 2 | 8.30<br>*100.0* | 12.50<br>*150.6* | 16.70<br>*100.0* | 12.50<br>*74.9* | 16.60<br>*100.0* | 25.00<br>*150.7* | 15.20<br>*100.0* | 8.33<br>*54.9* | 18.20<br>*100.0* | 16.66<br>*91.6* |
| 8 | 0.5 | 1 | 9.80<br>*100.0* | 12.50<br>*127.5* | 15.20<br>*100.0* | 12.50<br>*82.3* | 19.60<br>*100.0* | 25.00<br>*127.0* | 18.55<br>*100.0* | 8.33<br>*44.9* | 11.85<br>*100.0* | 16.66<br>*140.5* |
| 9 | 0.5 | 0.5 | 10.85<br>*100.0* | 12.50<br>*115.2* | 14.15<br>*100.0* | 12.50<br>*88.4* | 21.70<br>*100.0* | 25.00<br>*115.0* | 21.40<br>*100.0* | 8.33<br>*38.9* | 6.9<br>*100.0* | 16.66<br>*241.5* |

## TABLE 30.

## EFFECT OF THE PROPORTIONS OF A BENT UPON THE ACCURACY OF METHOD III.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column *A* and girder *a* are equal, and the ratio of the moment of inertia of girder *a* to the moment of inertia of girder *b* equals the ratio of the bending moment in girder *a* to the bending moment in girder *b*, as determined by method III.

For each bent the upper line is the moment in per cent of W x h, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

| Bent No. | Proportions of the Bent | | Moment at Top and Bottom of Column *A* | | Moment at Top and Bottom of Column *B* | | Moment at Right End of Girder *a* | | Moment at Left End of Girder *a* | | Moment at End of Girder *b* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Ratio of Story Height to Width of Bay *a* | Ratio of Width of Bay *a* to Width of Bay *b* | Slope-Deflection Method | Method III | Slope-Deflection Method | Method III | Slope-Deflection Method | Method III | Slope-Deflection Method | Method III | Slope-Deflection Method | Method III |
| 1 | 2 | 2 | 10.12<br>*100.0* | 12.50<br>*123.5* | 14.88<br>*100.0* | 12.50<br>*84.1* | 20.24<br>*100.0* | 25.00<br>*123.5* | 17.06<br>*100.0* | 13.48<br>*79.0* | 12.70<br>*100.0* | 11.52<br>*90.8* |
| 2 | 2 | 1 | 10.30<br>*100.0* | 12.50<br>*121.5* | 14.70<br>*100.0* | 12.50<br>*85.2* | 20.60<br>*100.0* | 25.00<br>*121.5* | 17.60<br>*100.0* | 5.0<br>*28.4* | 11.80<br>*100.0* | 20.00<br>*170.0* |
| 3 | 2 | 0.5 | 10.67<br>*100.0* | 12.50<br>*117.4* | 14.33<br>*100.0* | 12.50<br>*87.3* | 21.34<br>*100.0* | 25.00<br>*117.3* | 18.79<br>*100.0* | —5.0<br>....... | 9.87<br>*100.0* | 30.00<br>*304.5* |
| 4 | 1 | 2 | 9.55<br>*100.0* | 12.50<br>*131.0* | 15.45<br>*100.0* | 12.50<br>*80.9* | 19.10<br>*100.0* | 25.00<br>*131.0* | 17.00<br>*100.0* | 13.48<br>*79.3* | 13.90<br>*100.0* | 11.52<br>*83.0* |
| 5 | 1 | 1 | 9.80<br>*100.0* | 12.50<br>*127.7* | 15.20<br>*100.0* | 12.50<br>*82.3* | 19.60<br>*100.0* | 25.00<br>*127.6* | 17.60<br>*100.0* | 5.0<br>*28.4* | 12.80<br>*100.0* | 20.00<br>*156.3* |
| 6 | 1 | 0.5 | 10.26<br>*100.0* | 12.50<br>*122.0* | 14.74<br>*100.0* | 12.50<br>*84.9* | 20.52<br>*100.0* | 25.00<br>*122.0* | 18.98<br>*100.0* | —5.0<br>....... | 10.50<br>*100.0* | 30.00<br>*286.1* |
| 7 | 0.5 | 2 | 9.15<br>*100.0* | 12.50<br>*136.9* | 15.85<br>*100.0* | 12.50<br>*78.8* | 18.30<br>*100.0* | 25.00<br>*136.8* | 17.10<br>*100.0* | 13.48<br>*78.8* | 14.60<br>*100.0* | 11.52<br>*79.0* |
| 8 | 0.5 | 1 | 9.46<br>*100.0* | 12.50<br>*132.2* | 15.54<br>*100.0* | 12.50<br>*80.5* | 18.92<br>*100.0* | 25.00<br>*132.1* | 17.63<br>*100.0* | 5.0<br>*28.4* | 13.45<br>*100.0* | 20.00<br>*148.8* |
| 9 | 0.5 | 0.5 | 10.05<br>*100.0* | 12.50<br>*124.5* | 14.95<br>*100.0* | 12.50<br>*83.70* | 20.10<br>*100.0* | 25.00<br>*124.5* | 18.95<br>*100.0* | —5.0<br>....... | 10.95<br>*100.0* | 30.00<br>*274.5* |

## TABLE 31.

## EFFECT OF THE PROPORTIONS OF A BENT UPON THE ACCURACY OF METHOD IV.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column *A* and girder *a* are equal, and the ratio of the moment of inertia of girder *a* to the moment of inertia of girder *b* equals the ratio of the bending moment in girder *a* to the bending moment in girder *b*, as determined by method IV.

For each bent the upper line is the moment in per cent of *W* x *h*, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

| Bent No. | Proportions of the Bent | | Moment at Top and Bottom of Column *A* | | Moment at Top and Bottom of Column *B* | | Moment at Right End of Girder *a* | | Moment at Left End of Girder *a* | | Moment at End of Girder *b* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Ratio of Story Height to Width of Bay *a* | Ratio of Width of Bay *a* to Width of Bay *b* | Slope-Deflection Method | Method IV | Slope-Deflection Method | Method IV | Slope-Deflection Method | Method IV | Slope-Deflection Method | Method IV | Slope-Deflection Method | Method IV |
| 1 | 2 | 2 | 9.09<br>*100.0* | 8.33<br>*91.8* | 15.91<br>*100.0* | 16.66<br>*104.5* | 18.18<br>*100.0* | 16.66<br>*91.7* | 13.62<br>*100.0* | 16.66<br>*122.1* | 18.20<br>*100.0* | 16.66<br>*91.5* |
| 2 | 2 | 1 | 10.0<br>*100.0* | 8.33<br>*83.3* | 15.00<br>*100.0* | 16.66<br>*111.1* | 20.00<br>*100.0* | 16.66<br>*83.3* | 16.66<br>*100.0* | 16.66<br>*100.0* | 13.33<br>*100.0* | 16.66<br>*125.0* |
| 3 | 2 | 0.5 | 10.89<br>*100.0* | 8.33<br>*76.8* | 14.13<br>*100.0* | 16.66<br>*117.8* | 21.74<br>*100.0* | 16.66<br>*76.6* | 19.56<br>*100.0* | 16.66<br>*85.20* | 8.70<br>*100.0* | 16.66<br>*191.5* |
| 4 | 1 | 2 | 8.09<br>*100.0* | 8.33<br>*103.2* | 16.91<br>*100.0* | 16.66<br>*98.3* | 16.18<br>*100.0* | 16.66<br>*103.0* | 13.22<br>*100.0* | 16.66<br>*126.0* | 20.60<br>*100.0* | 16.66<br>*80.9* |
| 5 | 1 | 1 | 9.38<br>*100.0* | 8.33<br>*88.8* | 15.62<br>*100.0* | 16.66<br>*106.4* | 18.76<br>*100.0* | 16.66<br>*88.9* | 16.64<br>*100.0* | 16.66<br>*100.0* | 14.60<br>*100.0* | 16.66<br>*114.0* |
| 6 | 1 | 0.5 | 10.53<br>*100.0* | 8.33<br>*79.2* | 14.47<br>*100.0* | 16.66<br>*115.0* | 21.06<br>*100.0* | 16.66<br>*79.1* | 19.74<br>*100.0* | 16.66<br>*84.4* | 9.20<br>*100.0* | 16.66<br>*181.0* |
| 7 | 0.5 | 2 | 7.32<br>*100.0* | 8.33<br>*113.9* | 17.68<br>*100.0* | 16.66<br>*94.2* | 14.64<br>*100.0* | 16.66<br>*113.8* | 12.96<br>*100.0* | 16.66<br>*128.7* | 22.40<br>*100.0* | 16.66<br>*74.4* |
| 8 | 0.5 | 1 | 8.93<br>*100.0* | 8.33<br>*93.3* | 16.07<br>*100.0* | 16.66<br>*103.7* | 17.86<br>*100.0* | 16.66<br>*93.3* | 16.69<br>*100.0* | 16.66<br>*100.0* | 15.45<br>*100.0* | 16.66<br>*107.8* |
| 9 | 0.5 | 0.5 | 10.33<br>*100.0* | 8.33<br>*80.7* | 14.67<br>*100.0* | 16.66<br>*113.5* | 20.66<br>*100.0* | 16.66<br>*80.6* | 19.74<br>*100.0* | 16.66<br>*84.4* | 9.60<br>*100.0* | 16.66<br>*173.5* |

## TABLE 32.

## LOG OF THE TEST OF CELLULOID MODEL NO. 4.

| Story No. | Shear $W$ in Lb. | Horizontal Deflection at Center of Girder at Top of Story in Inches | | | Change in Slope Measured on an Arc Having a 19-inch Radius, in Inches. | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Force at Top of Model Acting Toward the Right | | | | Force at Top of Model Acting Toward the Left | | | |
| | | | | | $\theta$ at Top of Column $A$ | | $\theta$ at Top of Column $B$ | | $\theta$ at Top of Column $A$ | | $\theta$ at Top of Column $A$ | |
| | | Right-Hand Span | Middle Span | Left-Hand Span | Right-Hand Side of Model | Left-Hand Side of Model | Right-Hand Side of Model | Left-Hand Side of Model | Right-Hand Side of Model | Left-Hand Side of Model | Right-Hand Side of Model | Left-Hand Side of Model |
| 1 | 1.5 | .04 | .04 | .04 | .20 | .20 | .09 | .10 | .19 | .18 | .09 | .10 |
| | 3.0 | .08 | .08 | .08 | .40 | .40 | .20 | .21 | .40 | .40 | .19 | .23 |
| | 4.5 | .11 | .11 | .11 | .61 | .59 | .31 | .33 | .61 | .63 | .30 | .38 |
| 2 | 1.5 | .08 | .08 | .08 | .21 | .21 | .10 | .10 | .19 | .19 | .11 | .10 |
| | 3.0 | .16 | .16 | .16 | .43 | .43 | .21 | .20 | .41 | .41 | .24 | .22 |
| | 4.5 | .25 | .24 | .24 | .66 | .66 | .34 | .33 | .63 | .65 | .37 | .37 |
| 3 | 1.5 | .14 | .14 | .14 | .18 | .17 | .11 | .12 | .17 | .16 | .13 | .14 |
| | 3.0 | .28 | .29 | .28 | .37 | .35 | .26 | .26 | .36 | .35 | .26 | .29 |
| | 4.5 | .42 | .42 | .42 | .59 | .54 | .39 | .40 | .56 | .55 | .40 | .42 |
| 4 | 1.5 | .18 | .18 | .18 | .13 | .10 | .06 | .07 | .13 | .09 | .06 | .07 |
| | 3.0 | .36 | .36 | .37 | .25 | .19 | .13 | .15 | .26 | .20 | .13 | .15 |
| | 4.5 | .55 | .55 | .55 | .40 | .30 | .20 | .22 | .39 | .27 | .18 | .25 |

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# UNIVERSITY OF ILLINOIS
# ENGINEERING EXPERIMENT STATION

BULLETIN NO. 81 JULY, 1915

## INFLUENCE OF TEMPERATURE ON THE STRENGTH OF CONCRETE

BY A. B. MCDANIEL, ASSISTANT PROFESSOR OF CIVIL ENGINEERING.

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# INFLUENCE OF TEMPERATURE ON THE STRENGTH OF CONCRETE

## I. Introduction.

1. *Preliminary.*—The general use of concrete in various kinds of construction and at all seasons of the year renders important a knowledge of the effect of temperature upon the strength of this material. It is of special economic importance to the contractor or the builder to be informed concerning the strength of concrete at early ages under different temperature conditions so that he may know when to remove forms and what loads may be safely applied to the different parts of a structure.

2. *Scope of Bulletin.*—It is the purpose of this bulletin to furnish some information concerning the influence of temperature on the attainment of strength in concrete.

Three groups of tests were made, viz.; forty-five 6 by 6-in. cylinders; fifty-one 6-in. cubes; and sixty 8 by 16-in. cylinders.

The temperature conditions were limited by available facilities.

3. *Acknowledgment.*—The tests reported herein were made in the Laboratory of Applied Mechanics of the University of Illinois. The work was done under the supervision of the writer. Special acknowledgment is due to the Department of Theoretical and Applied Mechanics for the use of material and apparatus. The writer is indebted to A. N. Talbot, Professor in Charge of Theoretical and Applied Mechanics, and to Ira O. Baker, Professor of Civil Engineering, for their co-operation in planning the tests and in interpreting the data.

The tests of Groups I and II—1913 Series—were made by J. Albert Anderson and W. J. Bublitz, senior civil engineering students of the class of 1914; and furnished the subject matter of their baccalaureate thesis. The tests of Group III—1914 Series—were made by J. Albert Anderson, a graduate student in the Department of Civil Engineering; and special credit is due Mr. Anderson for the preparation of the tables and diagrams. All the tests were made with painstaking care and faithful attention to uniformity and accuracy of manipulation.

## II. Materials, Form of Test Pieces, and Methods of Storing and Testing.

4. *Materials.*—The materials were of the same character and quality as those used for other concrete and reinforced concrete specimens made and tested by the Engineering Experiment Station during the past five years. The quality of the materials may be taken as representative of that used in first-class concrete work in the Middle West.

Cement. All of the test specimens were made with Universal portland cement. Samples were taken at the beginning of each series and were tested for fineness, soundness, and tensile strength. The cement passed the requirements of the Standard Specifications of the American Society for Testing Materials. The tensile strength tests of neat and 1:3 mortar briquettes made of a sample of the cement used in Group III of the 1914 Series gave average values of 542 and 609 lb. per sq. in. for the neat cement at seven and twenty-eight days, respectively; and 174 and 295 lb. per sq. in. for the 1:3 mortar at seven and twenty-eight days, respectively.

Sand. The sand used came from a deposit of glacial drift near the Wabash River at Attica, Indiana. The sand was clean and well graded. The sand of the 1913 Series was somewhat coarser than that of the 1914 lot. The sand used in Group III—1914 Series—gave a density of 1.79, a specific gravity of 2.65, and contained 32 per cent voids.

Stone. The crushed limestone came from Kankakee, Illinois. The stone used in the 1913 Series contained 87 per cent material smaller than one-half inch and 46 per cent material smaller than one-fourth inch. The stone used in the 1914 series was well graded. It contained 49 per cent voids, and had a density of 1.35 and a specific gravity of 2.65. It was carefully screened over a ¼-in. screen before use, and contained 10 per cent of material smaller than one-fourth inch.

5. *Concrete.*—All the concrete was composed of one part cement, 2 parts sand, and 4 parts broken stone, by weight; corresponding to 1 part cement, 2.2 parts sand, and 3.6 parts broken stone, by volume. The materials for each specimen were weighed out separately and then mixed.

The mixing of the concrete for the 1913 series was done with a trowel in a large galvanized iron pan. The cement and sand were first mixed dry to a uniform color and spread out in a layer of uni-

### TABLE 1.
### DESCRIPTION OF TEST SPECIMENS

| Series | Group | Set | Specimens | | Number and Age of Specimens When Tested |
|---|---|---|---|---|---|
| | | | Number | Form | |
| 1913 | I | A<br>B<br>C | 15<br>15<br>15 | 6 x 6-in. cylinders | 5 specimens of each set; at 7, 14, and 28 days. |
| 1913 | II | D<br>E<br>F | 15<br>18<br>18 | 6-in. cubes | 3 specimens of each set; at 4, 7, 11, 14, and 28 days. |
| 1914 | III | G<br>H<br>I<br>M | 15<br>15<br>15<br>15 | 8 x 16-in. cylinders | 3 specimens of each set; at 3, 7, 10, 14, and 28 days. |

form thickness over the bottom of the pan. The stone was then added, and the whole mass given four complete turnings, which secured thorough incorporation of the dry materials. Water was added, and the material turned until thoroughly mixed. The concrete was gathered together in a compact mass, in one end of the mixing pan, so as to reduce evaporation losses to a minimum. The time of mixing of each specimen was kept as nearly constant as possible.

The concrete used in the 1914 Series was mixed in similar manner to that of the 1913 Series, but was mixed on the concrete floor of the laboratory with shovels.

6. *Molding and Storage of Test Specimens.*—The specimens were classified according to the form of test specimen and storage conditions. Table 1 gives the details of the classification.

### TABLE 2.
### DATA CONCERNING MOLDING OF SPECIMENS

| Type of Specimen | Set | Average Time of Molding, minutes | Average Temperature | | Weights of Materials | | | Water, per cent* |
|---|---|---|---|---|---|---|---|---|
| | | | Air | Concrete | Cement, lb. | Sand, lb. | Stone, lb. | |
| 6-in. cylinders | A | 8.5 | 32°F. | 70°F. | 2.17 | 4.34 | 8.68 | 10.0 |
| | B | 8.5 | 65 | 71 | 2.17 | 4.34 | 8.68 | 10.0 |
| | C | 8.5 | 84 | 70 | 2.17 | 4.34 | 8.68 | 10.0 |
| 6-in. cubes | D | 7.0 | 77 | 70 | 2.42 | 4.84 | 9.68 | 10.0 |
| | E | 7.0 | 75 | 70 | 2.42 | 4.84 | 9.68 | 11.0 |
| | F | 7.0 | 71 | 69 | 2.42 | 4.84 | 9.68 | 10.0 |
| 8 x 16-in. cylinders | G | | 68 | 69 | 10.2 | 20.4 | 40.8 | 9.3 |
| | H | | | | | | | |
| | I | | | | | | | |
| | M | | | | | | | |

*The concrete used in Groups I and II was of a medium or quaking consistency; while that used in Group III was wet, and was similar in consistency to that used in concrete building construction.

Molding. The specimens of Group I of the 1913 Series were molded in the storage rooms under the following temperatures: Set A at 32°F., Set B at 65°F., and Set C at 84°F. The specimens of Group II of the 1913 Series were molded in the cement laboratory at the following temperatures: Set D at 77°F., Set E at 75°F., and Set F at 71°F. The specimens of Group III—1914 Series—were molded in the concrete room of the Engineering Experiment Station at a temperature of 68°F. The specimens of Group II and Group III were moved to their respective storage rooms after a set of six hours.

The forms used for Group I were sheet-iron cylinders 6 in. in diameter and 6 in. high. The specimens of Group II were molded in three-gang cube forms made up of two 6-in. channels and plates placed 6 in. apart. The forms for the specimens of Group III were sections of standard 8-in. wrought iron pipe, 16 in. long. The forms were removed from the specimens after a storage of two days.

Table 2 shows the weight of the dry materials, the per cent of water in terms of the total dry materials, the temperature of the room and of the concrete, and the average time of molding.

Storage. The temperature of the storage room was determined by daily readings of the maximum and minimum thermometers. The temperatures for the several groups are shown in Fig. 1–10.

Set A was stored in the ice-storage room of the Smith Ice Company in Urbana, at an average temperature of 30°F. Set B was stored in the meat storage room of the Smith Ice Company in Urbana, at an average temperature of 48.5°F. Set C was stored in the cement laboratory of the University of Illinois at an average temperature of 72.8°F.

Set D was stored in the cement laboratory of the University of Illinois at an average temperature of 68°F. Set E was stored in the ice chest of the Dairy Department of the University of Illinois at an average temperature of 35.5°F. Set F was stored in the ice-storage room of the Twin City Ice and Cold Storage Company of Champaign, at an average temperature of 27.1°F.

Set G was stored in the ice-storage room of the Twin City Ice and Cold Storage Company at Champaign, at an average temperature of 26.5°F. Set H was stored in the ice chest of the Dairy Department of the University of Illinois at an average temperature of 34.7°F. Set I was stored in an interior heated room of the Twin City Ice and Cold Storage Company of Champaign, at an average temperature of 71.8°F. Set M was stored in a chamber of the conduit tunnel under the Floriculture building of the University of Illinois, at an average temperature of 95.6°F.

All the specimens while in storage were covered with several layers of moist sacking, which was sprinkled daily.

7. *Method of Testing.*—All the specimens of Group I were taken from their storage places to the Laboratory of Applied Mechanics of the University of Illinois the day before they were tested. They were measured and weighed, their bearing surfaces coated with plaster of paris, and then were left in the open air of the laboratory for about twenty hours under a temperature of about 70°F.

The specimens of Group II were tested after about one hour from the time of their removal from the storage rooms. Two specimens of Set F, designated as $F_{17}$ and $F_{18}$, after being stored under an average mean daily temperature of 27.1°F. for forty-four days, were stored in the testing laboratory under an average mean daily temperature of 70°F., the former for seven days and the latter for twenty-one days.

The specimens of Group III were brought to the testing laboratory from their storage places, weighed, measured, plastered, and tested within one hour. The specimens of Set G, which were stored under freezing temperatures, were allowed to thaw out before being tested.

In the tests a spherical-seated bearing block was used.

## III. THE DATA.

8. *Observed Results.*—The results of the tests are given in Tables 3 to 11, pages 8 to 16, and in Fig. 1–10.

9. *Standardized Strength.*—Since a cube or a cylinder having a height equal to its diameter, tested for compressive strength, may be expected to give a value which is higher than the representative compressive strength of the material, it seems desirable for the purposes of comparison to reduce the observed values for the cubes and short cylinders of Groups I and II to what may be considered as the equivalent values which would be obtained from cylinders of height equal to twice their diameter. To do this the values for the cubes and cylinders have been multiplied by 0.73, which is the ratio of strength of prisms to strength of cubes determined by the Committee on Specifications and Methods of Tests for Concrete Materials of the American Concrete Institute. The reduced values are designated as the standardized strengths in Table 3 to 11, and are shown by the lower curve in Fig. 1–10.

### TABLE 3.

### COMPRESSIVE STRENGTH—AGE 7 DAYS

Group I. 6 x 6-in. Cylinders

| Set | Average Diameter, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Standardized Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|
| A | 6.0 | 29 240 | 1030 | 890 | 650 | Cracked uniformly around circumferential area. |
| | 6.18 | 27 670 | 930 | | | |
| | 6.06 | 24 890 | 870 | | | |
| | 6.06 | 22 450 | 780 | | | |
| | 6.06 | 24 450 | 850 | | | |
| B | 6.12 | 36 130 | 1230 | 1100 | 800 | Cracked uniformly |
| | 6.00 | 34 770 | 1230 | | | " " |
| | 6.00 | 31 660 | 1120 | | | " " |
| | 5.87 | 25 030 | 930 | | | Skewed |
| | 5.87 | 26 950 | 980 | | | " |
| C | 5.87 | 27 080 | 1000 | 1200 | 880 | Visible voids |
| | 5.81 | 35 830 | 1350 | | | |
| | 5.94 | 37 660 | 1370 | | | |
| | 5.87 | 31 140 | 1150 | | | |
| | 6.00 | 32 250 | 1140 | | | |

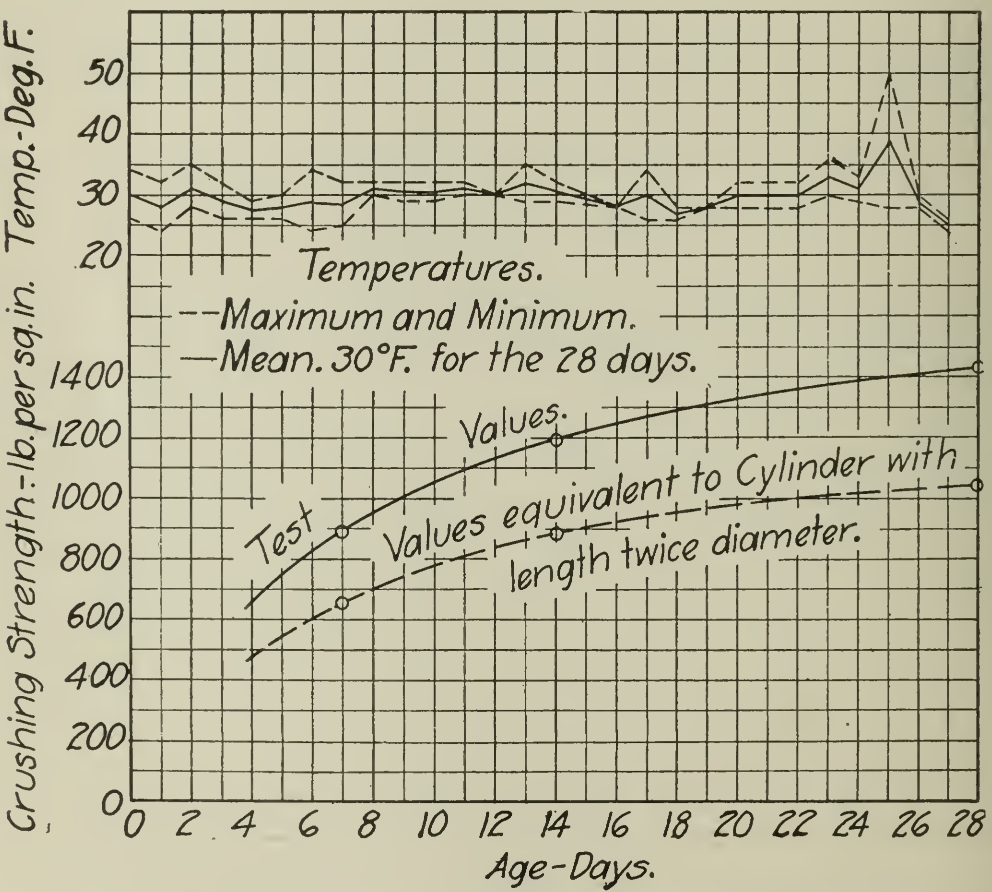

FIG. 1. SET A, GROUP I—1913 SERIES—6 x 6-IN. CYLINDERS.

### TABLE 4.

### COMPRESSIVE STRENGTH—AGE 14 DAYS

Group I. 6 x 6-in. Cylinders

| Set | Average Diameter, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Standardized Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|
| A | 6.0 | 38 020 | 1340 | 1190 | 870 | Cracked uniformly around circumferential area. |
| | 6.12 | 33 340 | 1140 | | | |
| | 6.0 | 39 220 | 1390 | | | |
| | 6.0 | 31 390 | 1110 | | | |
| | 5.87 | 26 700 | 980 | | | |
| B | 6.0 | 47 090 | 1670 | 1540 | 1130 | Cracked uniformly |
| | 5.94 | 50 460 | 1820 | | | " " |
| | 5.97 | 45 850 | 1640 | | | " " |
| | 5.94 | 30 000 | 1090 | | | Badly skewed |
| | 5.87 | 40 640 | 1500 | | | Slightly skewed |
| C | 5.87 | 46 170 | 1700 | 1660 | 1210 | Uniform throughout |
| | 6.0 | 50 000 | 1770 | | | |
| | 6.12 | 44 190 | 1510 | | | |
| | 6.0 | 44 420 | 1570 | | | |
| | 5.87 | 47 100 | 1740 | | | |

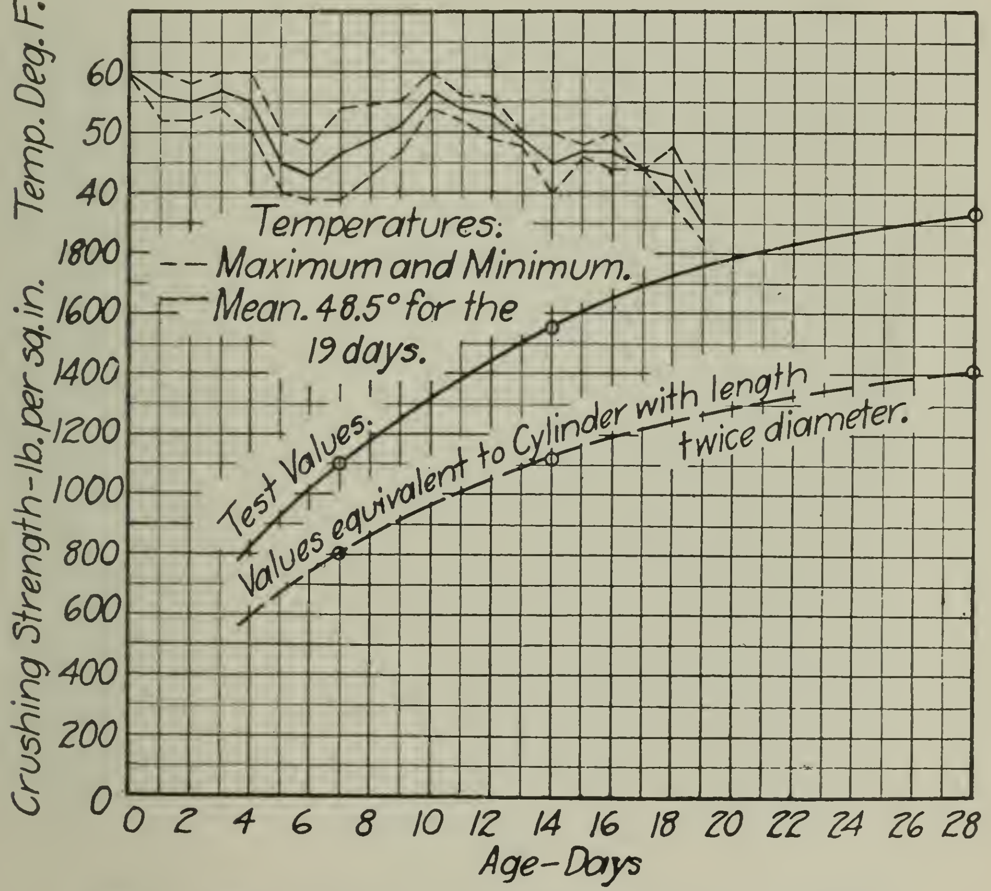

FIG. 2. SET B, GROUP I—1913 SERIES—6 x 6-IN. CYLINDERS.

### TABLE 5.

### COMPRESSIVE STRENGTH—AGE 28 DAYS

Group I. 6 x 6-in. Cylinders

| Set | Average Diameter, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Standardized Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|
| A | 6.06 | 39 340 | 1370 | 1430 | 1040 | All uniform. |
| | 6.06 | 37 730 | 1320 | | | |
| | 5.94 | 48 450 | 1750 | | | |
| | 6.12 | 37 660 | 1280 | | | |
| | 6.00 | 40 300 | 1430 | | | |
| B | 6.06 | 56 240 | 1960 | 1940 | 1410 | Area reduced by visible voids. |
| | 6.0 | 55 300 | 1950 | | | |
| | 6.06 | 54 600 | 1900 | | | |
| | 6.0 | 34 670 | 1230 | | | |
| | 6.0 | 40 000 | 1420 | | | |
| C | 5.97 | 55 720 | 2000 | 2090 | 1530 | |
| | 5.94 | 63 650 | 2310 | | | |
| | 5.87 | 60 260 | 2220 | | | |
| | 5.87 | 49 760 | 1840 | | | Slightly skewed |
| | 6.0 | 40 390 | *1430 | | | Badly skewed |

*Not used in calculating average.

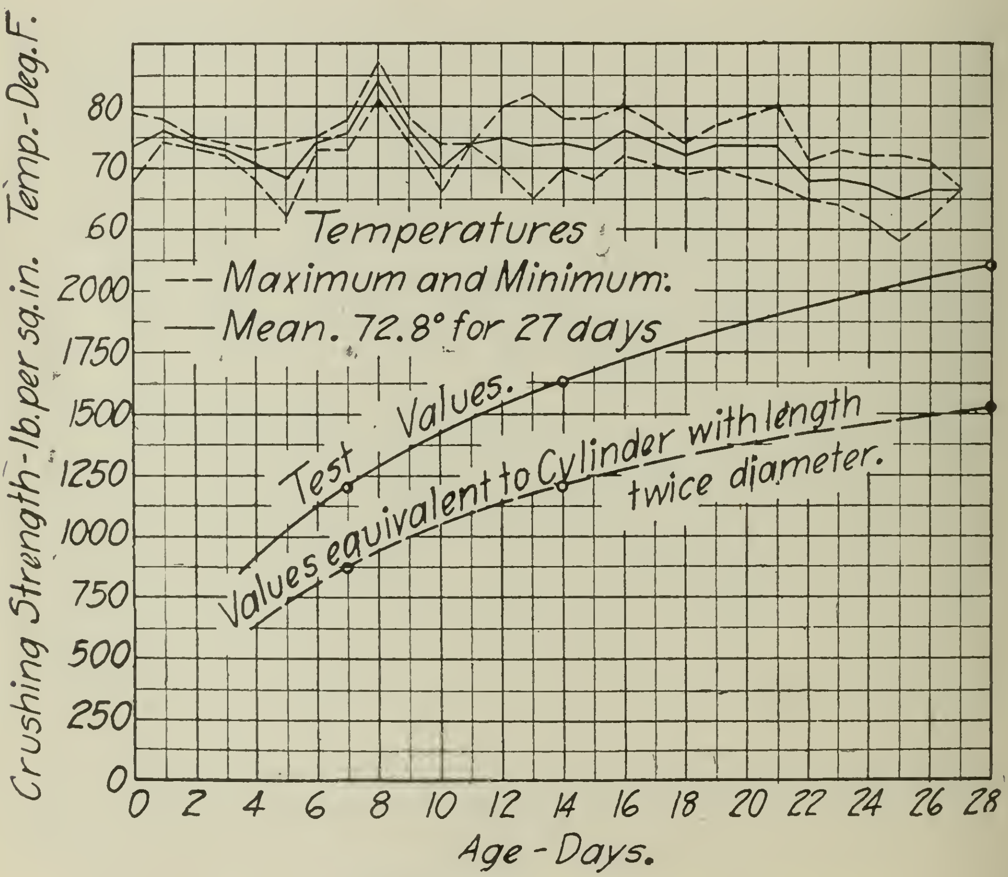

FIG. 3. SET C, GROUP I—1913 SERIES—6 x 6-IN. CYLINDERS.

## TABLE 6.

## COMPRESSIVE STRENGTH—AGE 4 DAYS

Group II. 6-in. Cubes

| Set | Weight lb. | Size, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Standardized Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|---|
| D | 18.50 | 6x6x6 | 28 400 | 790 | 780 | 570 | |
| | 18.50 | " | 22 450 | 620 | | | |
| | 18.50 | " | 33 340 | 920 | | | |
| E | 19.00 | " | 16 690 | 460 | 450 | 330 | |
| | 19.00 | " | 10 000 | 280 | | | |
| | 18.75 | " | 21 300 | 590 | | | |
| F | 18.75 | " | 15 680 | 440 | 390 | 280 | Slight coating of frost, but all had uniform break. |
| | 18.75 | " | 13 050 | 360 | | | |
| | 18.75 | " | 13 000 | 360 | | | |

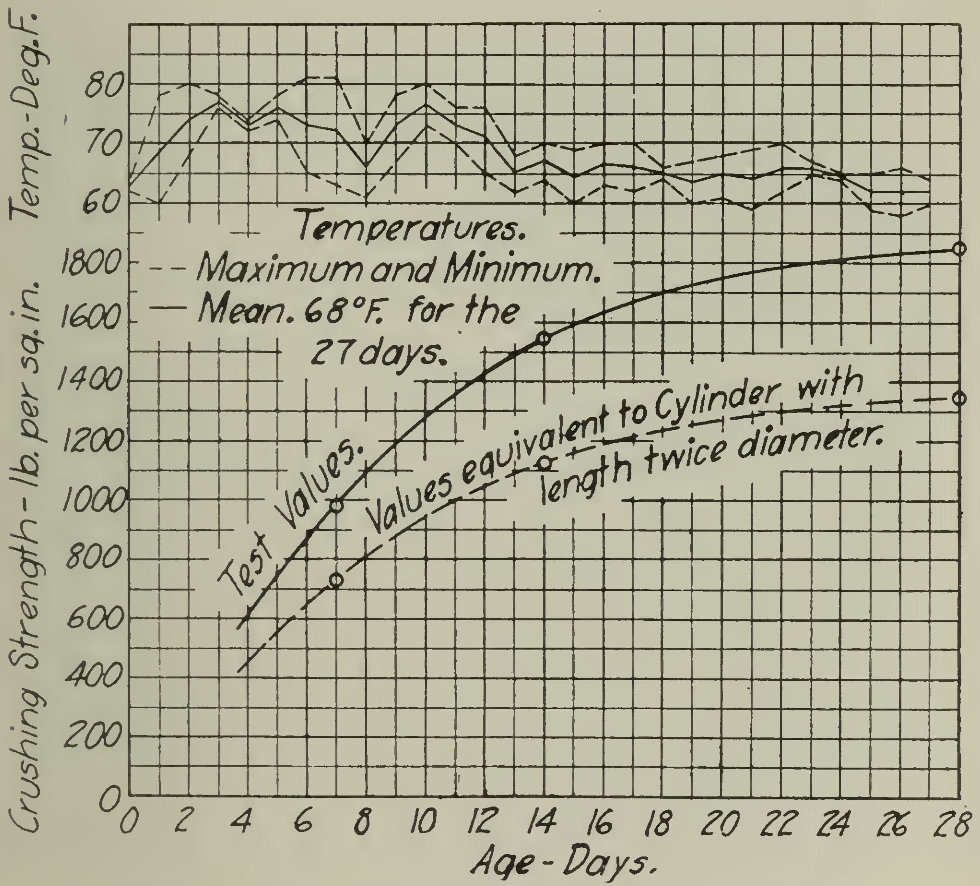

FIG. 4. SET D, GROUP II—1913 SERIES—6-IN. CUBES

TABLE 7.

COMPRESSIVE STRENGTH—AGE 7 DAYS

Group II. 6-in. Cubes

| Set | Weight, lb. | Size, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Standardized Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|---|
| D | 18.75<br>19.00<br>18.75 | 6x6x6<br>"<br>" | 39 390<br>35 930<br>31 300 | 1090<br>1000<br>860 | 980 | 720 | |
| E | | "<br>"<br>" | 17 100<br>19 820<br>14 060 | 470<br>550<br>390 | 470 | 340 | Broke uniformly |
| F | 18.50<br>18.75<br>18.75 | "<br>"<br>" | 20 880<br>19 230<br>20 760 | 580<br>530<br>580 | 560 | 410 | |

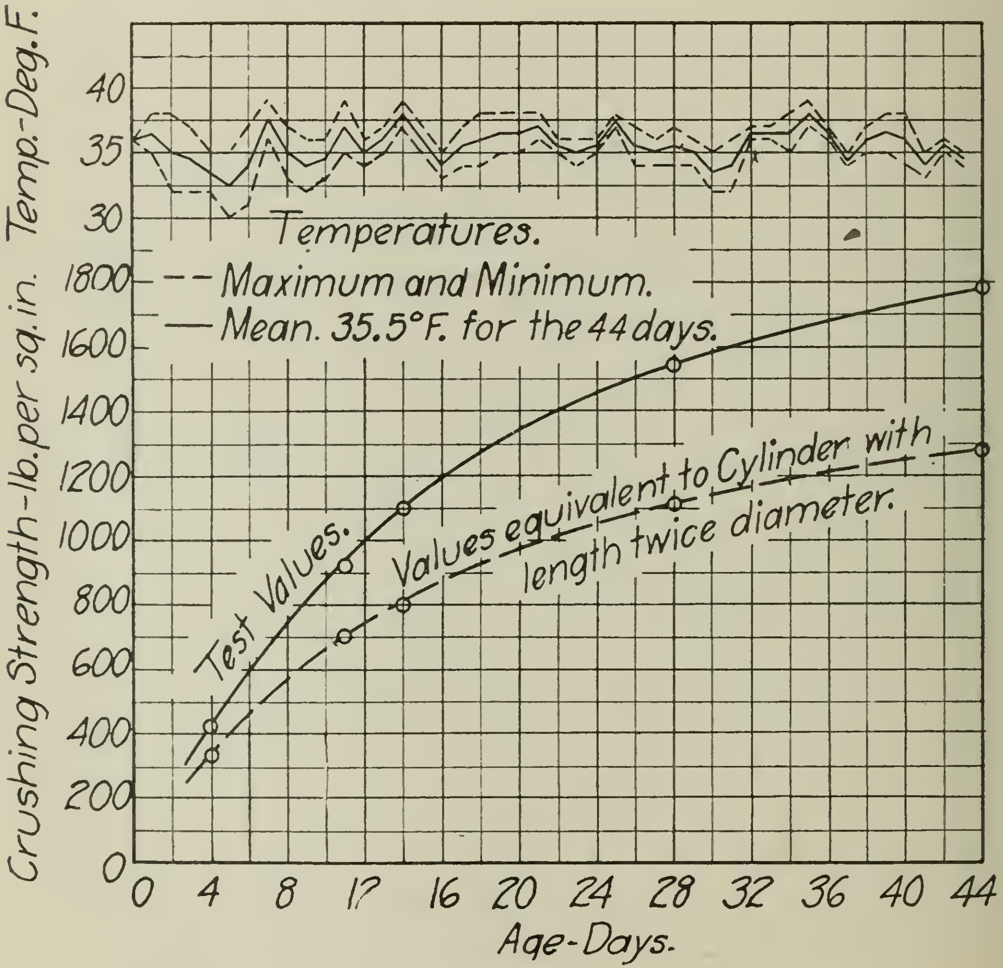

FIG. 5. SET E, GROUP II—1913 SERIES—6-IN. CUBES.

### TABLE 8.

### COMPRESSIVE STRENGTH—AGE 11 DAYS

Group II. 6-in. Cubes

| Set | Weight lb. | Size, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Standardized Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|---|
| D | 18.75 | 6x6x6 | 43 420 | 1200 | 1320 | 970 | |
| | 18.75 | " | 52 440 | 1460 | | | |
| | 18.75 | " | 47 300 | 1300 | | | |
| E | 19.00 | " | 42 310 | 1180 | 920 | 670 | Visible voids |
| | 19.00 | " | 31 060 | 860 | | | Broke at one corner |
| | 18.75 | " | 26 000 | 720 | | | |
| F | 18.75 | " | 15 080 | 420 | 500 | 370 | |
| | 18.75 | " | 17 760 | 490 | | | |
| | 18.75 | " | 21 820 | 610 | | | |

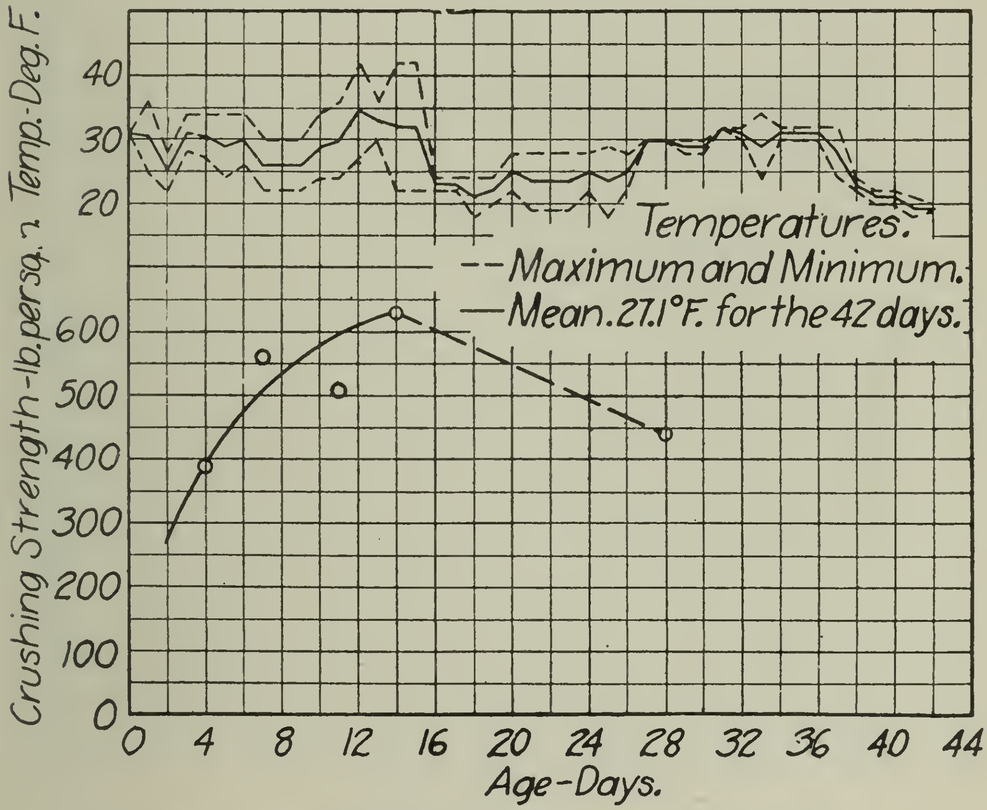

FIG. 6. SET F, GROUP II—1913 SERIES—6-IN. CUBES.

### TABLE 9.

### COMPRESSIVE STRENGTH—AGE 14 DAYS

Group II. 6-in. Cubes

| Set | Weight, lb. | Size, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Standardized Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|---|
| D | 19.00 | 6x6x6 | 58 710 | 1630 | 1540 | 1130 | |
| | 19.00 | ” | 62 810 | 1740 | | | |
| | 18.75 | ” | 45 530 | 1260 | | | |
| E | | ” | 38 630 | 1070 | 1100 | 800 | |
| | | ” | 40 300 | 1120 | | | |
| | | ” | 40 280 | 1120 | | | |
| F | 18.75 | ” | 19 330 | 540 | 640 | 470 | |
| | 18.75 | ” | 26 350 | 730 | | | |
| | 18.75 | ” | 22 900 | 650 | | | |

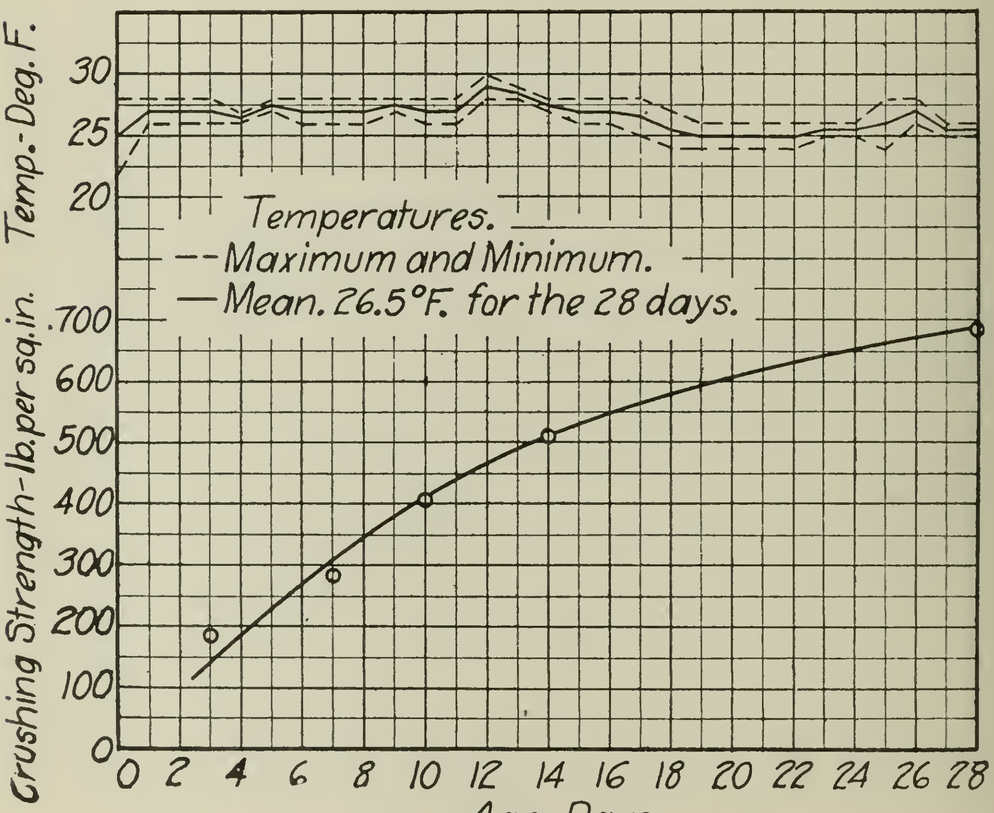

FIG. 7. SET G, GROUP III—1914 SERIES—8 x 16-IN. CYLINDERS.

### TABLE 10.

### COMPRESSIVE STRENGTH—AGE 28 DAYS

Group II. 6-in. Cubes

| Set | Weight, lb. | Size, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Standardized Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|---|
| D | 19.00<br>19.00<br>19.00 | 6x6x6<br>"<br>" | 64 000<br>67 460<br>68 440 | 1780<br>1870<br>1900 | 1850 | 1350 | |
| E | 18.75<br>18.75<br>18.75 | "<br>"<br>" | 52 590<br>58 270<br>56 350 | 1460<br>1620<br>1560 | 1550 | 1130 | |
| F | 19.00<br>19.00<br>18.75 | "<br>"<br>" | 16 540<br>15 160<br>10 400 | 460<br>420<br>*290 | 440 | 320 | |

*Specimen in bad condition; not included in average.

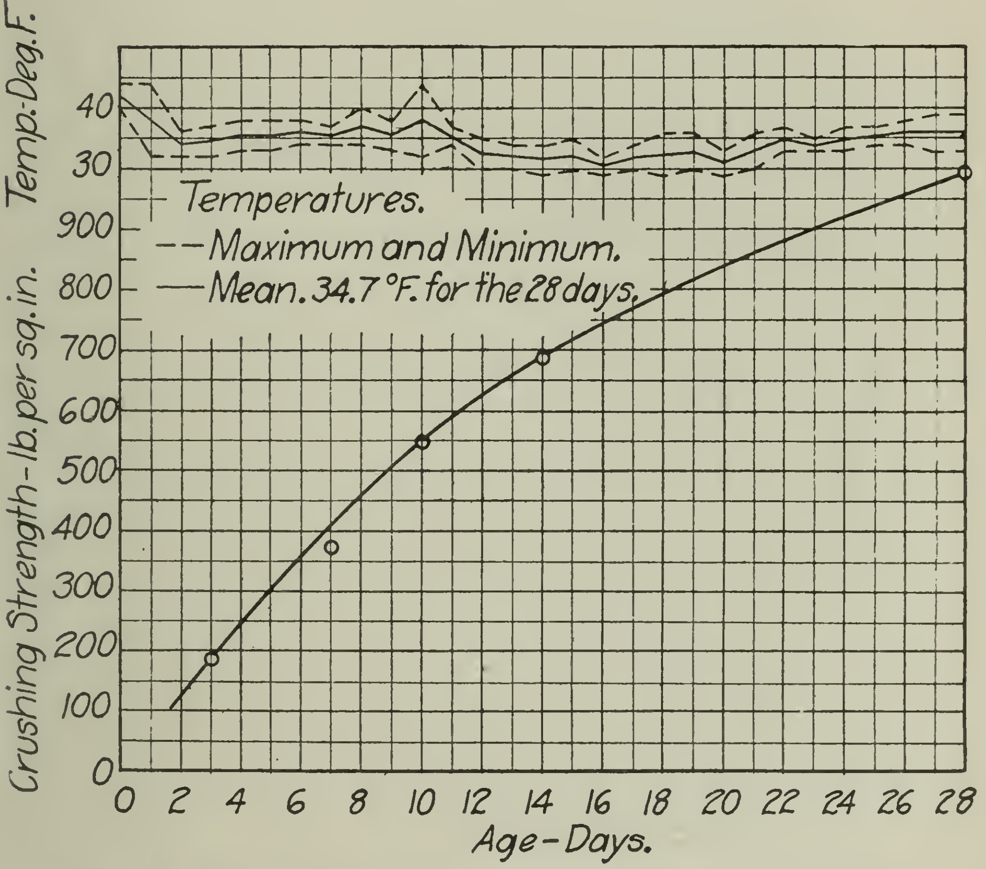

FIG. 8. SET H, GROUP III—1914 SERIES—8 x 16-IN. CYLINDERS.

## TABLE 11.

## COMPRESSIVE STRENGTH—AGE 42 DAYS

Group II. 6-in. Cubes.

| Set | Weight, lb. | Size, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Standardized Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|---|
| E | 18.75 | 6x6x6 | 66 710 | 1850 | 1780 | 1300 | Broke uniformly |
|  | 18.75 | ” | 62 880 | 1740 | 1780 | 1300 | One corner broke |
|  | 18.75 | ” | 62 240 | 1740 |  |  | Slightly skewed |
| F | 18.00 | ” | 15 240 | 420 |  |  | Specimens in a soft and crumbling condition |
|  |  | 5x5x6 | 31 720 | 1270* |  |  |  |
|  |  | 4x5x6 | 8 040 | 400† |  |  |  |

*Age when tested, 49 days.
†Age when tested, 63 days.

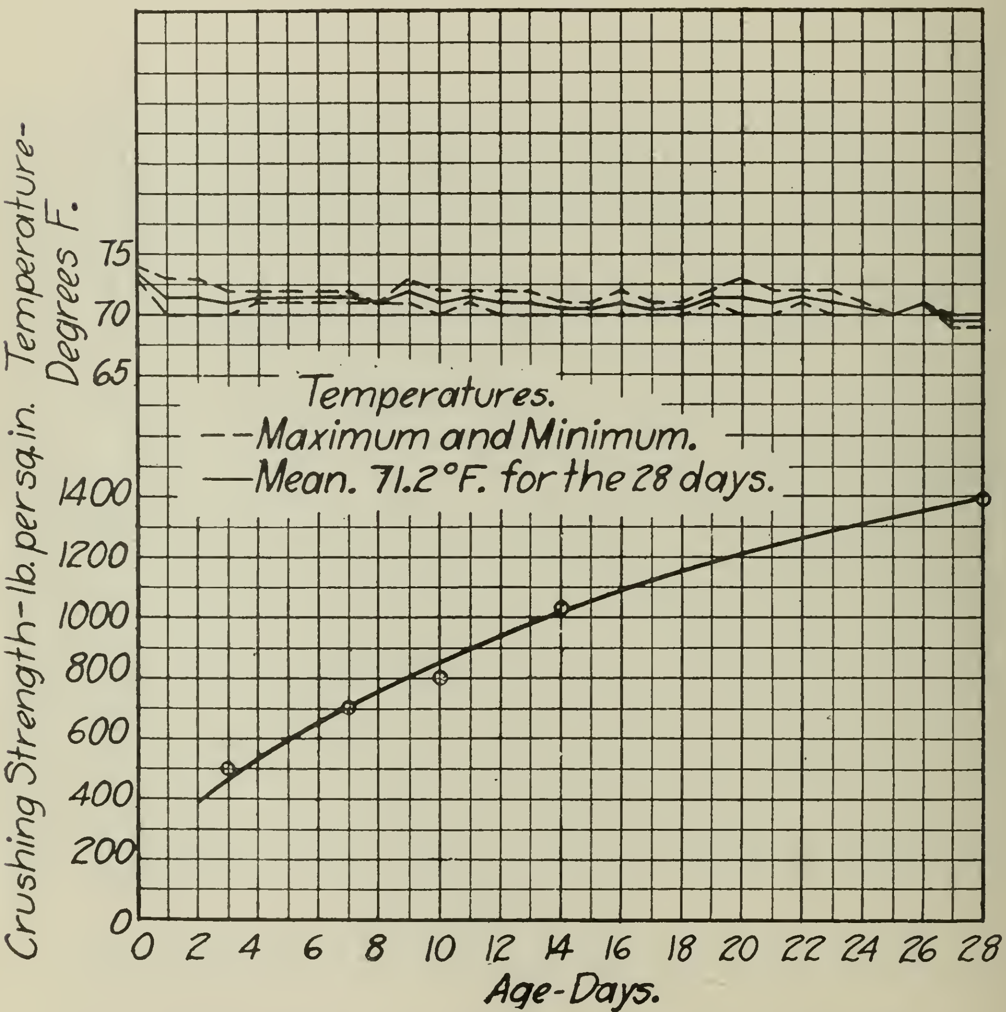

FIG. 9. SET I, GROUP III—1914 SERIES—8 x 16-IN. CYLINDERS.

### TABLE 12.

### COMPRESSIVE STRENGTH—AGE 3 DAYS

Group III—1914 Series—8 x 16-in. Cylinders

| Set | Weight lb. | Average Diameter, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|
| G | 67.75 | 7.94 | 8 880 | 180 | 190 | Crumbled |
| | 67.25 | 7.87 | 9 720 | 200 | | |
| | 70.25 | 8.06 | 9 340 | 180 | | Crumbled badly |
| H | 65.0 | 7.94 | 9 950 | 200 | 180 | |
| | 66.0 | 7.87 | 8 250 | 170 | | |
| | 65.0 | 8.0 | 9 250 | 180 | | Plaster loose on one end |
| I | 69.0 | 8.0 | 29 650 | 600 | 500 | |
| | 70.25 | 8.06 | 23 750 | 460 | | |
| | 69.75 | 8.06 | 22 600 | 440 | | |
| M | 64.0 | 8.0 | 24 000 | 480 | 500 | |
| | 65.0 | 7.94 | 30 850 | 620. | | |
| | 65.0 | 7.94 | 20 000 | 400. | | |

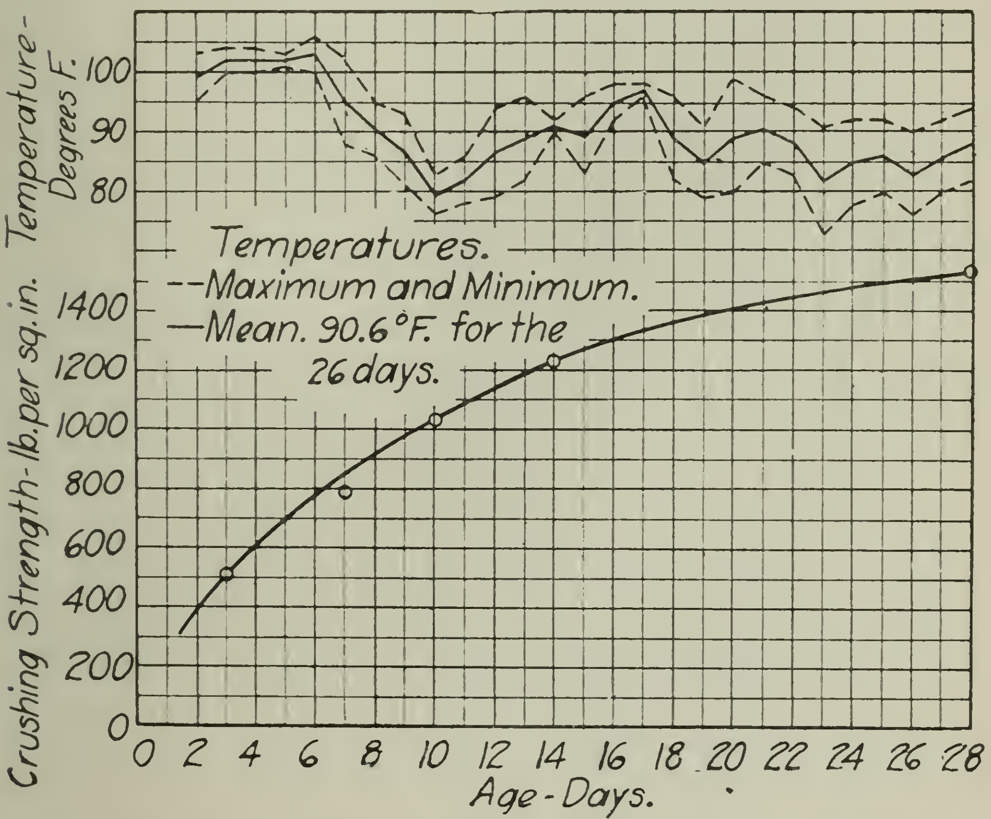

FIG. 10. SET M, GROUP III—1914 SERIES—8 x 16-IN. CYLINDERS.

## TABLE 13.

## COMPRESSIVE STRENGTH—AGE 7 DAYS

Group III—1914 Series—8 x 16-in. Cylinders

| Set | Weight, lb. | Average Diameter, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|
| G | 67.75 | 8.0 | 17 200 | 340 | 280 | |
| | 67.25 | 8.0 | 14 750 | 290 | | |
| | 65.50 | 8.0 | 10 700 | 210 | | |
| H | 66.0 | 7.87 | 14 450 | 300* | 370 | Skewed one inch, horizontal crack |
| | 69.5 | 8.06 | 18 920 | 370 | | |
| | 65.5 | 7.94 | 18 530 | 370 | | |
| I | 67.75 | 8.0 | 40 800 | 810 | 700 | |
| | 67.25 | 7.94 | 31 630 | 640 | | |
| | | 8.12 | 34 340 | 660 | | |
| M | 68.0 | 8.0 | 44 500 | 890 | 790 | |
| | 69.0 | 8.06 | 40 250 | 790 | | |
| | 69.0 | 8.06 | 35 150 | 690 | | |

*Not used in calculating average strength.

## TABLE 14.

## COMPRESSIVE STRENGTH—AGE 10 DAYS

Group III—1914 Series—8 x 16-in. Cylinders

| Set | Weight, lb. | Average Diameter, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|
| G | 66.5 | 8.0 | 18 670 | 370 | 400 | |
| | 67.5 | 7.87 | 18 280 | 370 | | |
| | 70.5 | 8.0 | 23 630 | 470 | | |
| H | 68.0 | 8.0 | 25 000 | 500 | 540 | 2—8x8 forms |
| | 68.5 | 8.12 | 30 680 | 600 | | |
| | 69.0 | 8.06 | 26 830 | 530 | | |
| I | 67.5 | 8.06 | 44 700 | 880 | 800 | Top crumbled |
| | 68.0 | 8.0 | 36 050 | 720 | | Skewed |
| | | | | | | Fractured in transit |
| M | 68.0 | 7.94 | 59 620 | 1200 | 1030 | |
| | 69.0 | 8.06 | 46 700 | 920 | | |
| | 69.5 | 8.06 | 49 540 | 970 | | 2—8x8 forms |

## TABLE 15.

## COMPRESSIVE STRENGTH—AGE 14 DAYS

Group III—1914 Series—8 x 16-in. Cylinders

| Set | Weight, lb. | Average Diameter, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|
| G | 70.0 | 8.0 | 26 430 | 520 | 510 | |
| | 68.5 | 7.94 | 25 330 | 510 | | |
| | 69.5 | 8.06 | 25 420 | 500 | | |
| H | 67.5 | 7.94 | 30 550 | 620 | 690 | |
| | 64.5 | 8.06 | 35 100 | 690 | | |
| | 66.5 | 8.0 | 37 750 | 750 | | |
| I | 66.0 | 7.94 | 51 000 | 1030 | 1040 | Skewed slightly |
| | 69.5 | 8.0 | 63 230 | 1260 | | Bearing faces not parallel |
| | 67.0 | 8.0 | 41 430 | 820 | | |
| M | 67.5 | 8.0 | 58 000 | 1150 | 1220 | |
| | 66.5 | 7.94 | 40 650 | 820* | | Visible voids |
| | 66.5 | 8.0 | 65 000 | 1290 | | |

*Not used in calculating average strength.

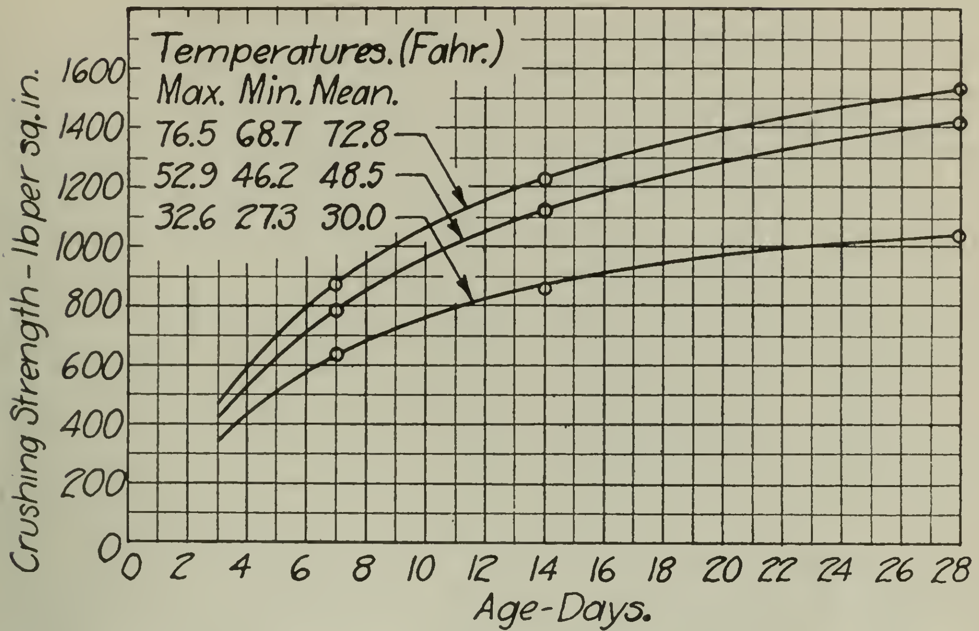

FIG. 11. GROUP I—1913 SERIES—6 x 6-IN. CYLINDERS. STANDARDIZED VALUES.

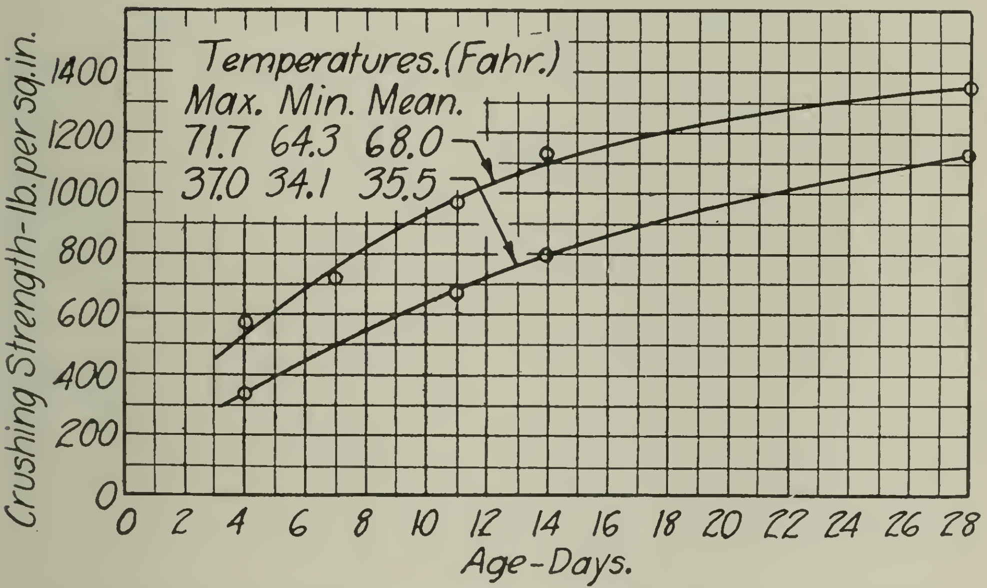

FIG. 12. GROUP II—1913 SERIES—6-IN. CUBES. STANDARDIZED VALUES.

## TABLE 16.

## COMPRESSIVE STRENGTH—AGE 28 DAYS

Group III—1914 Series—8 x 16-in. Cylinders

| Set | Weight, lb. | Average Diameter, in. | Crushing Strength, lb. | Strength, lb. per sq. in. | Average Strength, lb. per sq. in. | Remarks |
|---|---|---|---|---|---|---|
| G | 68.0<br>67.5<br>65.0 | 8.0<br>7.94<br>7.94 | 32 100<br>35 900<br>34 100 | 630<br>730<br>690 | 680 | |
| H | 65.0<br>65.0<br>64.0 | 8.06<br>8.0<br>7.87 | 45 900<br>51 600<br>51 400 | 900<br>1030<br>1050 | 990 | |
| I | 68.5<br>68.5<br>66.0 | 8.0<br>8.0<br>7.87 | 83 950<br>60 000<br>63 900 | 1670<br>1190<br>1290 | 1380 | Odd fracture |
| M | 66.0<br>66.0<br>66.0 | 8.0<br>8.12<br>8.0 | 63 500<br>101 900<br>68 200 | 1260<br>1960<br>1360 | 1530 | |

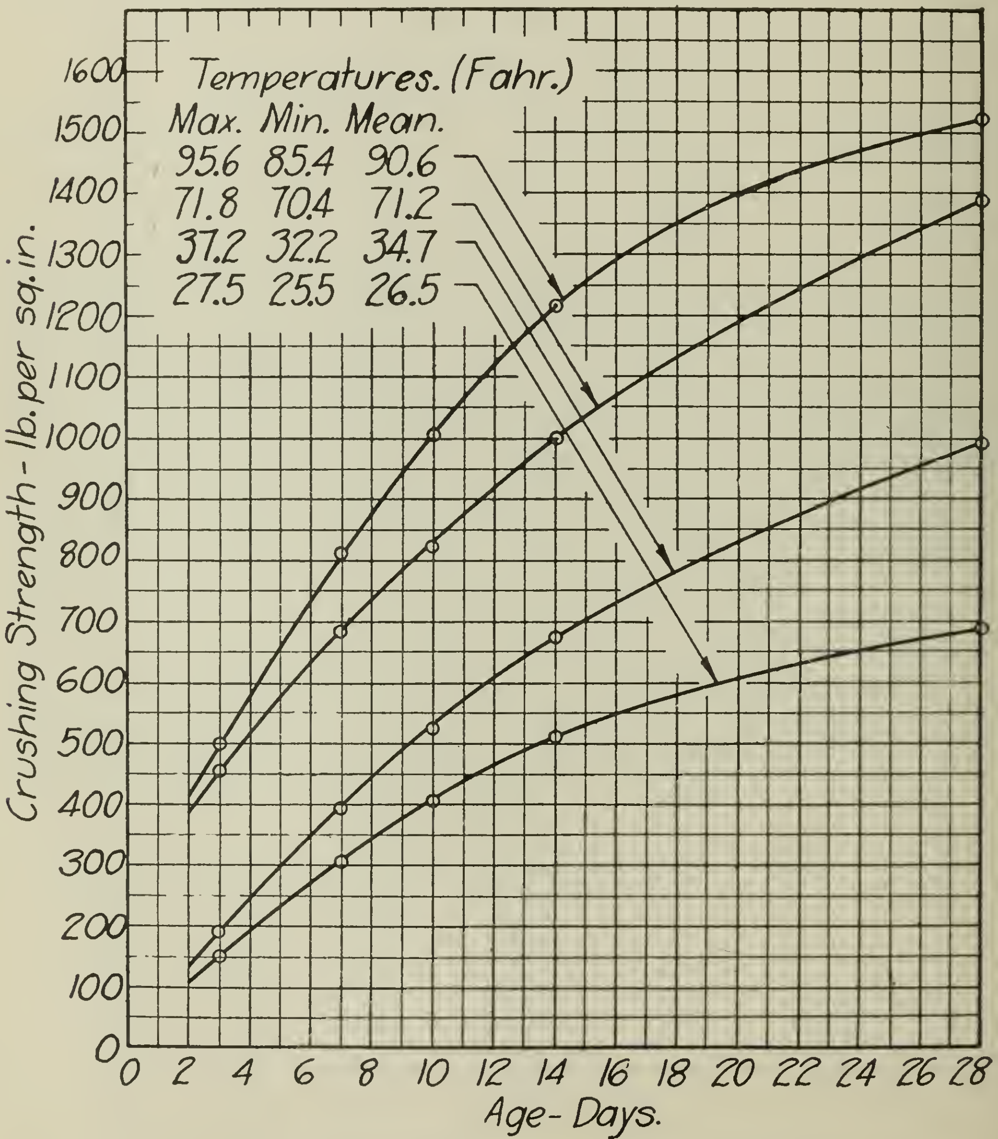

FIG. 13. GROUP III—1914 SERIES—8 x 16-IN. CYLINDERS.

10. *Group I.*—The results of the tests of Group I, 6 by 6-in. cylinders, are given in Table 3, 4, and 5; and the relation between strength and age is shown in Fig. 1, 2, and 3, pages 8, 9, and 10. The curves are drawn through the average values for each group of five specimens for 7, 14, and 28 days. At the top of each figure is shown the temperature conditions for that set; the maximum, the minimum, and the mean temperatures.

11. *Group II.*—The results of the tests of Group II, 6-in. cubes, are given in Tables 6–11; and the relation between strength and age is shown in Fig. 4, 5, and 6, pages 11, 12, and 13. The strength and temperature curves are drawn as stated for Group I.

The Sets D and E, Fig. 4 and 5, pages 11 and 12, were stored under substantially uniform temperature conditions, and give results of practically the same character as those of Group I.

The specimens of Set F were stored in a room where it was known the temperature would not be uniform. All of the specimens tested at 11 days were slightly disintegrated on the surface, and those tested at 28 days were badly disintegrated; while of those reserved to be tested at 42 days only one could be tested at that date, the remaining specimens, $F_{17}$ and $F_{18}$, being very badly disintegrated. Specimen $F_{17}$ was tested at 49 days, and $F_{18}$ at 63 days. Since there was only one specimen at each of these ages, and since none of the other groups contained specimens at corresponding ages, the results of these two tests are not plotted in Fig. 6, and are not further considered.

The results of Set F, indicate that the low temperature retarded the hardening action of the concrete, and that the alternations above and below freezing caused a softening and crumbling of the material.

12. *Group III.*—The results of the tests of Group III, 8 by 16-in. cylinders, are given in Tables 12–16; and the relation between strength and age is shown graphically in Fig. 7–10. It is noteworthy that under a temperature slightly below freezing the concrete gained strength continuously, see Fig. 7, page 14. It is also interesting to note that the curve for a mean temperature of 26.5°F. is substantially of the same character as that for a mean temperature of 71.2°,—compare Fig. 7 and Fig. 9.

13. *Summary.*—The results for the three sets of Group I are presented in Fig. 11, page 19; and the corresponding values for Groups II and III are given in Fig. 12 and 13. Fig. 11–13 show the relation between strength and age for the several mean temperatures.

In Group I the test specimens were cylinders 6 inches in diameter and 6 inches high, and in Group II the specimens were 6-inch cubes;

and owing to the effect of the restraint of the pressing surfaces of the testing machine, the results of these tests are not further considered.

In Group III the test specimens were cylinders 8 inches in diameter and 16 inches high, and the interpolated results for these tests are presented in Fig. 14 to show the relation between strength and temperature for the several ages. Fig. 14 may be employed to determine (1) the strength which the concrete attained at different ages under a constant temperature, (2) the age at which a particular strength was gained under the different temperatures, and (3) the strength which may be expected at different ages under different temperatures. The relative strength attained by concrete at different temperatures during hardening and at different ages may be expected to vary somewhat with differences in cements, aggregates, and consistencies; but

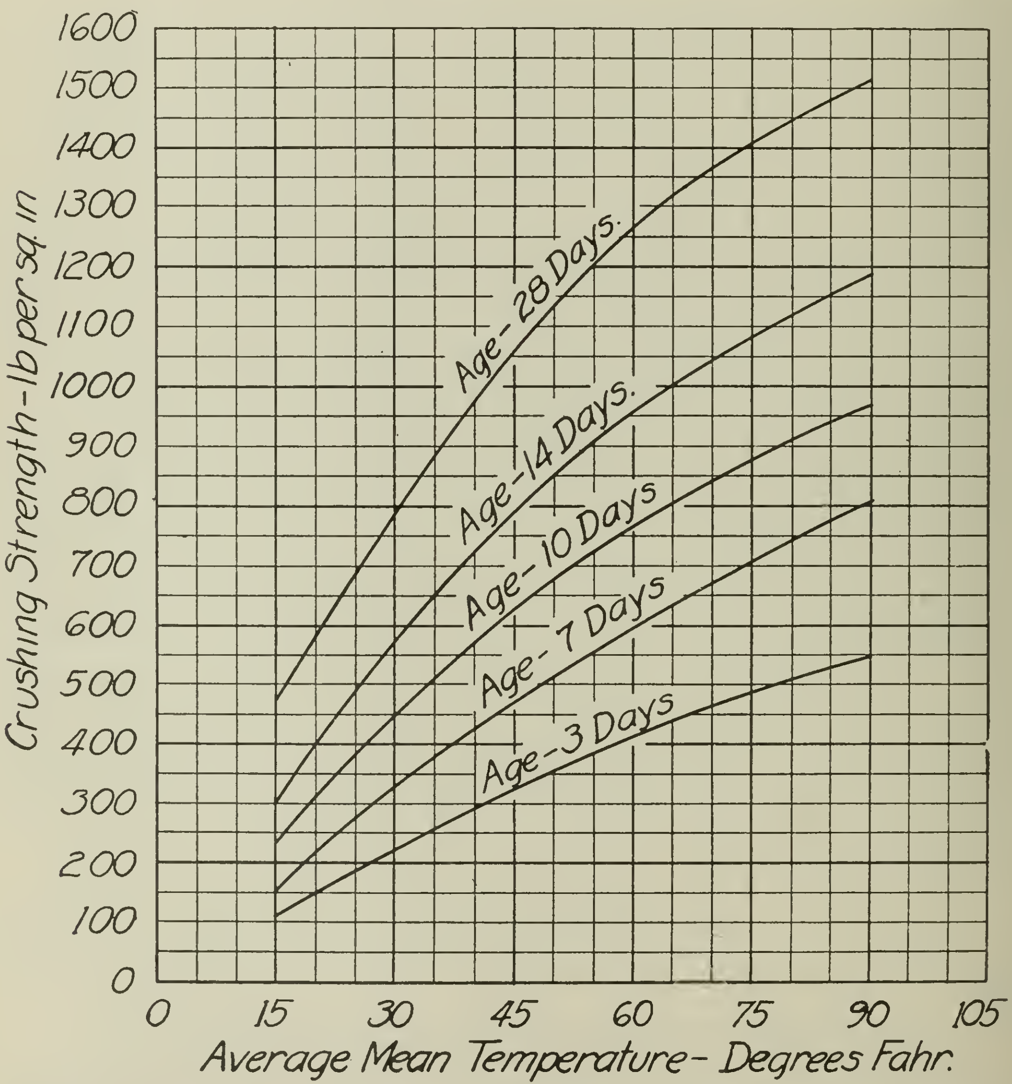

FIG. 14. RELATION OF STRENGTH TO TEMPERATURE FOR DIFFERENT AGES.

it is thought that the values in Fig. 14 may be taken to represent the effect of the variation in the temperatures during hardening upon the strength.

Fig. 15 has been drawn by taking values from the curves in Fig. 14. It shows in a general way, the relation between the strength at 28 days under 70°F. and the strength attained at various ages under varying temperatures. Fig. 15 can be used in substantially the same way as Fig. 14.

The tests summarized in Fig. 14 and 15 cover a wide range of temperature conditions, the average temperature varying from 20.4°F. to 90.6°F., and are fairly consistent; and hence it is believed these values are sufficiently accurate to furnish suggestive information which may be useful in determining the time when forms may be removed and loads applied.

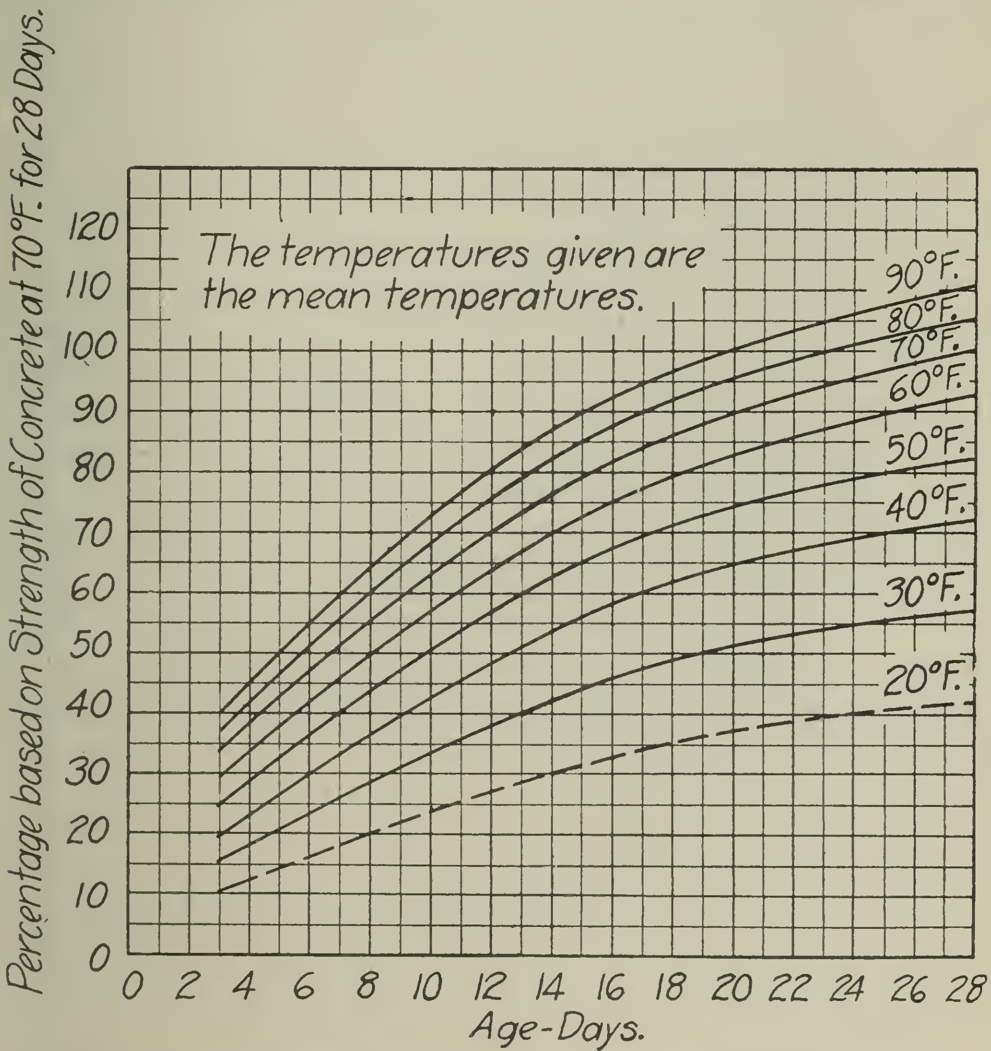

Fig. 15. Percentages of Strength for Different Temperatures.

## IV. Conclusions

14. *General Conclusions.*—It is believed the following general conclusions are justifiable.

1. Under uniform temperature conditions, there was an increase of strength with age within the limits of the tests. For any temperature the rate of increase decreases with the age of the specimen; and this rate of increase is less correspondingly at the lower temperature conditions. For the specimens tested, under normal hardening temperature conditions of from 60 to 70°F., the compressive strength of the concrete subjected to a uniform temperature at the ages of 7, 14, and 21 days may be taken as approximately 50 per cent, 75 per cent, and 90 per cent of the strength at twenty-eight days, respectively. For lower temperatures the percentage values are less; and for higher temperatures the percentages are higher. The relation between the percentage values at the ages of 7, 14, 21, and 28 days is nearly the same for temperature conditions from 30° to 70° F. However, the values for the lower temperatures should be used with caution.

2. Concrete which is maintained at a temperature of 60° to 70° F. will at the age of one week have practically double the strength of the same material which is kept at a temperature of 32° to 40° F.

3. Fig. 14 and 15 may be used to determine the representative strength of concrete similar to that used in these tests, for various temperature conditions and for ages up to 28 days. These diagrams may be used with a fair degree of approximation to ascertain the relative strengths which concrete of ordinary practice may be expected to attain at the different temperatures. It should be noted that generally in this investigation the specimens were stored under temperatures which were nearly uniform during the whole storage period. In set F the variations in temperature include a number of alternations above and below the freezing point and the specimens were seriously injured. The results accord with the well-known effect of freezing and thawing upon green concrete.

## PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

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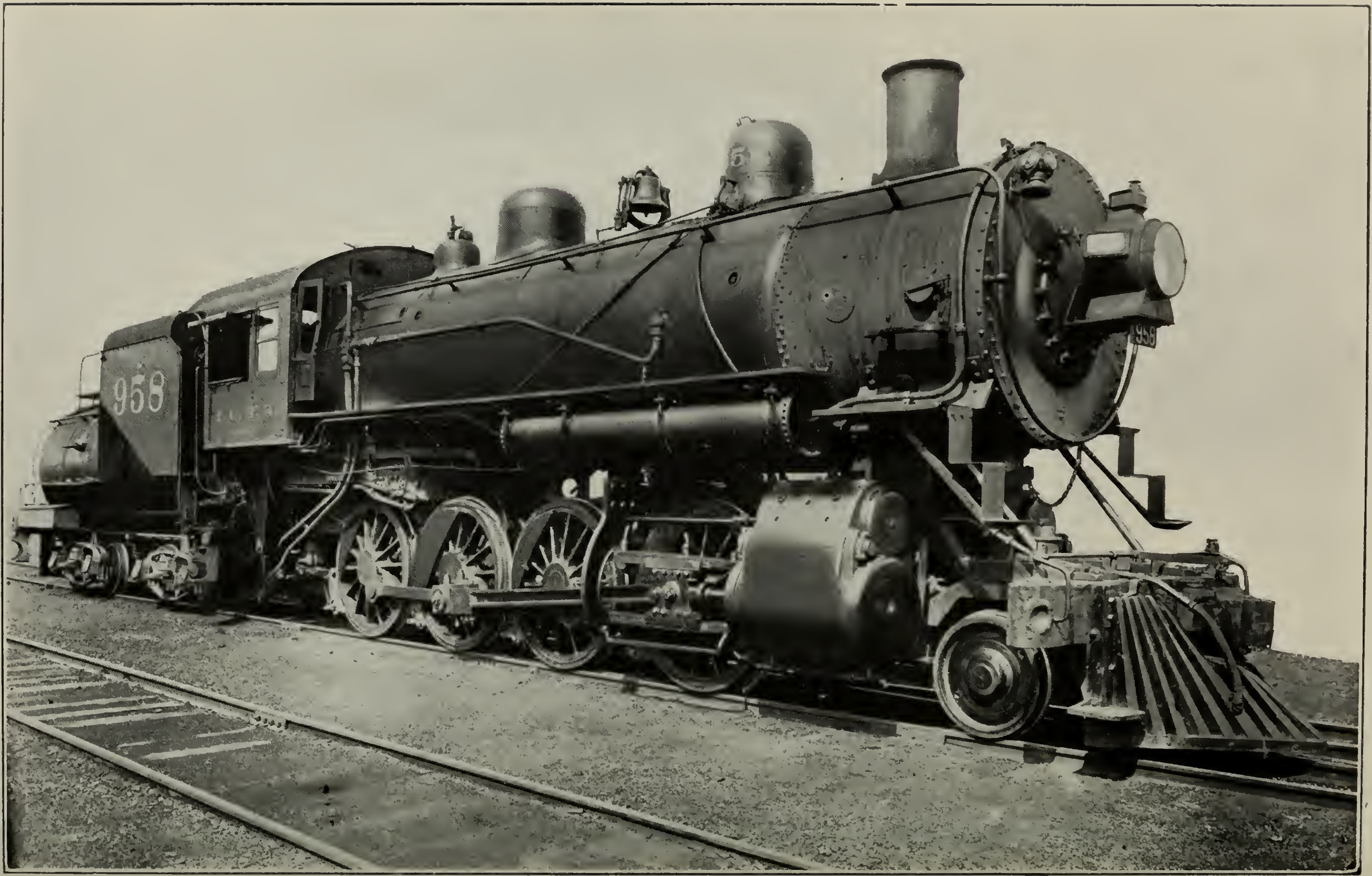

Fig 1. Illinois Central Railroad Locomotive 958.

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## UNIVERSITY OF ILLINOIS
## ENGINEERING EXPERIMENT STATION

BULLETIN NO. 82 SEPTEMBER, 1915

# LABORATORY TESTS OF A CONSOLIDATION LOCOMOTIVE

BY

EDWARD C. SCHMIDT,[1] JOHN M. SNODGRASS[2] AND ROBERT B. KELLER.[3]

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# LABORATORY TESTS OF A CONSOLIDATION LOCOMOTIVE.

## PART I.

### I. INTRODUCTION.

1. The tests the results of which are here recorded constitute the first work of the recently established locomotive laboratory of the University of Illinois. They relate to a typical consolidation locomotive which was loaned to the University by the Illinois Central Rrailroad.

In Part I of this report the aim has been to present as brief a statement of the conditions and results as is compatible with a clear understanding of the tests. Part II, on the other hand, consists of appendixes in which supplementary detail is fully recorded. In the presentation and discussion of the results in Part I, only the more important facts concerning boiler and engine performance have been included. There remain in the complete record of results given in Appendix 4 many facts which may be of use to those interested in the details of boiler and engine tests. In this, the first publication relating to the work of the laboratory, laboratory equipment and methods are described in detail in order to complete the record and to provide a basis for reference in future publications from which such detail will be omitted.

2. *Purpose of the Tests.*—The locomotive was first tested in the condition in which it was received from service. It was then subjected to certain repairs some of which affected its performance, and was again fully tested. The main purpose of the tests was to determine the general performance of the locomotive and the performance of its boiler and engines after the repairs were made and when the locomotive was in excellent condition.

3. *Acknowledgments.*—The locomotive was loaned for the tests through the interest and courtesy of Mr. W. L. Park, Vice President, and Mr. Morgan K. Barnum, General Superintendent of Motive Power, of the Illinois Central Railroad. During the progress of the tests Mr. R. W. Bell, then General Superintendent of Motive Power, and various members of his staff frequently gave assistance and advice to those in charge of the laboratory. It is a pleasure to record here our appreciation of these services.

Mr. Franklin W. Marquis, formerly Associate in the Department of Railway Engineering, was in immediate charge of the laboratory from its establishment until the completion of the first ten tests included in this report. He also had a large share in working out the details of the laboratory design. To him is due the successful solution of many of the problems which arose in putting the equipment in operation and in establishing the test procedure.

We would acknowledge also the assistance received from the numerous members of the laboratory staff and especially from Mr. H. H. Dunn.

## II. The Locomotive.

The locomotive tested is of the consolidation (2-8-0) type, built by the Baldwin Locomotive Works in 1909. It weighs 223 000 pounds, and has 22 in. x 30 in. simple cylinders using saturated steam. Its principal dimensions are given below, and a detailed description appears in Appendix 1.

| | |
|---|---|
| Total weight, in working order, lb. | 223 000 |
| Weight on drivers, lb. | 200 900 |
| Cylinders (simple), diameter and stroke, in. | 22 x 30 |
| Diameter of drivers, in. | 63 |
| Fire-box width, in. | 66 |
| Grate area, sq. ft. | 49.55 |
| Heating surface, tubes (fire side), sq. ft. | 3094 |
| Heating surface, total, sq. ft. | 3283 |
| Boiler pressure, lb. per sq. in. | 200 |

When it was received at the laboratory, the locomotive had been in service three and one-third years and had run 107 800 miles. Immediately preceding the tests the locomotive had been in service only five weeks after receiving general repairs, and was in good condition when it arrived at the laboratory. It was completely tested in this condition and the results of these tests are designated as Series I. The results of this series disclosed a performance not quite so good as had been anticipated and, in the endeavor to do whatever was possible to improve the performance, valves were reset and eccentric straps shimmed; cylinders and valve chambers were re-bored; new pistons and piston rings, new valve bull-rings and packing rings were applied; rod packing renewed; the exhaust nozzle-tip changed from 5¼ in. to 5⅞ in.; and a small leak in one of the steam pipe joints was stopped. Certain incidental repairs having no effect on performance were made at the same time. Following this work the locomotive was run the equivalent of about 1200 miles in wearing down the cylinders and packing before making the tests of Series 2. It should be emphasized that all of these repairs were resorted to only that nothing which would probably improve the performance be left undone, and that under ordinary service requirements they would have been regarded as quite unnecessary. After their completion the locomotive, then in excellent condition, was subjected to the tests which are designated as Series 2.

Locomotive 958 is a characteristic freight locomotive of whose type there are about twenty thousand on American railways, or one third of the total in service. Its weight and heating surface exceed the average values of these quantities for all consolidation locomotives by about twenty-five per cent. It is in most respects thoroughly representative of its type. Complete laboratory tests of simple consolidation locomotives are not common and include tests of only three different classes, all of which are somewhat smaller than the one here under consideration.*

## III. SUMMARY OF THE RESULTS.

While it is not possible to summarize all the results of the tests further than is done in the curves included beyond, it is feasible briefly to state at this point the main facts defining the range through which the locomotive was worked and to indicate the minimum or maximum values of a few of the more important quantities. The statements apply to the tests of Series 1 and 2 combined.

4. *The Boiler.*—The maximum amount of dry coal fired per hour during any of the tests was 11 127 lb. or 224.5 lb. per square foot of grate per hour, an amount much in excess of what is usual or desirable on hand-fired locomotives in service. The maximum quantity of cinders ejected into the front end and from the stack amounted to 27.4 per cent of the dry coal fired. This cinder loss also is quite unusual and it occurred under conditions which rarely prevail in service, the draft during this test being equivalent to 12.8 inches of water in front of the diaphragm.

During the test in which the heating surface was forced to its greatest activity, the total equivalent evaporation per hour was 57 954 lb., or 17.65 lb. per square foot of heating surface per hour. This rate of evaporation is altogether unusual in service and has been exceeded only rarely under test conditions. The best economic performance of the boiler was obtained in test No. 2024 during which the equivalent evaporation per pound of dry coal was 10.07 lb. There is some doubt however about the validity of this result which exceeds the next highest evaporation per pound of coal (8.96 lb.) by 12.4 per cent.

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*"Locomotive Tests and Exhibits" and bulletins No. 7, 8, 9, 12, 13, 15, and 16 published by the Pennsylvania Railroad Company.

These results were all obtained when using run-of-mine coal from Mission Field Mine, Vermilion County, Illinois, which varied in heating value from 11 835 B.t.u. to 12 848 B.t.u. per pound of dry coal.

5. *The Engines and the Locomotive.*—The maximum indicated horse power developed during the tests was 1654 which occurred in test No. 2093 with a cut-off of 48.6 per cent and a speed of 30.4 miles per hour. This is the greatest power which has been developed during laboratory tests with a locomotive of this type. The maximum drawbar horse power was 1431. The maximum tractive effort developed, 29 240 lb., is only 75 per cent of the rated maximum and is not significant because of the fact that, as in all laboratory tests, it was not feasible to work the locomotive at the lowest speeds and the greatest cut-offs.

The lowest water rate attained was 27.17 lb. of dry steam per indicated horse power per hour. This steam consumption is not so low as has been previously obtained in tests of locomotives of this type under similar conditions, being almost 17 per cent in excess of the lowest figure previously recorded. The minimum heat content of the dry coal fired per indicated horse power per hour was 50 872 B.t.u. and the minimum dry coal fired per hour per indicated horse power was 4.00 lb. The minimum dry coal fired per hour per drawbar horse power was 4.62 lb.

## IV. The Tests and the Test Program.

The locomotive was worked during the tests throughout a range of speed corresponding to that which would ordinarily prevail in service. At each of the various speeds the endeavor was made to vary the cut-off throughout as wide a range as the capacity of the boiler or of the grate would permit. The adhesion between the drivers and the supporting wheels in the laboratory is less however than the adhesion between the drivers and the rail on the road, and consequently it was impossible at low speeds to run at maximum cut-offs. The designations for speed and cut-off used in this section are approximate only, and represent the conditions predetermined for each test. The actual average values attained during the tests appear in Appendix 4. All tests were run with the throttle wide open.

### TABLE 1.

### TEST PROGRAM—SERIES 2.

SHOWING TESTS RUN AT VARIOUS SPEEDS AND CUT-OFFS.

| Approximate Speed | | Approximate Cut-off—Per cent of Stroke | | | | | |
|---|---|---|---|---|---|---|---|
| Rev. per Minute | Miles per Hour | 16 | 24 | 32 | 40 | 48 | 56 |
| 55 | 10 | | 2081<br>2086 | 2075<br>2097 | 2085<br>2096 | 2095<br>2098 | |
| 110 | 20 | 2080<br>2087 | 2077 | 2073 | 2072 | 2084 | 2094 |
| 165 | 30 | 2083 | 2078 | 2074<br>2092 | 2082 | 2093 | |
| 220 | 40 | 2088 | 2079 | 2076 | 2089 | | |

### TABLE 2.

### TEST PROGRAM—SERIES 1.

SHOWING TESTS RUN AT VARIOUS SPEEDS AND CUT-OFFS.

| Approximate Speed | | Approximate Cut-off—Per cent of Stroke | | | | | |
|---|---|---|---|---|---|---|---|
| Rev. per Minute | Miles per Hour | 16 | 20 | 24 | 32 | 40 | 48 |
| 55 | 10 | | | 2024 | 2028 | | |
| 83 | 15 | 2017<br>2021 | | 2018<br>2020 | 2019<br>2022 | 2031 | |
| 110 | 20 | 2026 | | 2027 | 2029 | 2035 | 2033 |
| 138 | 25 | 2009 | | 2012 | 2013 | 2023 | |
| 165 | 30 | | | 2030 | 2032 | 2037 | |
| 193 | 35 | 2016 | 2010 | 2015 | 2014 | 2034 | |

6. *Series 2.*—Series 2 comprises 25 tests and includes tests 2072 to 2098 (excepting only tests 2090 and 2091 which are referred to beyond). In this series the speed varied from 10 to 40 miles per hour or from 55 to 220 revolutions per minute, while the cut-off ranged from 16 per cent to 56 per cent of the stroke. The distribution of these tests at the different speeds and cut-offs is shown in Table 1.

As elsewhere explained, (see section II and Appendix 1) the locomotive during this group of tests was in excellent condition, valves having been reset, valve chambers and cylinders rebored, the packing for pistons and valves and rods renewed, a leak in one of the steam pipe joints stopped, and the exhaust nozzle tip changed from 5¼ in. to 5⅞ in.

7. *Series 1.*—Series 1 comprises 26 tests and includes tests 2009 to 2037 (excepting No. 2011, 2025, and 2036). Test 2025 is omitted from the record because of errors in water measurement, and tests 2011 and 2036 were discontinued before their completion—one on account of an injector failure, the other on account of a faulty valve in the line supplying oil to the absorption brakes.

In this group of tests the speed varied from 10 to 35 miles per hour or from 55 to 193 revolutions per minute, while the cut-off ranged from 16 per cent to 48 per cent of the stroke. The distribution of these tests at the different speeds and cut-offs is shown in Table 2. During Series 1 the locomotive was in the condition in which it was received at the laboratory, which is distinguished from the condition prevailing during Series 2 by the repairs above cited.

8. *Intermediate Tests.*—Immediately after the completion of the tests of Series 1, the valves of the locomotive were reset, the eccentric straps shimmed, rod packing replaced, and the valve rings and piston rings were renewed and refitted. After these changes eight tests—No. 2038 to 2045—were run.

These changes, intended to improve cylinder performance, did not materially affect it. Because of them, however, these tests are excluded from Series 1 and their results appear only in Appendix 4. They are not included in any of the figures presented in the report. Since during these eight tests the condition of the boiler was exactly the same as during Series 1, their results relating to boiler performance are comparable with those of that series.

During the progress of Series 2, two tests—No. 2090 and 2091—were run with the nozzle tip changed from 5⅞ in. to 5¼ in. With this exception all conditions prevailing in these two tests were the same as in Series 2. These tests are referred to beyond, and their results are separately presented in Appendix 4. They are excluded from Series 2.

In addition to the tests above mentioned, 26 runs (No. 2046-2071) were made for such purposes as to wear down the cylinder and valve chambers after re-boring, to make final choice of exhaust nozzle tip, etc. While these runs were given test numbers, they were incomplete and were not intended to be included in the report. Of the 64 tests made with the expectation that they would be embodied in the report, only the three referred to in paragraph 7 have been excluded from the record.

## V. Test Methods and Test Conditions.

9. *Methods and Equipment.*—The methods employed in conducting the tests and in deriving the results are explained in detail in Appendixes 3 and 5. They conform in general to those prescribed by the American Railway Master Mechanics' Association code for conducting laboratory tests of locomotives, published in the Proceedings of the Association for 1914. Whatever deviations from this code have been found desirable are indicated in the appendixes.

The laboratory equipment is described in Appendix 2. While this equipment differs in several details from that of other laboratories, the only difference which has materially affected test methods lies in the presence of a cinder separator, through which all the exhaust gases pass and in which the entire body of cinders is collected. Except during one group of tests conducted at the Pennsylvania Railroad testing plant, when temporary provision was made to collect all the cinders issuing from the stack, the cinder discharge has been determined in other laboratory tests merely by sampling the exhaust gas stream.

The design of this cinder separator is illustrated in Appendix 2. Its operation has been entirely successful. Repeated examinations of the exhaust gases as they issued from the separator, and unsuccessful attempts to collect solid matter in the neighborhood of the laboratory stack have made it clear that the separator collects and retains even the finest cinders under all test conditions.*

10. *Conditions.*—As previously stated the coal used during all the tests came from Mission Field Mine, Vermilion County, Illinois. For all tests to and including No. 2091 run-of-mine coal was used. During tests 2092, 2093, 2094, and 2095 a mixture of run-of-mine and screened lump was used, which in appearance, analysis, and performance was not materially different from the run-of-mine alone. During the last three tests, (2096-2098) on account of a shortage in the supply of run-of-mine coal, 1½-in. screenings were used. Because of this difference in conditions, all data and all results involving coal are excluded from the record of these three tests.

The locomotive during all tests was fired by C. Welker, a skilled fireman, detailed for this purpose by the Illinois Central Railroad from their regular force. Previous to his engagement at the laboratory, he had had four and one-half years' experience as fireman on this road and upon the completion of the tests returned to their service. Dur-

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*The term *cinders* is here used to mean particles of appreciable size as distinguished from impalpable dust. Samples of the stack cinders representing the entire range in rate of combustion contained from 10 to 18 per cent of material which passed a 200 mesh screen.

ing some of the tests he was assisted by one of three other firemen who were also detailed at various times from the local Illinois Central force. None of these men had had less than one year's experience. Mr. Welker in these tests, as in those in which he acted alone, remained in charge and responsible for the character of the work.

The condition of the locomotive has been briefly stated in Section II and is more fully explained in Appendix 1. The test program and the conditions of speed and cut-off have been presented in section IV.

## VI. The Results of the Tests of Series 2.

All the data and the results of the tests of Series 2 are presented in detail in the tables of Appendix 4. There are included in this section only the more important data and results relating to the performance of the boiler, the engines, and the locomotive. These facts are here presented in both tabular and graphical form. In establishing the relations between results chief reliance is placed upon the figures; and the tabular matter, which is a repetition of parts of Appendix 4, is included for convenience of reference only. Except where otherwise specifically stated, the curves in the figures have been produced by averaging the coordinates of various groups of points, plotting these average values, and passing as nearly as possible through the points thus determined a smooth curve. The test designations which appear in the tables indicate first the approximate speed in revolutions per minute, next the nominal cut-off in per cent, and finally the amount of throttle opening. Thus in test 2072, designated as 110-40-F, the speed was about 110 revolutions per minute, the cut-off approximately 40 per cent, and the throttle—as in all the tests—was ''full'' or wide open.

### A. Boiler Performance.

The more significant data and results pertaining to the performance of the boiler in Series 2 are collected from Appendix 4 and presented here in Tables 3 and 4, which include nearly all the facts used in producing the figures relating to boiler performance. In both of these tables the tests are arranged in the order of the increasing amounts of dry coal fired per hour per square foot of grate (code No. 627). If this arrangement is borne in mind, some of the relations may be more definitely and quite as conveniently studied in the tables as in the curves.

In attempting to draw from these results inferences concerning the performance of locomotives in service, it should be remembered that

## TABLE 3.

## BOILER PERFORMANCE—SERIES 2.

| Test No. | Laboratory Designation | Average Boiler Pressure, lb. per sq. in. | Duration of Test, Minutes | Dry Coal Fired per Hour, lb. |  | Draft, in. of Water |  |  |  | Temperature, Deg. F. |  | Quality of the Steam in the Dome |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  |  |  | Total | Per sq. ft. of Grate | In the Ash-pan | In the Fire-box | Back of the Diaphragm | In Front of the Diaphragm | In the Fire-box | In the Front-end |  |
|  | Code Item☞ | 380 |  | 626 | 627 | 397 | 396 | 395 | 394 | 374 | 367 | 407 |
| 2081 | 55–24–F | 198.2 | 150 | 1975 | 39.9 | 0.2 | 0.7 | 1.5 | 2.2 | 1407 | 507 | 0.9963 |
| 2086 | 55–24–F | 199.1 | 170 | 2068 | 41.7 | 0.3 | 0.7 | 1.4 | 2.2 |  | 506 | 0.9963 |
| 2075 | 55–32–F | 198.1 | 140 | 2327 | 47.0 | 0.3 | 0.9 | 1.8 | 2.8 | 1661 | 543 | 0.9952 |
| 2080 | 110–16–F | 198.8 | 130 | 2422 | 48.9 | 0.2 | 1.0 | 2.0 | 2.9 | 1418 | 534 | 0.9956 |
| 2087 | 110–16–F | 199.2 | 150 | 2474 | 49.9 | 0.2 | 0.9 | 1.9 | 2.9 |  | 524 | 0.9956 |
| 2085 | 55–40–F | 197.9 | 120 | 3058 | 61.7 | 0.3 | 1.1 | 2.5 | 4.0 |  | 545 | 0.9962 |
| 2077 | 110–24–F | 196.0 | 110 | 3281 | 66.2 | 0.4 | 1.3 | 2.5 | 4.1 | 1570 | 565 | 0.9947 |
| 2095 | 55–48–F | 198.1 | 60 | 3334 | 67.3 | 0.4 | 1.5 | 3.0 | 5.3 |  | 567 | 0.9943 |
| 2083 | 165–16–F | 198.7 | 100 | 3338 | 67.4 | 0.4 | 1.6 | 2.7 | 4.3 | 1267 | 563 | 0.9934 |
| 2088 | 220–16–F | 197.8 | 100 | 3353 | 67.7 | 0.3 | 1.2 | 2.9 | 4.5 |  | 563 | 0.9919 |
| 2073 | 110–32–F | 197.6 | 80 | 4359 | 87.9 | 0.5 | 1.6 | 3.5 | 5.7 | 1662 | 595 | 0.9919 |
| 2078 | 165–24–F | 196.4 | 70 | 4707 | 95.0 | 0.5 | 1.9 | 3.9 | 6.0 | 1597 | 595 | 0.9915 |
| 2092 | 165–32–F | 198.4 | 50 | 5640 | 113.8 | 0.6 | 2.2 | 5.1 | 8.2 |  | 643 | 0.9894 |
| 2079 | 220–24–F | 197.4 | 60 | 5783 | 116.7 | 0.4 | 2.3 | 4.8 | 7.0 | 1688 | 614 | 0.9889 |
| 2072 | 110–40–F | 196.7 | 60 | 5927 | 119.6 | 0.7 | 2.0 | 4.9 | 8.0 | 1643 | 620 | 0.9895 |
| 2074 | 165–32–F | 197.1 | 60 | 6015 | 121.3 | 0.5 | 2.4 | 5.3 | 8.5 | 1662 | 637 | 0.9861 |
| 2076 | 220–32–F | 196.0 | 35 | 7831 | 158.0 | 0.5 | 2.8 | 5.7 | 9.2 | 1785 | 675 | 0.9844 |
| 2084 | 110–48–F | 194.0 | 50 | 7914 | 159.7 | 0.8 | 2.9 | 6.3 | 10.0 |  | 653 | 0.9896 |
| 2094 | 110–56–F | 196.3 | 25 | 8434 | 170.2 | 0.8 | 3.0 | 7.3 | 12.1 |  | 679 | 0.9887 |
| 2082 | 165–40–F | 195.2 | 50 | 8994 | 181.5 | 0.6 | 3.4 | 7.3 | 11.2 | 1458 | 673 | 0.9886 |
| 2093 | 165–48–F | 191.5 | 30 | 10216 | 206.2 | 0.9 | 3.4 | 7.8 | 12.8 |  | 702 | 0.9879 |
| 2089 | 220–40–F | 194.9 | 35 | 11127 | 224.5 | 0.7 | 3.5 | 7.1 | 11.9 |  | 703 | 0.9884 |

### TABLE 4.

### BOILER PERFORMANCE—SERIES 2.

| Test No. | Dry Coal Fired, per Hour per sq. ft. of Grate, lb. | Equivalent Evaporation, lb. | | | Calorific Value per lb. of Dry Coal, B. t. u. | Cinders | | | | | Efficiency of the Boiler Including the Grate, per cent |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Per Hour | Per Hour per sq. ft. of Heating Surface | Per lb. of Dry Coal | | Accumulated in the Front-end per Hour, lb. | Discharged from the Stack per Hour, lb. | Total per Hour, lb. | Total in Per cent of the Dry Coal Fired | Calorific Value in Per cent of the B.t.u. Contained in the Dry Coal | |
| | 627 | 645 | 648 | 658 | 458 | 422 & 345 | 423 & 345 | 424 & 345 | 426 | 460 & 458 | 666 |
| 2081 | 39.9 | 17 277 | 5.26 | 8.75 | 12 700 | 7.2 | 62 | 69 | 3.5 | 44.93 | 66.89 |
| 2086 | 41.7 | 17 308 | 5.27 | 8.37 | 12 586 | 6.0 | 110 | 116 | 5.6 | 46.69 | 64.49 |
| 2075 | 47.0 | 19 954 | 6.08 | 8.57 | 12 718 | 5.6 | 165 | 171 | 7.3 | 55.17 | 65.46 |
| 2080 | 48.9 | 20 923 | 6.37 | 8.64 | 12 848 | 6.9 | 97 | 104 | 4.3 | 44.99 | 65.28 |
| 2087 | 49.9 | 20 846 | 6.35 | 8.43 | 12 272 | 7.2 | 118 | 126 | 5.1 | 55.34 | 66.63 |
| 2085 | 61.7 | 25 270 | 7.70 | 8.26 | 12 622 | 8.5 | 217 | 226 | 7.4 | 77.50 | 63.54 |
| 2077 | 66.2 | 26 417 | 8.05 | 8.05 | 12 633 | 7.7 | 245 | 253 | 7.7 | 66.50 | 61.82 |
| 2095 | 67.3 | 28 614 | 8.72 | 8.58 | 12 315 | 10.0 | 250 | 260 | 7.8 | 70.78 | 67.61 |
| 2083 | 67.4 | 26 448 | 8.06 | 7.92 | 12 660 | 12.6 | 259 | 271 | 8.2 | 73.70 | 60.72 |
| 2088 | 67.7 | 26 995 | 8.22 | 8.05 | 12 095 | 10.2 | 273 | 283 | 8.5 | 71.29 | 64.58 |
| 2073 | 87.9 | 33 719 | 10.27 | 7.74 | 12 751 | 9.0 | 553 | 562 | 12.9 | 75.69 | 58.92 |
| 2078 | 95.0 | 34 431 | 10.49 | 7.31 | 12 517 | 13.7 | 648 | 662 | 14.1 | 76.16 | 56.66 |
| 2092 | 113.8 | 41 770 | 12.72 | 7.41 | 12 620 | 14.5 | 841 | 855 | 15.1 | 81.42 | 56.95 |
| 2079 | 116.7 | 38 155 | 11.62 | 6.60 | 12 767 | 14.0 | 873 | 887 | 15.3 | 78.13 | 50.18 |
| 2072 | 119.6 | 40 590 | 12.36 | 6.85 | 12 460 | | 1140 | | | | 53.36 |
| 2074 | 121.3 | 41 701 | 12.70 | 6.93 | 12 575 | 3.0 | 949 | 952 | 15.8 | 79.55 | 53.50 |
| 2076 | 158.0 | 46 838 | 14.27 | 5.98 | 12 519 | 24.1 | 1822 | 1847 | 23.4 | 88.86 | 46.41 |
| 2084 | 159.7 | 46 913 | 14.29 | 5.93 | 12 305 | 24.1 | 1499 | 1523 | 19.2 | 88.90 | 46.76 |
| 2094 | 170.2 | 54 336 | 16.55 | 6.44 | 12 426 | 31.0 | 1931 | 1962 | 23.5 | 85.46 | 50.33 |
| 2082 | 181.5 | 49 007 | 14.93 | 5.45 | 12 626 | 22.9 | 1976 | 1999 | 22.1 | 87.08 | 41.87 |
| 2093 | 206.2 | 57 954 | 17.65 | 5.67 | 12 551 | 20.0 | 2782 | 2802 | 27.4 | 83.51 | 43.82 |
| 2089 | 224.5 | 54 989 | 16.75 | 4.94 | 12 356 | 22.4 | 2979 | 3002 | 26.8 | 91.95 | 38.77 |

during the tests the boiler was forced somewhat beyond the limits which would ordinarily be maintained in service; so that the maximum test values of such measures of boiler activity as draft, rate of combustion, and rate of evaporation are somewhat greater than the values which would be maintained on the road for any except very short periods.

11. *General Conditions.*—The average boiler pressure varied during the tests of this series from 191.5 to 199.2 pounds, and the feed temperature ranged between 44.7 and 63.6 degrees. As is common under the uniform conditions of load which are maintained in laboratory tests, the quality of the steam was high and nearly uniform throughout the series, the lowest quality being 0.984 and the highest 0.9963.

The calorific value of the fuel varied between the limits of 10 487 and 11 660 B.t.u. per pound of coal as fired, and from 12 095 to 12 848 B.t.u. per pound of dry coal. The ash in the coal as fired varied from 9.64 to 13.96 per cent.

Of the 25 tests of Series 2, seventeen were of more than one hour's duration. In the remaining eight tests the test period was less than one hour, being in one test only 25 minutes. Even in this test, however, the coal burned amounted to 4095 pounds.

12. *Draft.*—The relation between the draft values and the rate of combustion is indicated in Fig. 2, and their relation to rate of evaporation in Fig. 3. Inspection of the curve of firebox draft in these figures reveals close agreement between the values represented by the individual points and the average value represented by the curve. This fact may be accepted as an indication of the uniformity with which the fire was managed during the tests.

In test 2093 the drafts in front of the diaphragm, back of the diaphragm, in the firebox, and in the ashpan were 12.8, 7.8, 3.4, and 0.9 inches of water respectively. The rate of combustion in this test was 118.66 pounds of coal per hour. The drafts cited are the maxima attained during this series except in the case of firebox draft which was exceeded by 0.1 of an inch in one other test.

13. *Firebox and Front-end Temperatures.*—The temperature of the gases in the firebox varied between 1267 and 1785 degrees during the first twelve tests of this series. This temperature was not recorded during the remaining tests because of a break-down in the pyrometer equipment. The relation of this temperature to both rate of combustion and rate of evaporation is exhibited by the upper curves of Fig. 4 and 5 respectively.

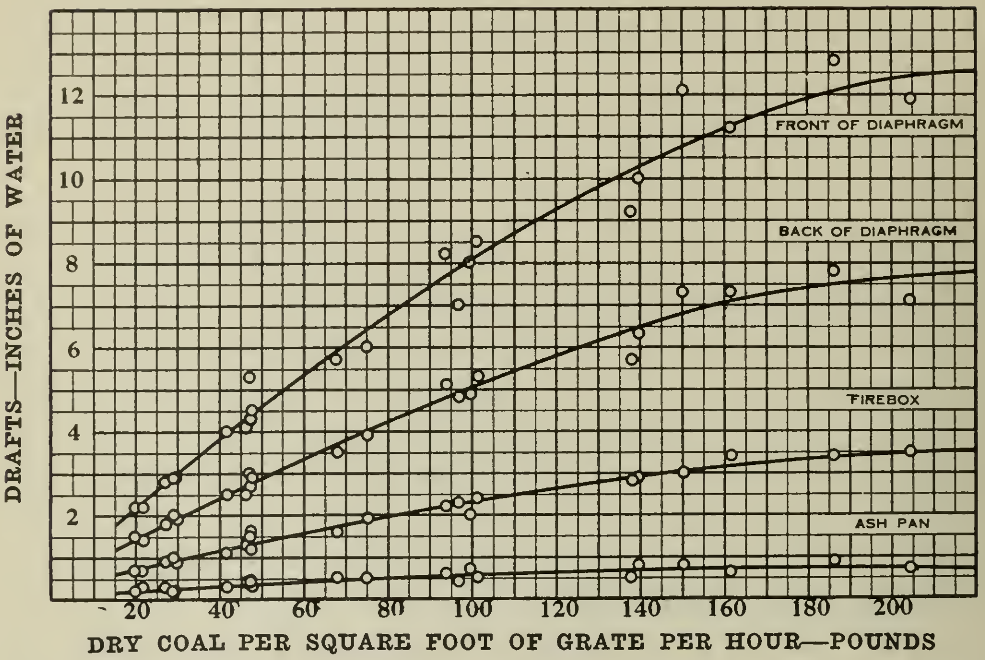

Fig. 2. The Relations Between Draft and Rate of Combustion.

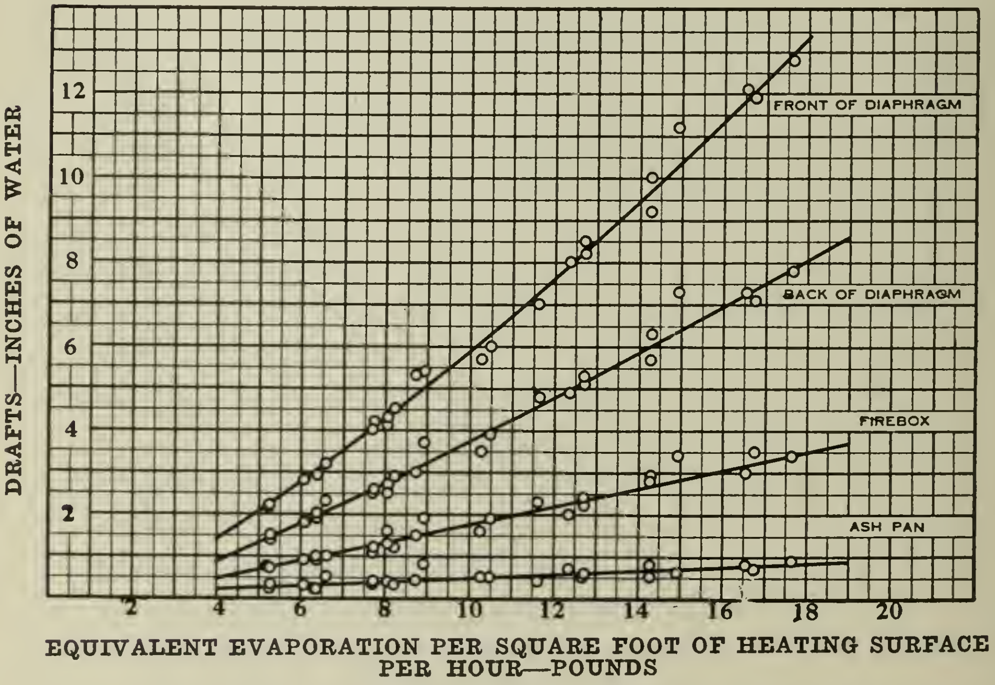

Fig. 3. The Relations Between Draft and Rate of Evaporation.

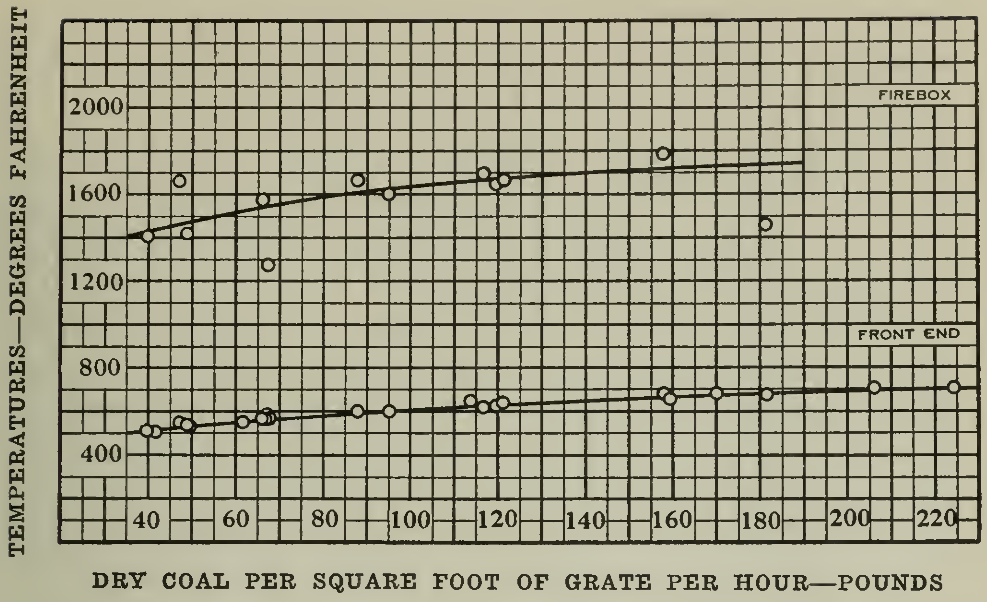

Fig. 4. The Relations Between Firebox and Front-end Temperatures and Rate of Combustion.

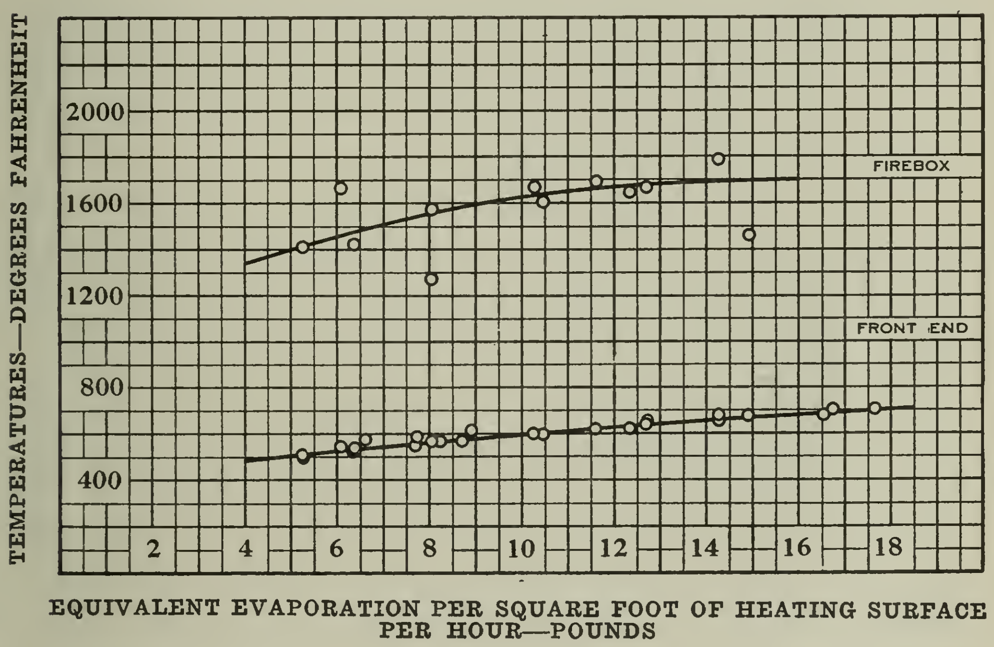

Fig. 5. The Relations Between Firebox and Front-end Temperatures and Rate of Evaporation.

The temperature of the gases in the front-end ranged between 506 and 702 degrees and increased very regularly as the activity of the grate and of the heating surface was increased. The relation of front-end temperature to rate of combustion appears in the lower curve of Fig. 4, and its relation to rate of evaporation in Fig. 5.

14. *Coal Consumption.*—The smallest amount of fuel fired during any of the tests was 3799 pounds of moist coal or 3334 pounds of dry coal. The greatest amount per test was 8506 pounds of moist coal or 7495 pounds of dry coal. The rate of firing ranged from 1975 pounds of dry coal per hour in test 2081 to 11 127 pounds of dry coal per hour in test 2089. The rate of combustion varied between 39.9 and 224.5 pounds of dry coal per square foot of grate per hour.

15. *Evaporation.*—The equivalent evaporation per hour varied between the limits of 17 277 and 57 954 pounds. The rate of increase in equivalent evaporation per hour with respect to the hourly consumption of dry coal is exhibited in Fig. 6. In this figure four of the highest values of evaporation are somewhat more divergent from the average represented by the curve than are the values for other tests. These four are all tests of short duration in which the measurement of the coal may be on this account slightly less accurate than in the other tests. Two of them, however, are tests in which the coal used was the mixture of lump and run-of-mine referred to in section V, and this fact may perhaps partially account for their divergence.

The equivalent evaporation per square foot of heating surface per hour varied in this series from 5.26 pounds to 17.65 pounds. Fig. 7 shows the relation of the rate of evaporation to the amount of dry coal fired per square foot of grate per hour.

16. *Boiler Horse Power.*—Under the usual convention of 34.5 pounds of equivalent evaporation per hour per horse power, the boiler of this locomotive developed a maximum horse power of 1680. This maximum is equivalent to one horse power for each 1.95 square feet of heating surface, or for each 0.295 of a square foot of grate area.

17. *Economic Performance.*—The equivalent evaporation per pound of dry coal ranged from a minimum of 4.94 to a maximum of 8.75 pounds. This range represents as good a performance as would be expected from the grade of coal used. The lower evaporations per pound of coal were of course obtained with the higher rates of combustion and evaporation. The rate of this decrease in evaporation per pound of dry coal is shown in Fig. 8 and 9, the former showing the decrease with respect to increase in the rate of combustion and the latter with respect to increase in the rate of evaporation. Either of these figures may serve as an index of the general performance of the boiler.

18. *Boiler Efficiency.*—By efficiency is meant, in this connection, the ratio of the heat absorbed by the boiler to the heat contained in

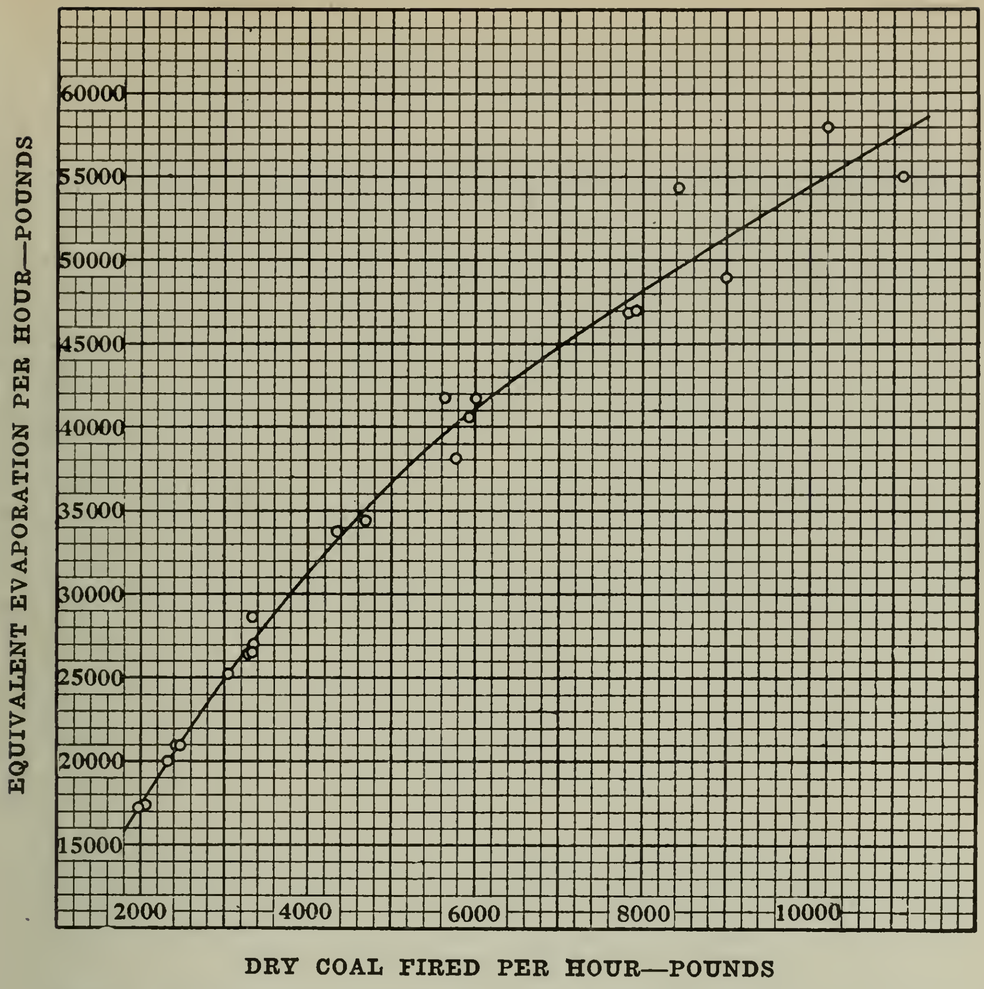

Fig. 6. The Relation Between Hourly Evaporation and Hourly Coal Consumption.

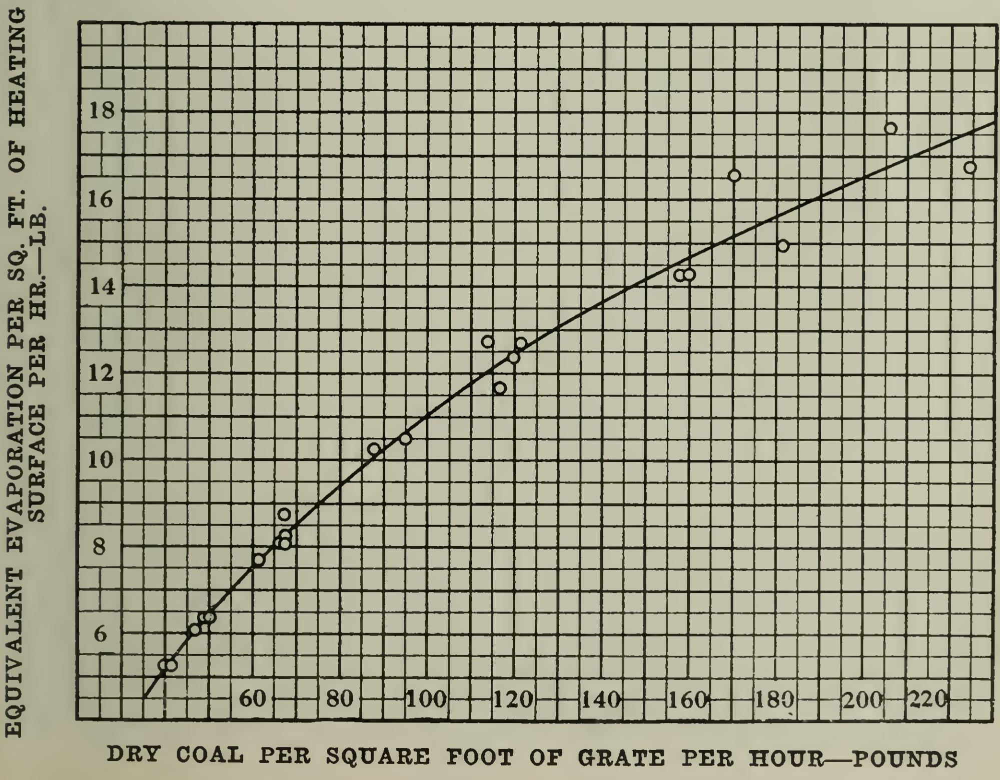

Fig. 7. The Relation Between Rate of Evaporation and Rate of Combustion.

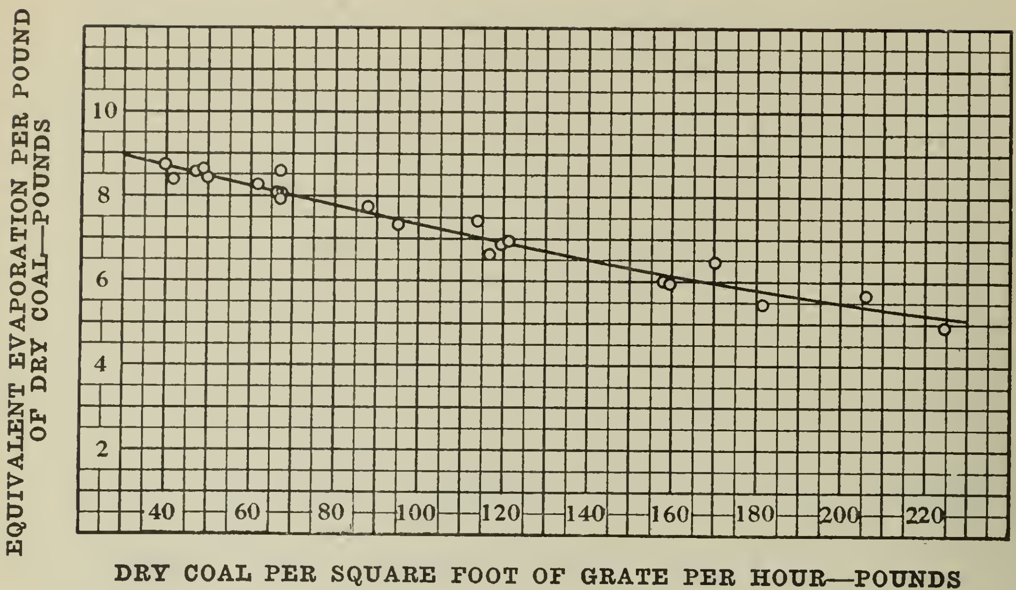

FIG. 8. THE RELATION BETWEEN EVAPORATION PER POUND OF COAL AND RATE OF COMBUSTION.

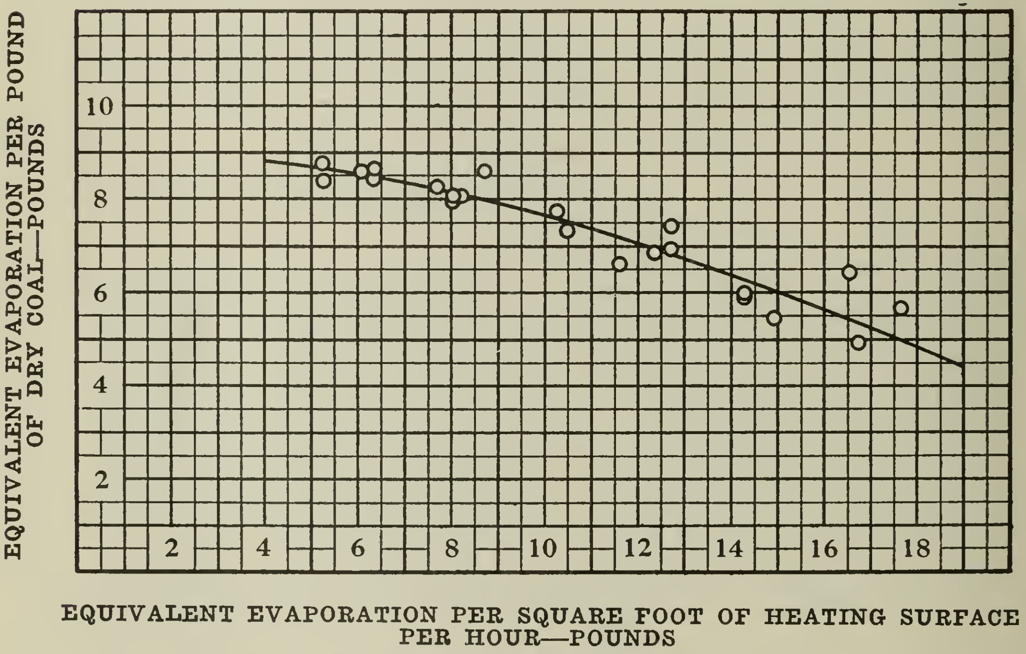

FIG. 9. THE RELATION BETWEEN EVAPORATION PER POUND OF COAL AND RATE OF EVAPORTAION.

the coal in the condition in which it was supplied to the fire, and it represents therefore the combined efficiencies of the furnace and of the boiler proper, in producing and utilizing the heat. The maximum efficiency, 67.61 per cent, was obtained in test 2095 with a rate of combustion of 67.3 pounds of dry coal per square foot of grate per hour, which is not quite the lowest rate of combustion occurring during this

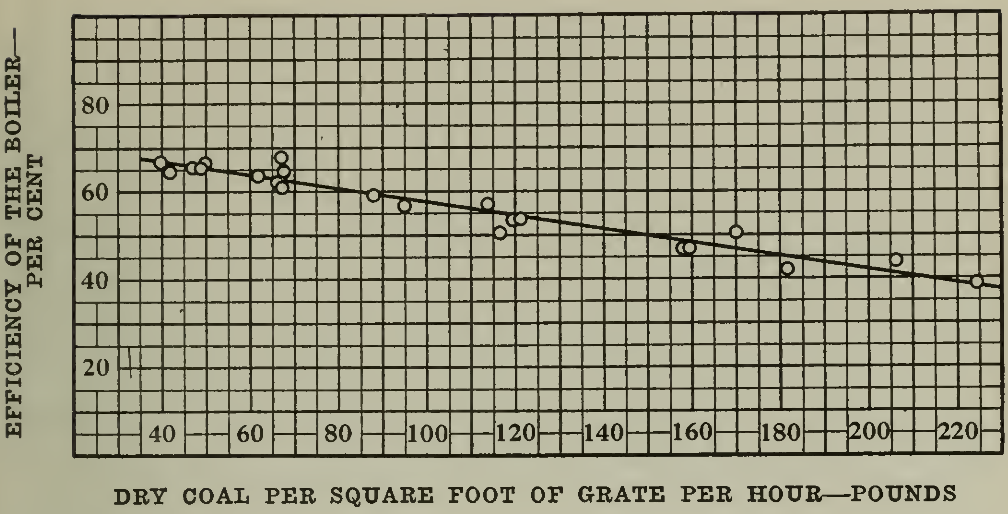

FIG. 10. THE RELATION BETWEEN BOILER EFFICIENCY AND RATE OF COMBUSTION.

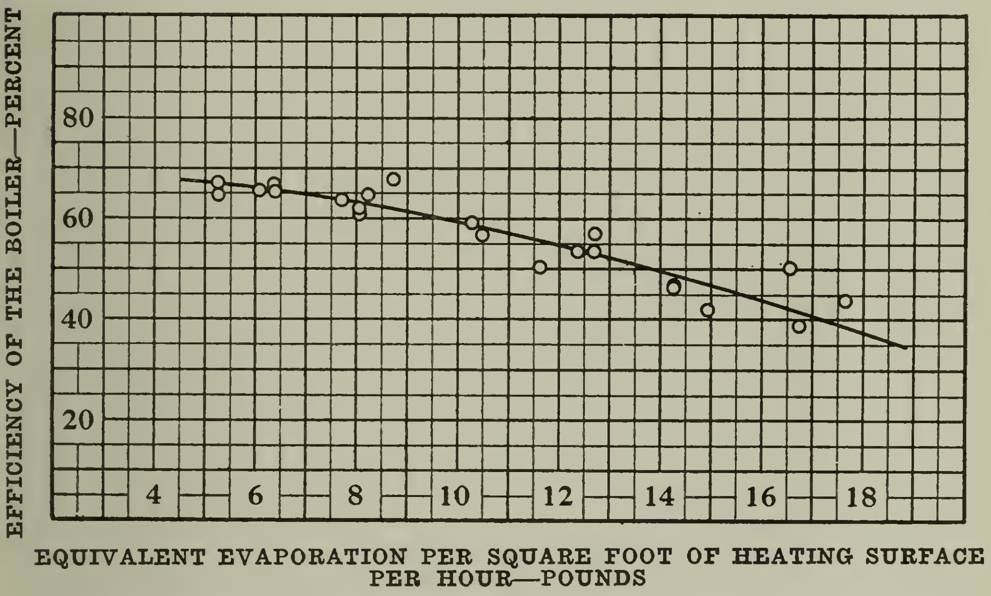

FIG. 11. THE RELATION BETWEEN BOILER EFFICIENCY AND RATE OF EVAPORATION.

series. The minimum efficiency, 38.77 per cent, was obtained in test 2089 in which the highest rate of combustion prevailed, namely 224.5 pounds of dry coal per square foot of grate per hour. The relations of efficiency to rate of combustion and to rate of evaporation are shown in Fig. 10 and 11, respectively.

19. *Cinder Losses.*—The data relating to the cinder losses which occurred during these tests have an especial significance in view of the methods by which they were obtained. The locomotive was equipped with a self-cleaning front-end and the maximum amount of cinders there collected during any test was only 21 pounds. In no test did the weight of front-end cinders amount to more than 0.4 of one per cent of the dry coal fired. For this reason no attempt is made to distinguish between cinders accumulated in the front end and those discharged from the stack, in the discussion here presented, although they are so distinguished in Table 4. In the discussion only the total amounts of cinders formed are referred to. These are substantially the same as the amounts discharged from the stack.

The minimum cinder loss occurred in test 2081, during which cinders were formed at the rate of 69 pounds per hour. The draft in this test was equivalent to 2.2 inches of water in front of the diaphragm, and the rate of combustion was 39.9 pounds of dry coal per square foot of grate per hour. The greatest cinder loss amounted to 2984 pounds per hour and occurred in test 2089, in which the corresponding draft was 11.9 inches of water and the rate of combustion 224.5 pounds of dry coal per square foot of grate per hour. Fig. 12 shows the increase in cinders formed per hour as the amount of coal burned per hour increases.

These cinder losses are more conveniently expressed as percentages of the weight of dry coal fired, and they are so presented in Table 4. Inspection of this table shows the minimum loss to have amounted to 3.5 per cent of the dry coal fired. This loss occurred in test 2081 in which, as above stated, the draft in front of the diaphragm was 2.2 inches of water and 39.9 pounds or dry coal were burned per square foot of grate per hour. The maximum cinder loss amounted to 27.4 per cent of the dry coal fired and occurred in test 2093, when the draft was 12.8 inches and the rate of combustion 206.2 pounds of dry coal per square foot of grate per hour. With few exceptions the cinders increase in amount with every increase in the rate of combustion. Fig. 13 shows the relation between cinder loss in per cent of the dry coal fired and the rate of combustion. The cinder losses have been expressed as percentages of the dry coal rather than of the coal as fired, because of the accidental variations in the moisture content of the latter. For this reason the amounts of dry coal fired have seemed here as elsewhere to offer the more logical basis for computation. If the cinder losses had been based upon moist instead of dry coal they would have been de-

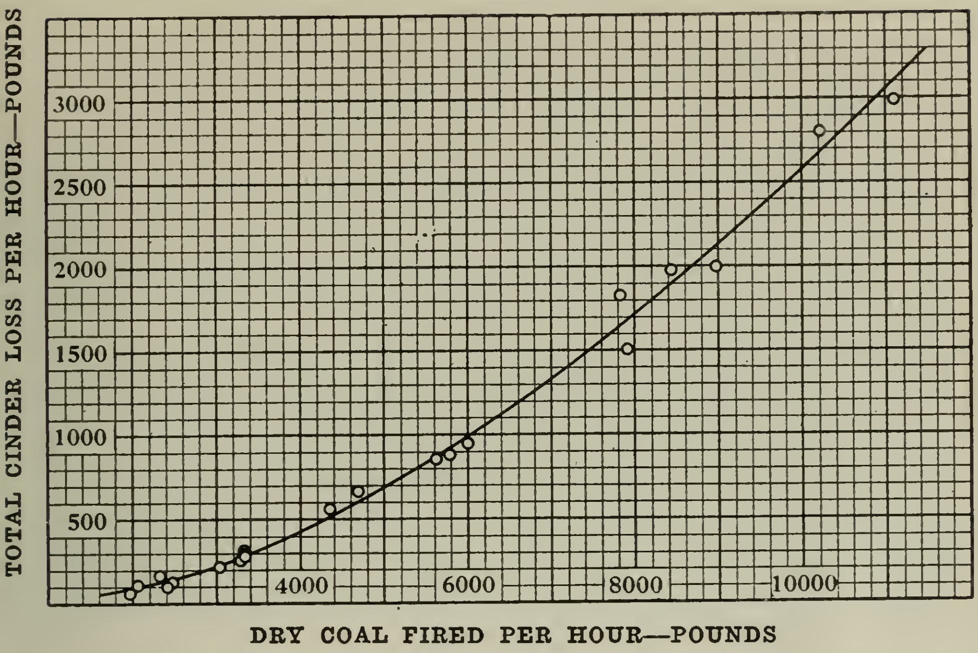

FIG. 12. THE RELATION BETWEEN HOURLY CINDER DISCHARGE AND HOURLY COAL CONSUMPTION.

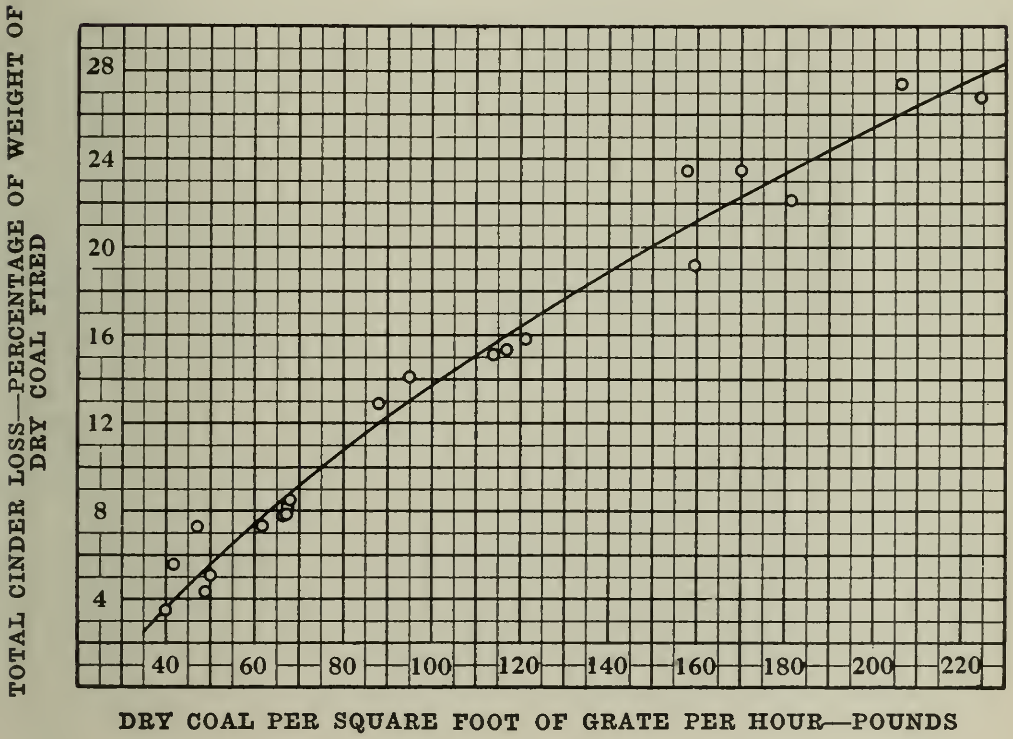

FIG. 13. THE RELATION BETWEEN CINDER DISCHARGE AND RATE OF COMBUSTION.

fined, of course, by smaller percentages. So based, the cinder losses in this series varied between 3.1 and 23.6 per cent.

In service the dry coal fired per square foot of grate per hour probably would rarely exceed 120 pounds. At this rate of combustion Fig. 13 indicates a cinder loss of about 16 per cent. On the road, therefore, except during rare and short intervals, the cinder discharge for this locomotive would probably range between 3 and 16 per cent of the weight of the dry coal fired, when using coal similar to that used during these tests.

The immediate cause of the cinder discharge is of course the intense draft which is essential in locomotive boiler operation. The relation of the cinder loss to draft is shown in Fig. 14 in which the ordinates represent cinder discharge in percentages of the dry coal, and the abscissae the draft in the front-end in front of the diaphragm. Fig. 14 shows the cinder discharge to have been almost directly proportional to draft.

The heating value of the cinders varied greatly, depending on the speed of their transit through the furnace and boiler. This speed is obviously influenced largely by the draft, which in turn determines also the rate of combustion. The increase in the heating value of the cinders as the rate of combustion increases is shown in Fig. 15, in which the ordinates represent the heating value of the cinders expressed as percentages of the B.t.u. contained in the dry coal, and the abscissae represent the rates of combustion. In test 2081 with the lowest rate of combustion and the smallest cinder loss, the calorific value of the cinders was only 44.9 per cent of the calorific value of the dry coal. In test 2089 with the highest rate of combustion and next to the greatest cinder discharge, the calorific value of the cinders was 92 per cent of that of the dry coal, the cinders in this case having passed through the boiler practically unburnt.

20. *Heat Balances.*—The heat balances for the tests of Series 2 are presented in Table 5, in which the various items of the balance are expressed in percentages of the heating value of the coal in the condition in which it was fired. The tests are arranged in the table in the order of the increasing amounts of equivalent evaporation per square foot of heating surface per hour.

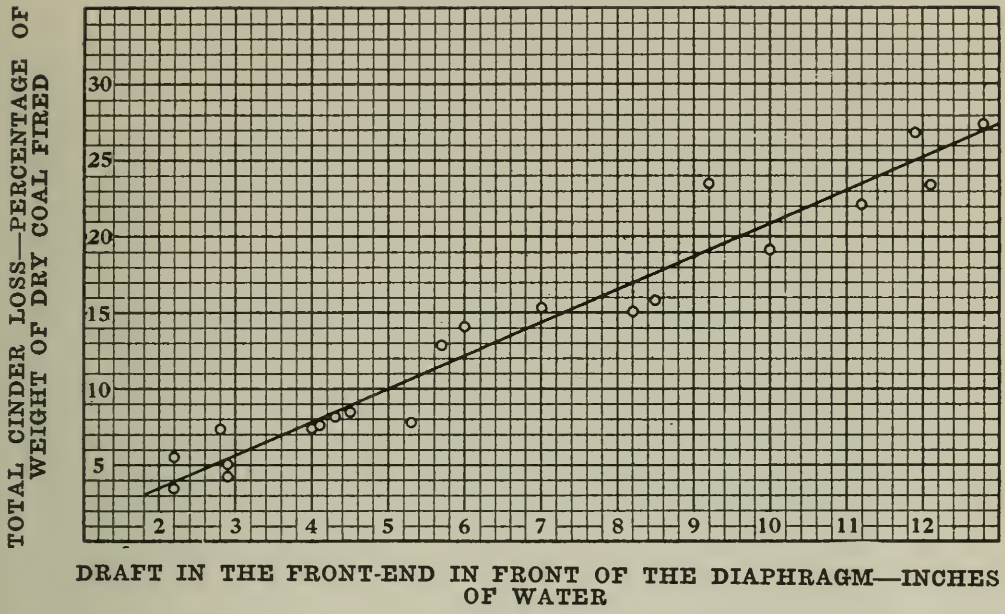

FIG. 14. THE RELATION BETWEEN CINDER DISCHARGE AND DRAFT.

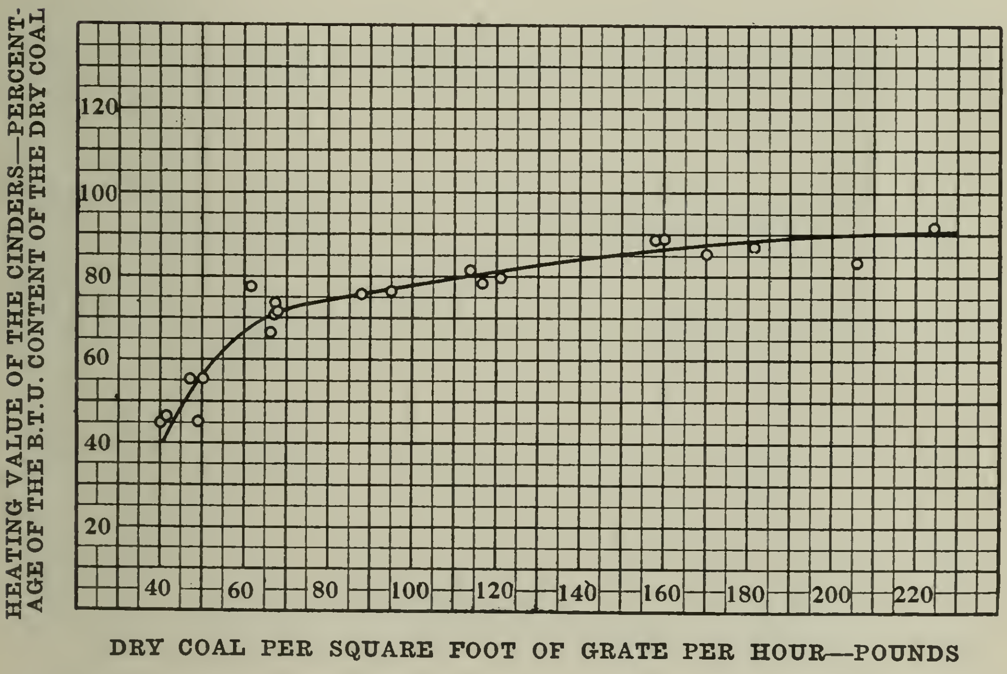

FIG. 15. THE RELATION BETWEEN THE HEATING VALUE OF THE CINDERS AND RATE OF COMBUSTION.

### TABLE 5.
### HEAT BALANCE FOR SERIES 2.

| Test No. | Equivalent Evaporation per sq. ft. of Heating Surface per Hour, lb. | Percentages of the Heating Value of the Coal as Fired | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Absorbed by the Boiler | To Moisture in the Coal | To Moisture in the Air | To Hydrogen in the Coal | To Escaping Gases | To Incomplete Combustion | To Combustible in the Front-end Cinders | To Combustible in the Stack Cinders | To Combustible in the Ash | To Radiation and Not-accounted for |
| | 648 | 881 | 882 | 883 | 884 | 885 | 886 | 887 | 888 | 890 | 899 |
| 2081 | 5.26 | 66.9 | 1.3 | 0.3 | 4.6 | 18.9 | 0.0 | 0.2 | 1.5 | 3.2 | 3.2 |
| 2086 | 5.27 | 64.5 | 1.4 | 0.2 | 4.5 | 15.8 | 0.6 | 0.1 | 2.4 | 3.3 | 7.2 |
| 2075 | 6.08 | 65.5 | 1.5 | 0.3 | 4.6 | 17.9 | 0.7 | 0.1 | 3.9 | 1.4 | 4.3 |
| 2087 | 6.35 | 66.6 | 1.4 | 0.3 | 4.6 | 15.9 | 0.0 | 0.1 | 2.6 | 3.5 | 5.0 |
| 2080 | 6.37 | 65.3 | 1.5 | 0.3 | 4.6 | 17.7 | 1.3 | 0.1 | 1.7 | 2.8 | 4.7 |
| 2085 | 7.70 | 63.5 | 1.5 | 0.2 | 4.6 | 15.6 | 0.0 | 0.2 | 5.3 | 3.3 | 5.7 |
| 2077 | 8.05 | 61.8 | 1.5 | 0.2 | 4.6 | 16.0 | 0.3 | 0.1 | 4.9 | 2.0 | 8.5 |
| 2083 | 8.06 | 60.7 | 1.6 | 0.2 | 4.6 | 15.3 | 0.0 | 0.2 | 5.6 | 4.1 | 7.6 |
| 2088 | 8.22 | 64.6 | 1.6 | 0.2 | 4.5 | 14.5 | 0.0 | 0.1 | 5.7 | 3.2 | 5.6 |
| 2095 | 8.72 | 67.6 | 1.5 | 0.2 | 4.7 | 15.3 | 0.0 | 0.2 | 5.2 | 2.9 | 2.4 |
| 2073 | 10.27 | 58.9 | 1.6 | 0.2 | 4.7 | 14.8 | 0.5 | 0.1 | 9.4 | 2.4 | 7.4 |
| 2078 | 10.49 | 56.7 | 1.6 | 0.2 | 4.7 | 16.0 | 0.0 | 0.1 | 10.4 | 1.7 | 8.6 |
| 2079 | 11.62 | 50.2 | 1.0 | 0.2 | 4.7 | 15.0 | 0.0 | 0.1 | 11.7 | 4.3 | 13.5 |
| 2072 | 12.36 | | | | | | | | | | |
| 2074 | 12.70 | 53.5 | 1.3 | 0.2 | 4.7 | 14.0 | 2.1 | 0.0 | 12.3 | 2.4 | 9.5 |
| 2092 | 12.72 | 57.0 | 1.8 | 0.2 | 4.8 | 13.7 | 0.2 | 0.1 | 12.0 | 3.0 | 7.3 |
| 2076 | 14.27 | 46.4 | 1.9 | 0.2 | 4.9 | 12.8 | 0.7 | 0.1 | 20.4 | 2.7 | 10.0 |
| 2084 | 14.29 | 46.8 | 1.6 | 0.3 | 4.8 | 14.6 | 0.0 | 0.1 | 16.3 | 2.8 | 12.7 |
| 2082 | 14.93 | 41.9 | 1.4 | 0.2 | 4.9 | 13.2 | 0.0 | 0.1 | 18.3 | 2.9 | 17.1 |
| 2094 | 16.55 | 50.3 | 1.8 | 0.2 | 4.9 | 12.0 | 0.3 | 0.2 | 19.4 | 3.0 | 7.9 |
| 2089 | 16.75 | 38.8 | 1.5 | 0.1 | 4.9 | 11.1 | 1.2 | 0.1 | 23.5 | 2.0 | 16.8 |
| 2093 | 17.65 | 43.8 | 1.7 | 0.1 | 4.9 | 11.0 | 0.6 | 0.1 | 22.3 | 5.0 | 10.4 |

## TABLE 6.

## ENGINE AND GENERAL PERFORMANCE—SERIES 2.

| Test No. | Laboratory Designation | Duration of Test, Minutes | Revolutions, Average per Minute | Speed in Miles per Hour | Piston Speed in Feet per Minute | Position of Throttle | Average Boiler Pressure, lb. per sq. in. | Drawbar Pull, lb. | Average Cut-off, Per cent of Stroke | Average Least Back Pressure, lb. per sq. in. | Average Mean Effective Pressure, lb. per sq. in. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Code Item☞ | | 352 | 353 | 354 | 363 | 380 | 487 | 499 | 615 | 678 |
| 2081 | 55–24–F | 150 | 50.6 | 9.2 | 252.5 | Full | 198.2 | | 24.1 | 0.7 | 77.1 |
| 2086 | 55–24–F | 170 | 51.3 | 9.3 | 256.1 | Full | 199.1 | 15 532 | 23.4 | 0.6 | 77.1 |
| 2075 | 55–32–F | 140 | 50.6 | 9.2 | 252.3 | Full | 198.1 | 20 483 | 32.1 | 1.7 | 98.9 |
| 2097 | 55–32–F | 110 | 52.1 | 9.5 | 260.0 | Full | 198.8 | 20 820 | 32.3 | 2.1 | 101.5 |
| 2085 | 55–40–F | 120 | 51.1 | 9.3 | 255.1 | Full | 197.9 | 24 833 | 41.3 | 1.8 | 117.8 |
| 2096 | 55–40–F | 90 | 51.5 | 9.4 | 257.0 | Full | 196.1 | 24 980 | 40.4 | 1.7 | 120.2 |
| 2095 | 55–48–F | 60 | 51.3 | 9.3 | 255.9 | Full | 198.1 | 28 922 | 49.2 | 1.8 | 135.4 |
| 2098 | 55–48–F | 50 | 51.7 | 9.4 | 258.2 | Full | 198.2 | 29 240 | 49.1 | 2.0 | 137.5 |
| 2080 | 110–16–F | 130 | 110.4 | 20.0 | 550.7 | Full | 198.8 | | 16.9 | 2.0 | 43.9 |
| 2087 | 110–16–F | 150 | 111.1 | 20.2 | 554.4 | Full | 199.2 | 8 135 | 16.6 | 2.3 | 43.7 |
| 2077 | 110–24–F | 110 | 110.7 | 20.1 | 552.2 | Full | 196.0 | 12 512 | 24.0 | 3.2 | 62.4 |
| 2073 | 110–32–F | 80 | 109.4 | 19.9 | 545.9 | Full | 197.6 | 16 961 | 29.6 | 6.5 | 80.8 |
| 2072 | 110–40–F | 60 | 109.6 | 19.9 | 546.8 | Full | 196.7 | 20 877 | 41.5 | 9.3 | 97.6 |
| 2084 | 110–48–F | 50 | 110.4 | 20.0 | 550.8 | Full | 194.0 | 22 403 | 48.4 | 12.2 | 105.7 |
| 2094 | 110–56–F | 25 | 110.9 | 20.1 | 553.3 | Full | 196.3 | 25 225 | 57.0 | 16.3 | 119.1 |
| 2083 | 165–16–F | 100 | 170.3 | 30.9 | 849.8 | Full | 198.7 | 7 078 | 18.4 | 3.9 | 37.2 |
| 2078 | 165–24–F | 70 | 169.0 | 30.7 | 843.3 | Full | 196.4 | 10 188 | 24.0 | 7.4 | 52.0 |
| 2074 | 165–32–F | 60 | 169.6 | 30.8 | 846.3 | Full | 197.1 | 13 486 | 28.8 | 12.5 | 64.7 |
| 2092 | 165–32–F | 50 | 168.5 | 30.6 | 840.8 | Full | 198.4 | 13 701 | 30.4 | 11.6 | 65.3 |
| 2082 | 165–40–F | 50 | 169.7 | 30.8 | 846.7 | Full | 195.2 | 14 783 | 41.4 | 18.3 | 74.5 |
| 2093 | 165–48–F | 30 | 167.4 | 30.4 | 835.5 | Full | 191.5 | 17 660 | 48.6 | 22.1 | 84.2 |
| 2088 | 220–16–F | 100 | 234.2 | 42.5 | 1168.6 | Full | 197.8 | 5 568 | 15.9 | 5.1 | 28.1 |
| 2079 | 220–24–F | 60 | 231.9 | 42.1 | 1157.2 | Full | 197.4 | 8 270 | 23.4 | 9.8 | 41.5 |
| 2076 | 220–32–F | 35 | 229.9 | 41.7 | 1147.2 | Full | 196.0 | 10 396 | 32.2 | 15.8 | 51.5 |
| 2089 | 220–40–F | 35 | 230.7 | 41.9 | 1151.2 | Full | 194.9 | 11 831 | 43.5 | 22.0 | 58.7 |

## TABLE 7.

## ENGINE AND GENERAL PERFORMANCE—SERIES 2.

| Test No. | Laboratory Designation | Indicated Horse Power, Total | Dry Coal Consumed per Indicated Horse Power per Hour, lb. | Dry Steam Consumed per Indicated Horse Power per Hour, lb. | Drawbar Horse Power | Dry Coal Consumed per Drawbar Horse Power per Hour, lb. | Dry Steam Consumed per Drawbar Horse Power per Hour, lb. | Machine Friction of Locomotive in Terms of Horse Power | Machine Efficiency of Locomotive, per cent. | Efficiency of Locomotive, per cent. |
|---|---|---|---|---|---|---|---|---|---|---|
|  | Code Item☞ | 711 | 734 | 736 | 743 | 744 | 745 | 770 | 778 | 779 |
| 2081 | 55–24–F | 450.5 | 4.36 | 31.53 |  |  |  |  |  |  |
| 2086 | 55–24–F | 456.0 | 4.51 | 31.18 | 386.0 | 5.33 | 36.83 | 70.0 | 84.7 | 3.80 |
| 2075 | 55–32–F | 578.6 | 4.00 | 28.40 | 501.4 | 4.62 | 32.76 | 77.2 | 86.7 | 4.33 |
| 2097 | 55–32–F | 610.6 |  |  | 525.2 |  | 33.91 | 85.4 | 86.0 |  |
| 2085 | 55–40–F | 694.3 | 4.39 | 29.69 | 614.5 | 4.96 | 33.55 | 79.8 | 88.5 | 4.07 |
| 2096 | 55–40–F | 713.6 |  |  | 622.8 |  | 33.46 | 90.8 | 87.3 |  |
| 2095 | 55–48–F | 804.9 | 4.15 | 29.37 | 718.0 | 4.66 | 32.92 | 86.9 | 89.2 | 4.44 |
| 2098 | 55–48–F | 822.2 |  |  | 732.2 |  | 32.56 | 90.0 | 89.1 |  |
| 2080 | 110–16–F | 558.5 | 4.31 | 30.87 |  |  |  |  |  |  |
| 2087 | 110–16–F | 560.3 | 4.39 | 30.65 | 437.6 | 5.63 | 39.24 | 122.7 | 78.1 | 3.69 |
| 2077 | 110–24–F | 795.7 | 4.10 | 27.36 | 670.2 | 4.87 | 32.48 | 125.5 | 84.2 | 4.14 |
| 2073 | 110–32–F | 1019.3 | 4.26 | 27.20 | 898.3 | 4.83 | 30.87 | 121.0 | 88.1 | 4.13 |
| 2072 | 110–40–F | 1233.8 | 4.79 | 27.19 | 1107.8 | 5.33 | 30.28 | 126.0 | 89.8 | 3.83 |
| 2084 | 110–48–F | 1347.5 | 5.88 | 28.69 | 1197.2 | 6.62 | 32.30 | 150.3 | 88.9 | 3.13 |
| 2094 | 110–56–F | 1521.4 | 5.58 | 29.39 | 1354.1 | 6.27 | 33.02 | 167.3 | 89.0 | 3.27 |
| 2083 | 165–16–F | 730.4 | 4.52 | 29.65 | 583.6 | 5.65 | 37.10 | 146.8 | 79.9 | 3.56 |
| 2078 | 165–24–F | 1011.2 | 4.63 | 28.09 | 833.8 | 5.62 | 34.06 | 177.4 | 82.5 | 3.62 |
| 2074 | 165–32–F | 1264.3 | 4.73 | 27.17 | 1107.3 | 5.40 | 31.03 | 157.0 | 87.6 | 3.75 |
| 2092 | 165–32–F | 1267.3 | 4.46 | 27.34 | 1117.6 | 5.06 | 31.01 | 149.7 | 88.2 | 3.99 |
| 2082 | 165–40–F | 1457.3 | 6.15 | 27.88 | 1214.6 | 7.38 | 33.45 | 242.7 | 83.4 | 2.73 |
| 2093 | 165–48–F | 1633.5 | 6.31 | 29.62 | 1431.6 | 7.19 | 33.80 | 201.9 | 87.6 | 2.82 |
| 2088 | 220–16–F | 756.1 | 4.42 | 29.40 | 631.3 | 5.29 | 35.21 | 124.8 | 83.5 | 3.98 |
| 2079 | 220–24–F | 1109.9 | 5.17 | 28.25 | 928.5 | 6.18 | 33.84 | 181.4 | 83.7 | 3.23 |
| 2076 | 220–32–F | 1364.3 | 5.71 | 28.30 | 1157.1 | 6.73 | 33.36 | 207.2 | 84.8 | 3.02 |
| 2089 | 220–40–F | 1559.9 | 7.10 | 29.18 | 1321.6 | 8.38 | 34.42 | 238.3 | 84.7 | 2.46 |

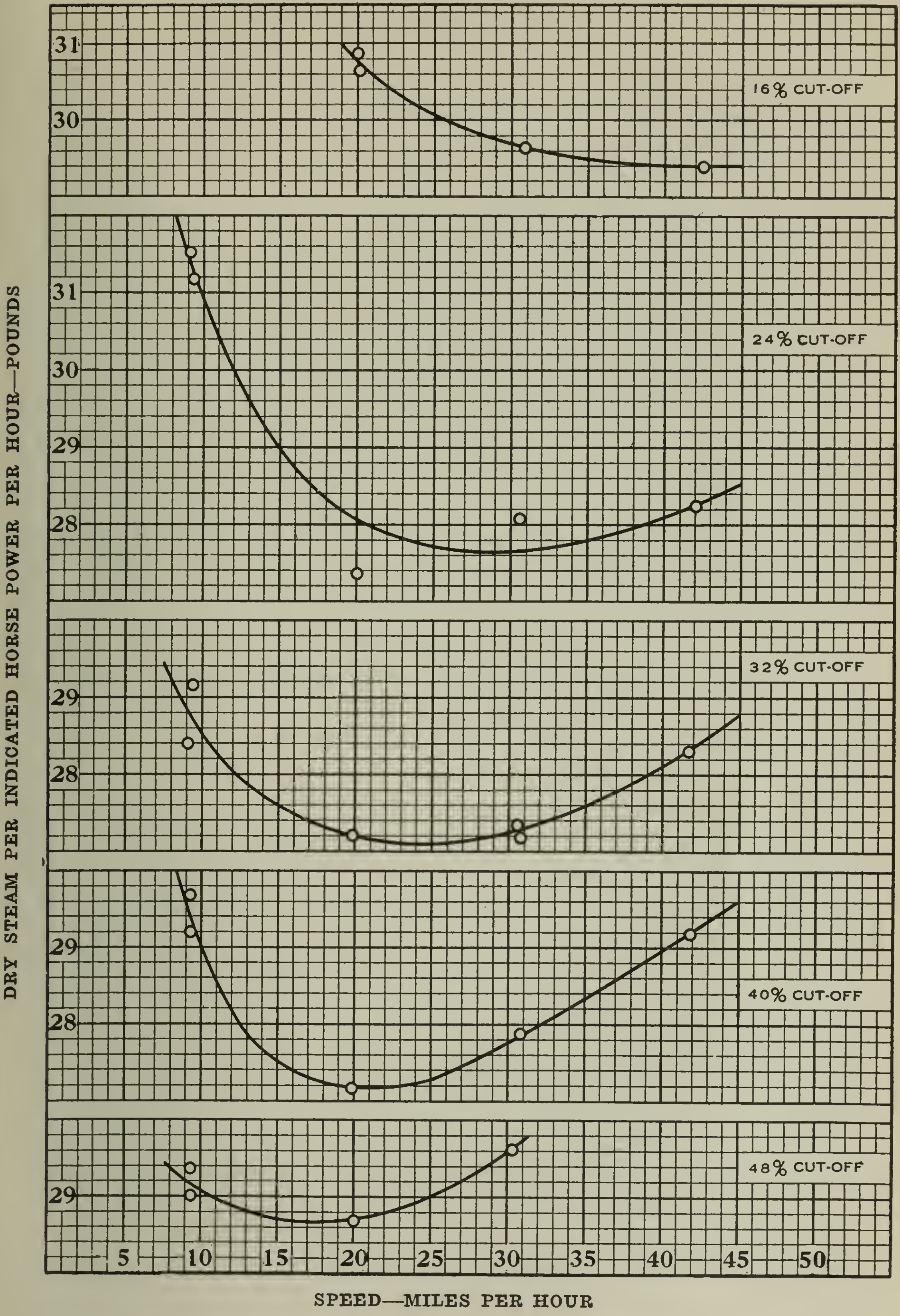

FIG. 16. THE RELATION BETWEEN STEAM CONSUMPTION AND SPEED, AT VARIOUS CUT-OFFS.

### B. Engine Performance.

Tables 6 and 7 present information relating to the general conditions and the more important results concerning engine and general performance for all tests of Series 2. These data are arranged in groups with reference to the speed of the locomotive and, within each group, are arranged with reference to the indicated horse power developed, the first test in each group giving the lowest horse power developed at the group speed. Appendix 4 contains data and results for all tests including information concerning cylinder performance as shown by average values taken from indicator diagrams. Fig. 56 and 57, there included, show representative indicator diagrams.

The nominal speeds at which the locomotive was operated were 10, 20, 30, and 40 miles per hour, and the data indicate that the actual speeds obtained closely approximated these figures. The nominal cut-offs at which the locomotive was operated were 16, 24, 32, 40, 48, and 56 per cent of the stroke. The actual cut-offs, as determined from the indicator cards, do not vary greatly from the nominal cut-offs. All tests at a given nominal cut-off were made with the reverse lever in the same notch of the reverse-lever quadrant. In the discussion which follows relative to engine and general performance, speed and cut-off are referred to in terms of the nominal values. All points plotted upon the figures are, however, located with regard to the actual speed and cut-off as determined from test data.

The data in general indicate uniform conditions in those particulars in which uniformity was sought. Test conditions as to nominal speed and nominal cut-off were duplicated in six cases. Such duplicated tests show, in general, satisfactory agreement as regards both test conditions and derived results.

21. *Dry Steam per Indicated Horse Power Hour and per Drawbar Horse Power Hour.*—Fig. 16 and 17 present the relation between dry steam per indicated horse power hour and speed. In Fig. 16 this relation has been shown with a separate water-rate scale for each group of tests at a given cut-off. In Fig. 17 the same curves have been referred to a single water-rate scale in order that the curves may be compared more readily. No curve is drawn for 56 per cent cut-off, since only one test was made at that cut-off.

The minimum water-rate was 27.17 pounds of dry steam, and occurred in test 2074 at a speed of 30 miles per hour and 32 per cent cut-off. The maximum water rate was 31.53 pounds of dry steam, and occurred in test 2081 at a speed of 10 miles per hour and 24 per cent cut-off. The difference between the minimum and maximum water-

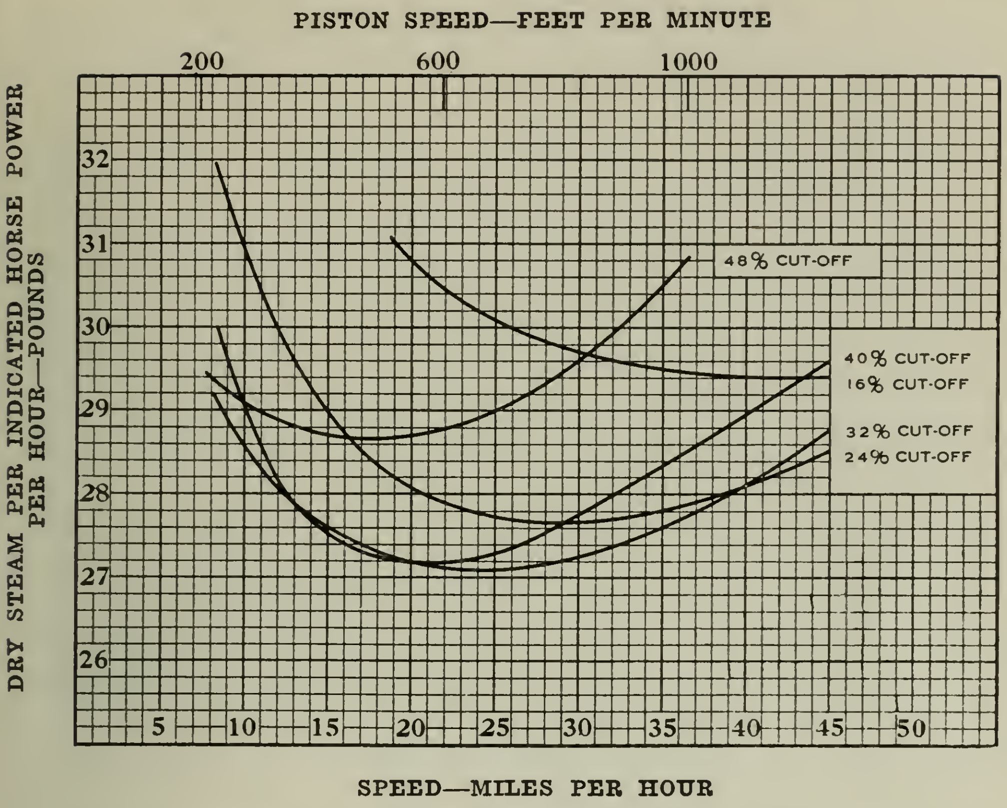

Fig. 17. The Relation Between Steam Consumption and Speed, at Various Cut-offs.

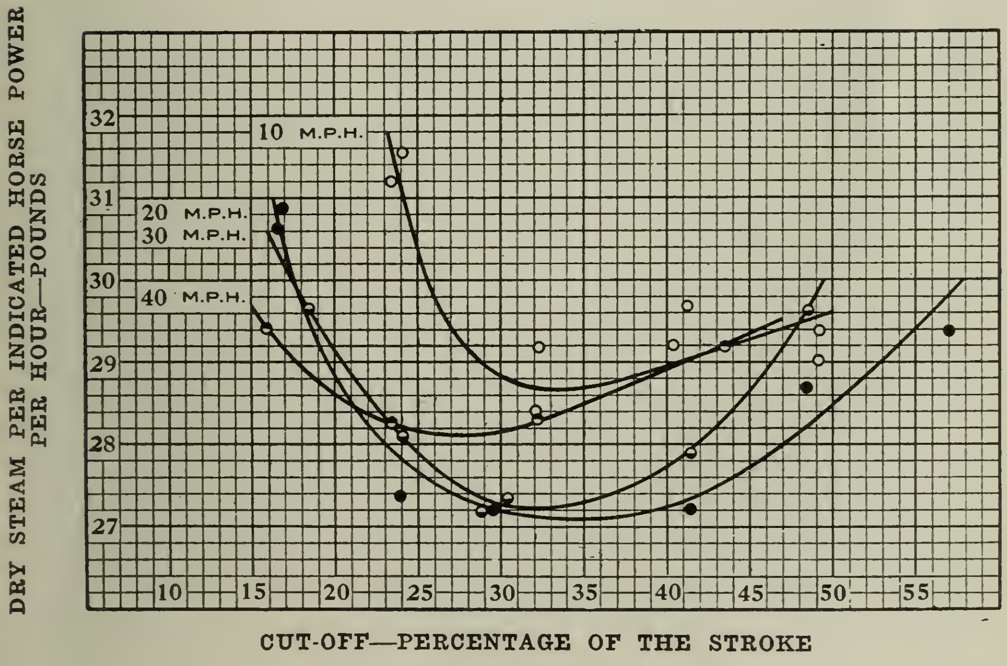

Fig. 18. The Relation Between Steam Consumption and Cut-off, at Various Speeds.

rates for all tests was only 4.36 pounds of dry steam. The corresponding differences between minimum and maximum rates at a given cut-off are in general much smaller.

A decrease in steam consumption per indicated horse power per hour as speed increases is shown until the speed has become from 20 to 30 miles per hour. Further increase of speed is then accompanied by increased steam consumption, as shown by all curves with the exception of that for tests at 16 per cent cut-off. The tests at both short and long cut-off show comparatively high water-rates. The best performance is shown by the curve for tests at 32 per cent cut-off. The tests at 40 per cent cut-off show rather better performance for freight service conditions than those made at 24 per cent cut-off.

Fig. 18 presents curves showing the dry steam consumed per indicated horse power per hour in its relation to cut-off. A curve is shown for each of the four nominal speeds at which tests were made—10, 20, 30, and 40 miles per hour. The tests made at 10 and at 40 miles per hour show much higher water-rates than do the tests made at 20 and 30 miles per hour. This is particularly true for cut-offs between 20 and 50 per cent. At short cut-off the 40 miles per hour curve shows a lower water-rate than the 20 and 30 miles per hour curves. At long cut-off the 10 miles per hour curve appears likewise to show a lower water-rate than the 20 and 30 miles per hour curves.

Except during short periods at starting and on heavy grades, the speed of this locomotive in service would probably vary between about 15 and 35 miles per hour, and the cut-off would range from say 50 to 20 per cent. Under these conditions of speed and cut-off the steam consumption varies between approximately 27 and 29 pounds of steam per indicated horse power per hour. It is probable that this range fairly represents the general average water rate for a very considerable number of freight locomotives in service.

22. *Indicated Horse Power and Drawbar Horse Power.*—Fig. 19 presents indicated horse power in its relation to speed, each curve of the figure representing all of the tests made at a particular nominal cut-off. In addition to the relationship just mentioned this figure shows clearly the range of the tests as to speed, cut-off, and load. It further shows the range covered within each group of tests when the tests are grouped either according to constant speed or to constant cut-off. The six different conditions of speed and cut-off at which duplicate tests were made are evident, and the proximity of the two points representing each pair of such tests indicates the uniformity obtained as to speed and indicated horse power developed.

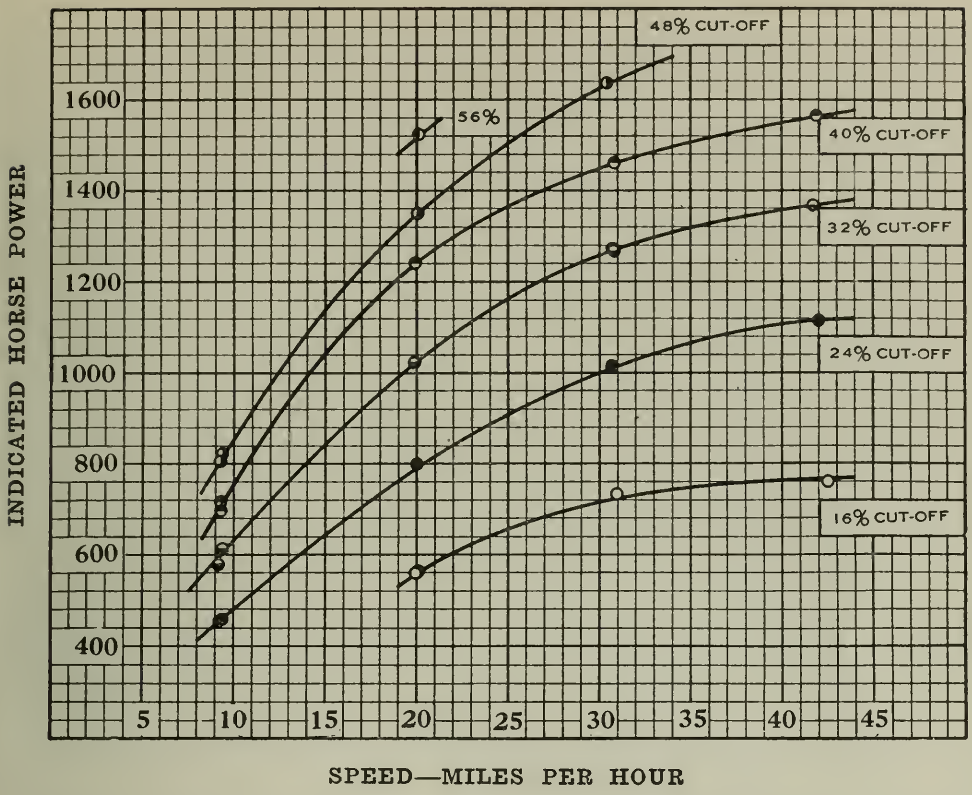

FIG. 19. THE RELATION BETWEEN INDICATED HORSE POWER AND SPEED, AT VARIOUS CUT-OFFS.

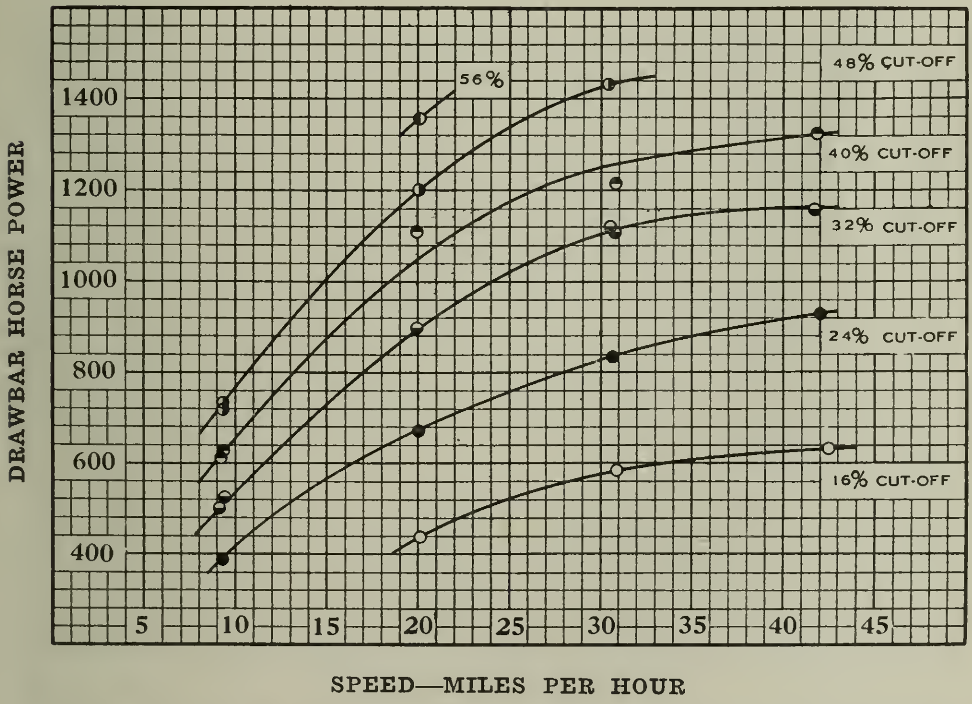

FIG. 20. THE RELATION BETWEEN DRAWBAR HORSE POWER AND SPEED, AT VARIOUS CUT-OFFS.

The maximum indicated horse power was developed in test 2093 at a speed of 30 miles per hour and at 48 per cent cut-off. The average rate of working for this test was 1633.5 indicated horse power. The lowest rate of working was for test 2081, being 450.5 indicated horse power while running at 10 miles per hour with 24 per cent cut-off. The dry steam supplied to the engines per hour when developing 1633.5 indicated horse power was 48 387 pounds. The moist steam delivered to the engines for the same test was 48 812 pounds per hour.

The relations between the horse power developed at the locomotive drawbar and the speed are shown in Fig. 20, in which each of the curves presents this relation for a particular cut-off. The maximum rate of 1431.6 drawbar horse power and the minimum rate of 386.0 drawbar horse power were developed during tests 2093 and 2086 respectively, the former test being at 30 miles per hour and 48 per cent cut-off, and the latter at 10 miles per hour and 24 per cent cut-off. Owing to incomplete dynamometer records for tests 2081 no record is available for the drawbar horse power developed for this test. Tests 2081 and 2086 were made under similar conditions of speed and cut-off.

The plotted points and curves of Fig. 19 and 20 show the engines to have been tested throughout a range of speed, cut-off, and load which would cover all ordinary service conditions above a speed of 10 miles per hour.

23. *Machine Friction.*—The diagrams in Fig. 21 present information concerning machine friction and its relation to indicated horse power for speeds of 10, 20, 30, and 40 miles per hour. Upon each diagram is shown the relation between the indicated horse power developed and machine friction expressed in horse power and also the relation between the indicated horse power developed and machine friction expressed in per cent of indicated horse power. Obviously the ordinates of each pair of curves in this figure bear to each other a definite numerical relation, and the curves have been so drawn that they satisfy this relation and also fairly represent the plotted values for the individual tests.

The range in machine friction is, for the entire series, from 70 to 242.7 horse power. These values were obtained in tests 2086 and 2082 during which 456.0 and 1457.3 indicated horse power respectively were developed. Test 2086 was at 10 miles per hour and 24 per cent cut-off, and test 2082 was at 30 miles per hour and 40 per cent cut-off. Expressed as percent of the indicated horse power developed, the minimum machine friction was 10.2 per cent and occurred in test 2072;

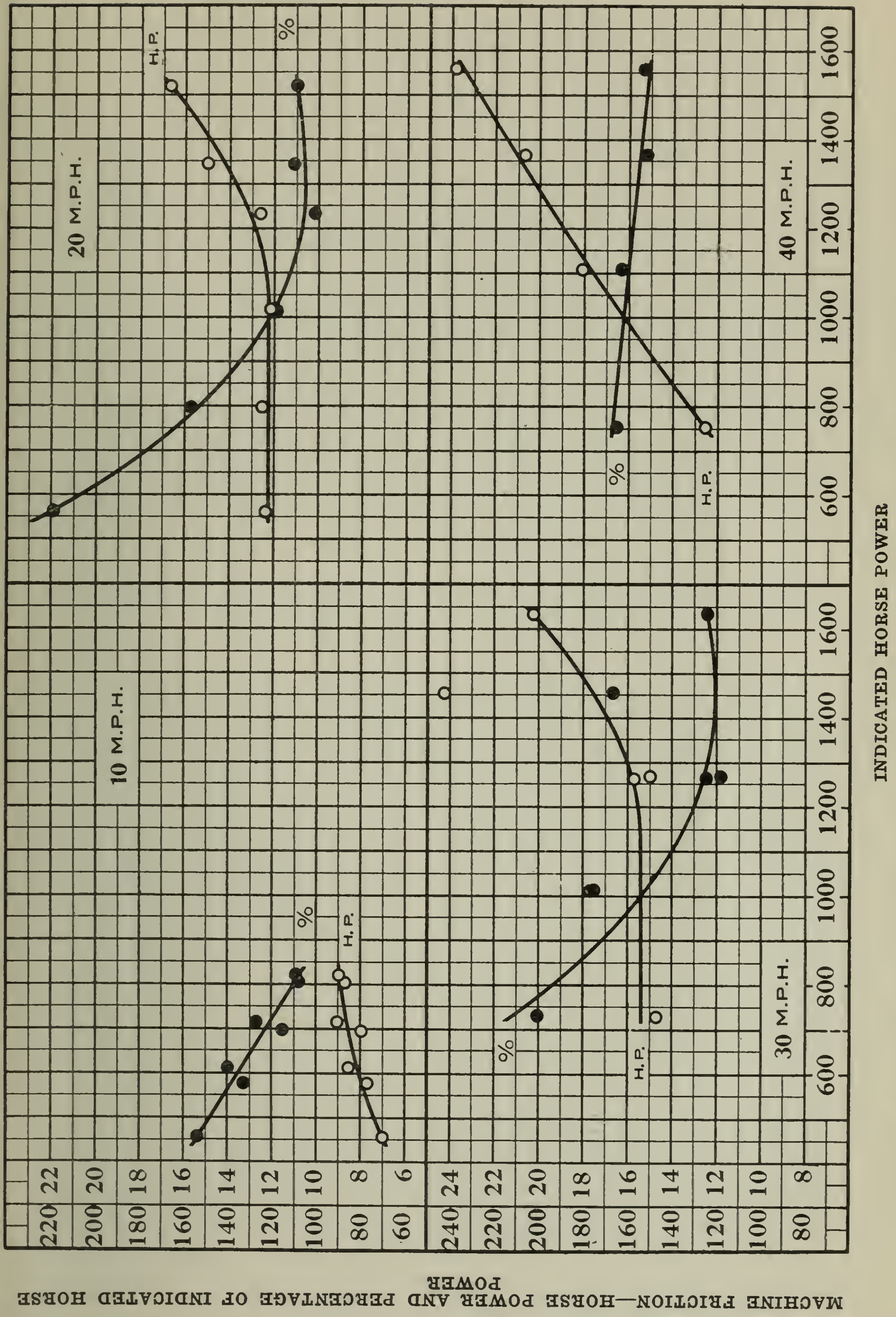

Fig. 21. The Relation Between Machine Friction and Indicated Horse Power, at Various Speeds.
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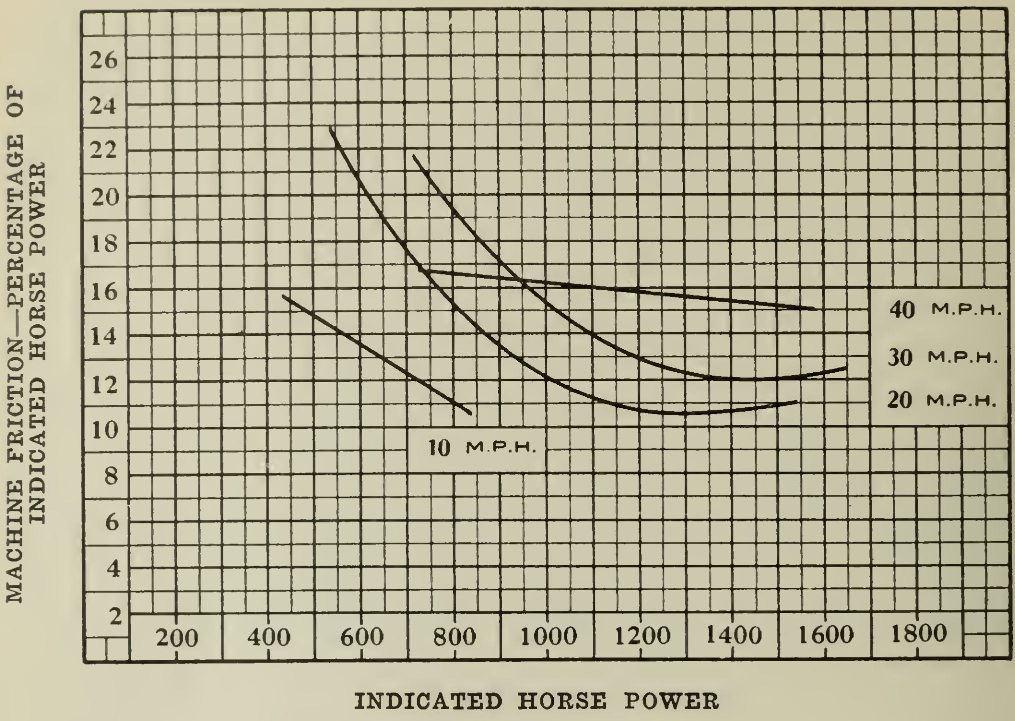

Fig. 22. The Relation Between Machine Friction and Indicated Horse Power, at Various Speeds.

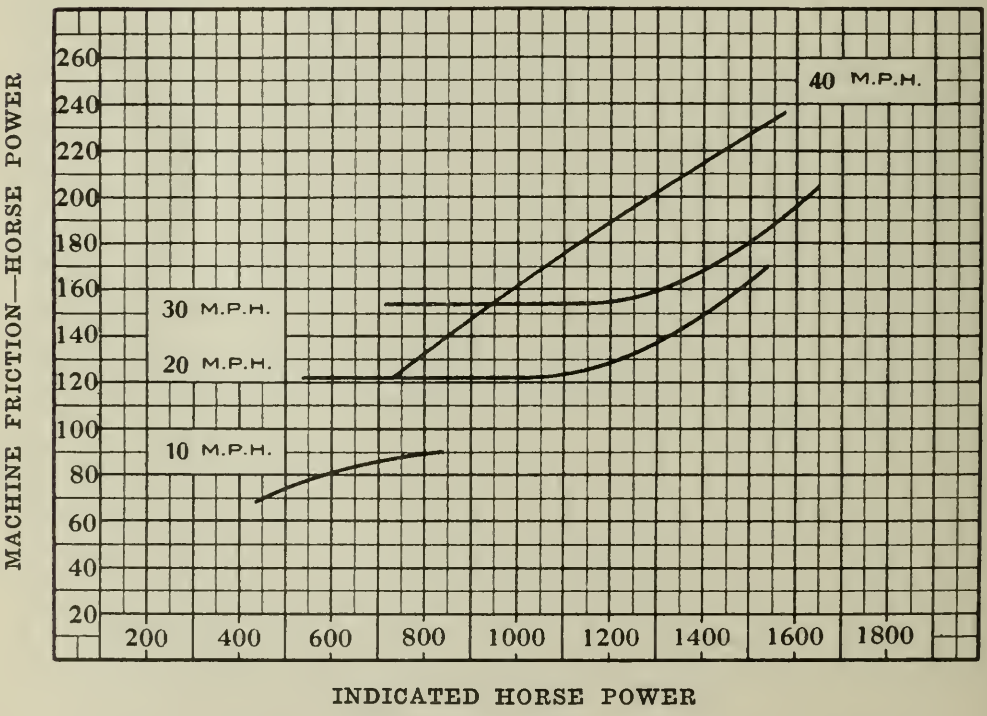

Fig. 23. The Relation Between Machine Friction Power and Indicated Horse Power, at Various Speeds.

the maximum was 21.9 per cent and occurred in test 2087. The former test was at 20 miles per hour and 40 per cent cut-off, developing 1233.8 indicated horse power, and the latter test at 20 miles per hour and 16 per cent cut-off, developing 560.3 indicated horse power.

While the curves differ more or less for different speeds, they all show the machine friction horse power to increase with increasing indicated horse power. In general, the ratio of machine friction horse power to indicated horse power decreases with increasing load. The rate of decrease of this ratio appears to be quite rapid for loads under 1000 horse power, but at greater loads the ratio becomes fairly constant for a given speed and ranges from 10 per cent to 15 per cent for the different speeds.

Fig. 22 presents upon a single diagram the four curves showing the relation between the machine friction in percentage of indicated horse power and the indicated horse power, which are included in Fig. 21. The curves so grouped indicate that as the speed increased, the percentage of power which was absorbed by machine friction also increased.

Fig. 23 likewise presents upon a single diagram the four curves showing the relation between machine friction and indicated horse power, the machine friction being expressed in terms of horse power. A general increase in machine friction both with increase of speed and with increase of indicated horse power is shown. The curves of Fig. 23, taken as a whole, show a tendency for the machine friction horse power to be fairly constant at a given speed or to increase rather slowly as the load increases up to about 1000 horse power. With loads greater than 1000 to 1200 horse power, the increase in machine friction is more rapid.

Fig. 24, 25, and 26 again present machine friction in its relation to indicated horse power. In these figures however each curve represents the data for all tests at a given nominal cut-off, instead of at a given speed as in the three preceding diagrams. The curves have been located and are presented in a manner similar to that used in connection with Fig. 21, 22, and 23.

These curves indicate that for a given cut-off machine friction horse power increased with the load, and at a rate approximately proportional to the increase of the load. The curves further indicate that, within the range of the tests, machine friction horse power increased with decreasing cut-off when the load remained constant. The per cent of indicated horse power absorbed in machine friction increased rap-

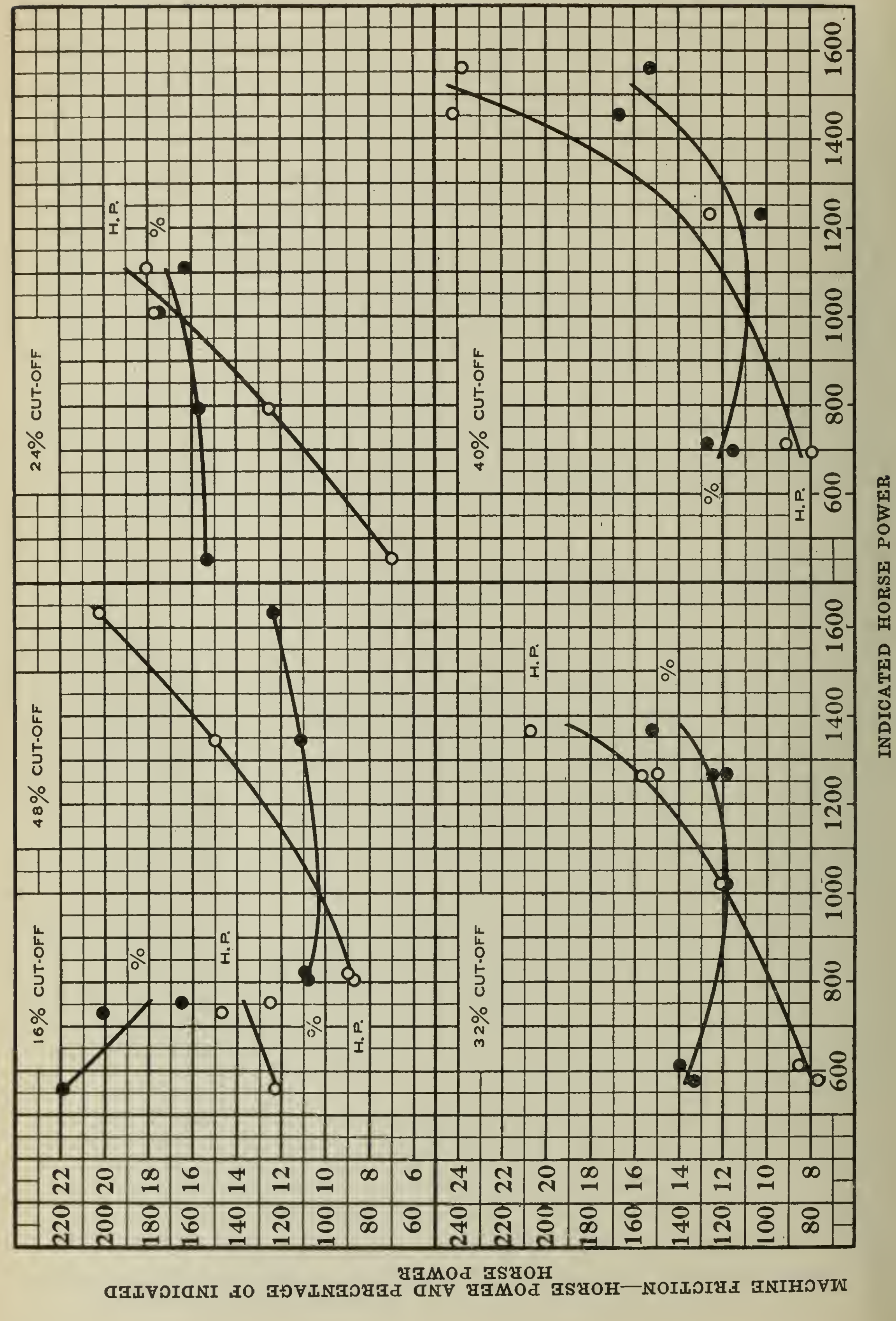

Fig. 24. The Relation Between Machine Friction and Indicated Horse Power, at Various Cut-offs.

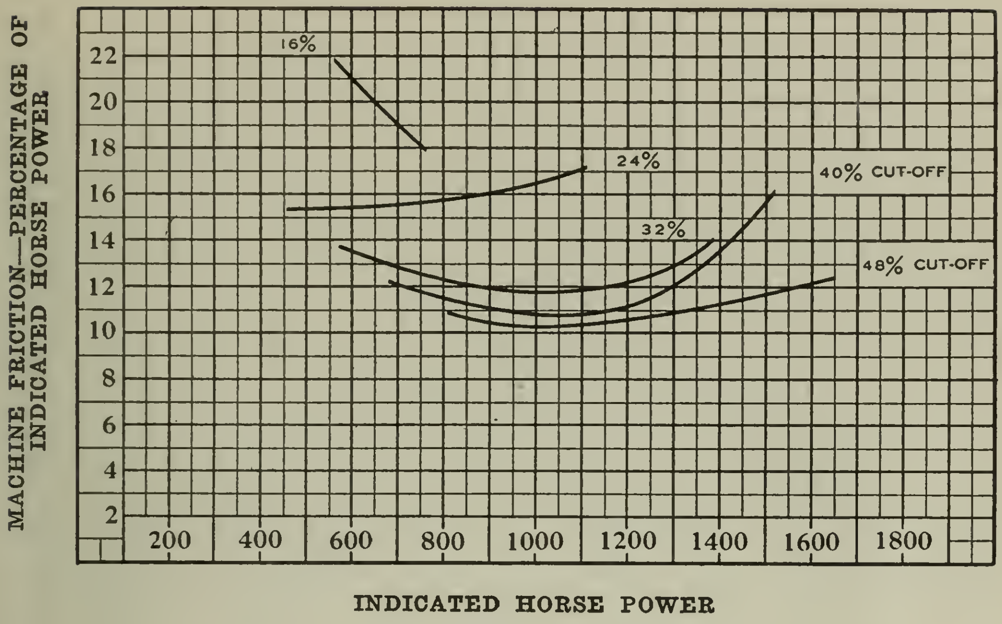

FIG. 25. THE RELATION BETWEEN MACHINE FRICTION AND INDICATED HORSE POWER, AT VARIOUS CUT-OFFS.

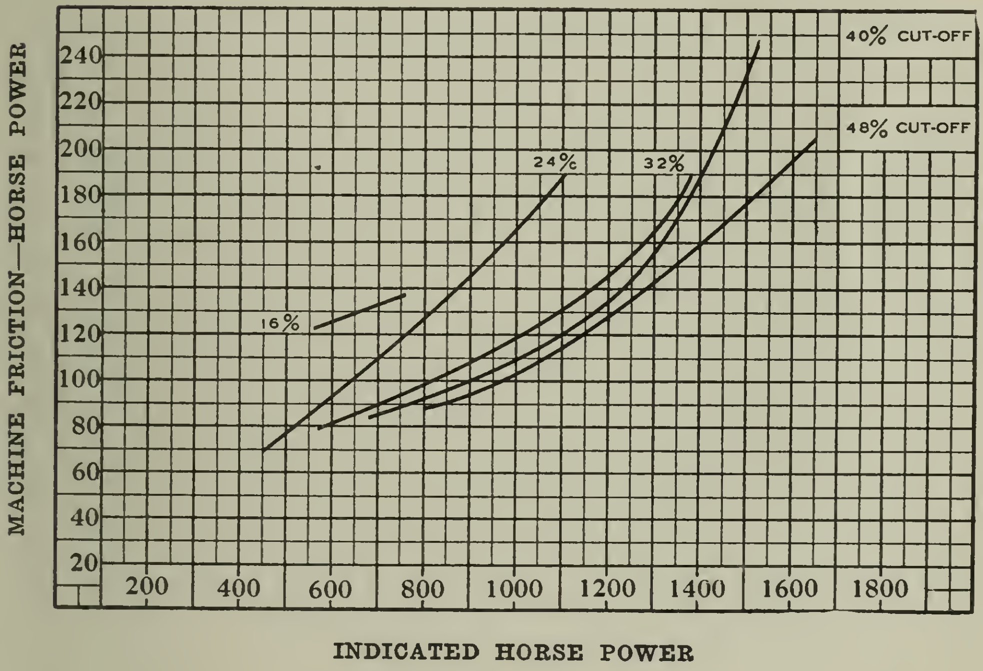

FIG. 26. THE RELATION BETWEEN MACHINE FRICTION POWER AND INDICATED HORSE POWER, AT VARIOUS CUT-OFFS.

idly with decreasing cut-off at constant load, and increased but slightly with increasing load at constant cut-off.

The locomotive tested carried 100.45 tons upon the drivers. The following table presents machine friction in terms of tractive force in pounds per ton upon drivers. The values given in the table were calculated from the curves of Fig. 23, making use of the minimum, maximum, and average values for machine friction horse power as shown by the various curves. In making these calculations nominal speed was employed.

TABLE 8.

MACHINE FRICTION.—SERIES 2.

| Speed in Miles per Hour | Machine Friction Expressed as Tractive Force in Pounds per Ton of Weight upon Drivers. | | |
|---|---|---|---|
| | Minimum | Average | Maximum |
| 10 | 25 | 31 | 34 |
| 20 | 23 | 25 | 31 |
| 30 | 19 | 20 | 26 |
| 40 | 12 | 17 | 22 |
| Averages | 20 | 23 | 28 |

For the locomotive tested the values given in Table 8 show 20 to 23 pounds tractive force per ton of weight on drivers to be values fairly representative of machine friction for practically all speeds when the load does not exceed 1000 to 1200 indicated horse power. For conditions of low speed, however, and for all speeds where the indicated horse power is comparatively high and the maximum tractive effort is approached, the machine friction is materially greater, as shown by the values given in the table for the speed of 10 miles per hour, and by the maximum values, which vary from 22 to 34 lb. per ton of weight upon the drivers.

Fig. 27 and 28 present machine friction in its relation to speed in miles per hour. A curve is drawn for each nominal cut-off. The values presented are the same as have been shown in preceding curves concerning machine friction, and some of the relations already considered in connection with Fig. 21 to 26 may be seen in Fig. 27 and 28. While the curves of Fig. 27 and 28 exhibit considerable lack of uniformity, the former figure shows machine friction horse power to increase more or less uniformly with increasing speed for all cut-offs; and Fig. 28 indicates that at constant speed the ratio of machine friction horse power to indicated horse power decreases as the cut-off increases, and that for any given cut-off this ratio is fairly constant throughout the range of speed shown.

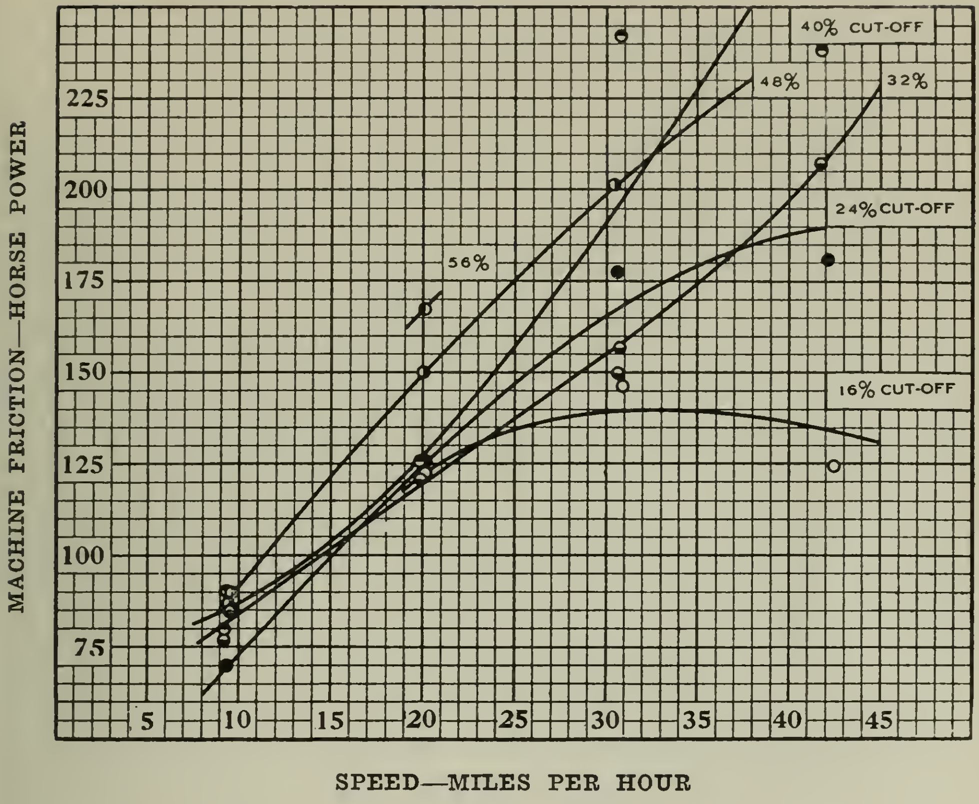

FIG. 27. THE RELATION BETWEEN MACHINE FRICTION POWER AND SPEED, AT VARIOUS CUT-OFFS.

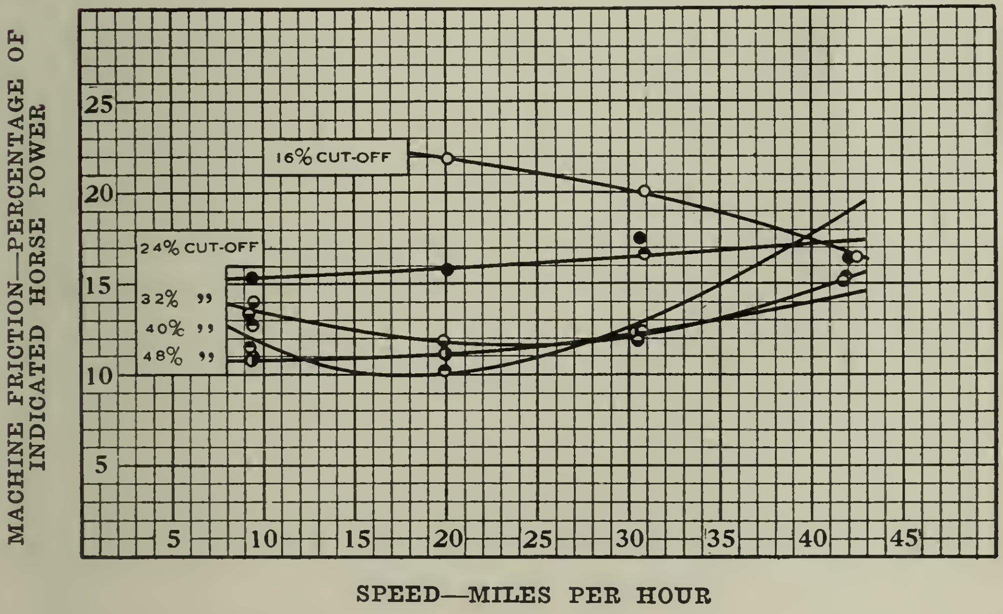

FIG. 28. THE RELATION BETWEEN MACHINE FRICTION AND SPEED, AT VARIOUS CUT-OFFS.

The facts here presented seem to warrant the conclusions that, for this locomotive, the percentage of indicated horse power absorbed by the friction of the machinery varies rather definitely with respect to speed, to cut-off, and to load; and that this machine friction entails a loss in tractive force between the cylinders and the locomotive drawbar which varies from about 15 to about 30 pounds for each ton of weight carried on the driving wheels.

### C. GENERAL PERFORMANCE.

24. *Coal Consumption per Indicated Horse Power Hour and per Drawbar Horse Power Hour.*—The curves of Fig. 29 show the relation between speed and the amount of dry coal consumed per indicated horse power per hour. Each of the curves there drawn applies to a particular cut-off. In a similar manner the relation between speed and the dry coal consumed per drawbar horse power per hour is presented in Fig. 30.

The most economical performance was obtained in test 2075, made at a speed of 10 miles per hour and at 32 per cent cut-off. During this test 4.00 pounds and 4.62 pounds of dry coal per indicated horse power hour and per drawbar horse power hour respectively were used. The highest coal rate occurred in test 2089, made at a speed of 40 miles per hour and at 40 per cent cut-off, during which 7.10 pounds and 8.38 pounds of dry coal per indicated horse power hour and per drawbar horse power hour respectively were used. Both figures show a more rapid increase in coal consumption with increase of speed at long cut-off than with increase of speed at short cut-off. This is in conformity with the results presented in Fig. 19 where the relation between indicated horse power and speed is shown. The curves of Fig. 29 and 30 show that the economy was fairly constant, or increased slowly as speed was increased from 10 to 20 miles per hour. Tests at 24 per cent cut-off show an economy apparently better at 15 to 20 miles per hour than at 10 miles per hour. As speed increased above 20 miles per hour the coal consumption increased more rapidly than at lower speeds, with the exception of the tests made at 16 per cent cut-off.

25. *General Efficiency.*—By general efficiency is meant, in this connection, the ratio of the heat equivalent of the work done at the locomotive drawbar to the heat content of the coal. This ratio is a measure of the economic performance of the locomotive as a whole. General efficiency and its relation to speed are shown in Fig. 31, in which a separate curve is presented for each nominal cut-off.

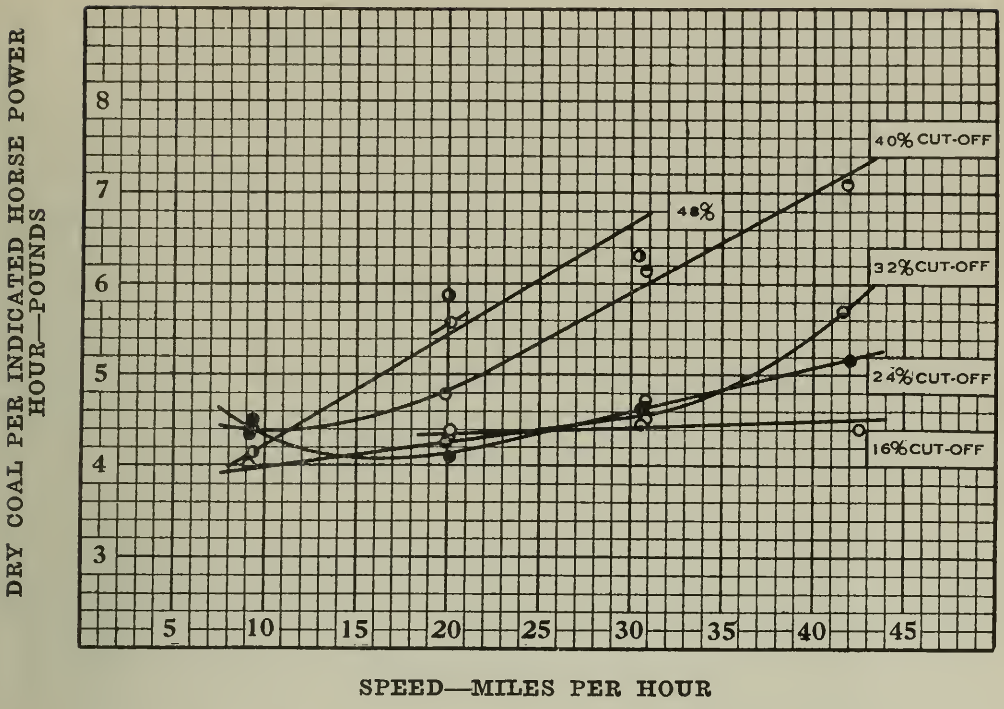

FIG. 29. THE RELATION BETWEEN COAL CONSUMED PER INDICATED HORSE POWER HOUR AND SPEED, AT VARIOUS CUT-OFFS.

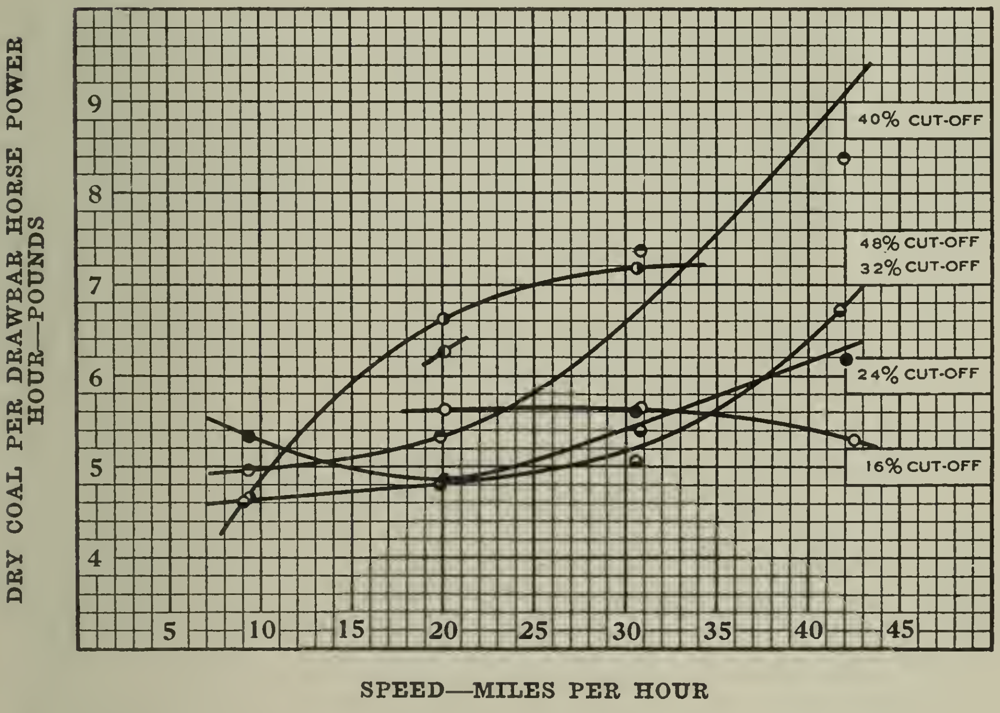

FIG. 30. THE RELATION BETWEEN COAL CONSUMED PER DRAWBAR HORSE POWER HOUR AND SPEED, AT VARIOUS CUT-OFFS.

The maximum efficiency obtained was 4.44 per cent, and occurred in test 2095 which was made at a speed of 10 miles per hour with 48 per cent cut-off, while developing 804.9 indicated horse power. The minimum efficiency obtained was 2.46 per cent, and occurred in test 2089 made at a speed of 40 miles per hour with 40 per cent cut-off, while developing 1559.9 indicated horse power. This group of curves shows substantially the same relations as were shown by Fig. 29 and 30 which presented coal consumption per indicated horse power hour and drawbar horse power hour. Collectively the curves indicate a fairly constant efficiency of about 4 per cent, for speeds from 10 to 20 miles per hour. As the speed increases above 20 miles per hour the efficiency decreases from about 4 per cent to about 3 per cent.

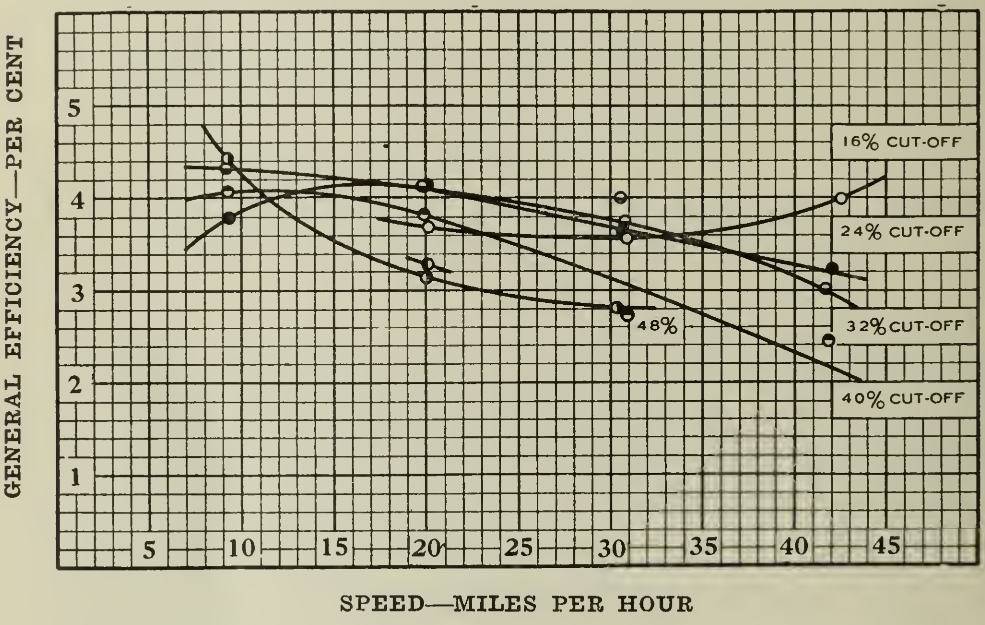

FIG. 31. THE RELATION BETWEEN GENERAL EFFICIENCY AND SPEED, AT VARIOUS CUT-OFFS.

## VII. THE RESULTS OF THE TESTS OF SERIES I.

Series 1 comprises tests 2009 to 2037 inclusive. The conditions which prevailed during the tests of this series have been set forth in sections IV and V, and the differences in the condition of the locomotive during Series 1 as compared with Series 2 are stated in section II and in Appendix 1. It is sufficient here to recall the fact that during Series 1 the locomotive was in the condition in which it was delivered to the laboratory. The repairs made between Series 1 and 2 did not prove to have affected radically the performance of the locomotive, and most of the relations presented in the preceding section relating to Series 2 remain substantially the same for Series 1. For these reasons it has seemed unnecessary to present the results for Series 1 in very great detail. All the results are given in Appendix 4; but only the more important measures of performance are here included and discussed.

### D. Boiler Performance.

26. *The Range of Performance.*—The more significant data and results pertaining to the performance of the boiler during Series 1 are given in Table 9, in which the tests are arranged in the order of the increasing amounts of dry coal fired per hour. In order to exhibit the range of the boiler performance, the minimum and maximum values of the main data and results are assembled in the table immediately following. As in all the tables, the quantities cited are the average values prevailing during the tests.

| | *Minimum* | *Maximum* |
|---|---|---|
| Duration of test, minutes | 40 | 180 |
| Boiler pressure, lb. per sq. in. | 189.9 | 198.1 |
| Feed water temperature, deg. F. | 57.7 | 72.2 |
| Quality of the steam in the dome | 0.9833 | 0.9956 |
| Calorific value of coal as fired, B.t.u. | 9929 | 11 376 |
| Calorific value of dry coal, B.t.u. | 11 835 | 12 757 |
| Ash content of coal as fired—per cent | 10.68 | 14.27 |
| Draft in front of diaphragm, inches of water | 2.2 | 10.7 |
| Firebox temperature, deg. F. | 1552 | 2081 |
| Front-end temperature, deg. F. | 494 | 761 |
| Dry coal fired per test, lb | 3568 | 10031 |
| Dry coal fired per hour, lb | 1814 | 7767 |
| Dry coal fired per hour per square foot of grate, lb | 36.6 | 156.8 |
| Equivalent evaporation per hour, lb. | 16 934 | 46 380 |
| Equivalent evaporation per hour per square foot of heating surface, lb. | 5.16 | 14.13 |
| Total cinder loss, per hour, lb | 66 | 1509 |
| Total cinder loss, per cent of dry coal | 3.4 | 20.8 |

27. *Economic Performance.*—In Series 1 the equivalent evaporation per pound of dry coal ranged from a minimum of 5.97 pounds to a maximum of 10.07 pounds. This latter value, applying to test 2024,

## TABLE 9.

## BOILER PERFORMANCE—SERIES 1.

| Test No. | Laboratory Designation | Average Boiler Pressure, lb. per sq. in. | Dry Coal Fired per Hour, lb. |  | Duration of Test, Minutes | Quality of the Steam in the Dome | Cinders, lb. |  | Calorific Value per lb. of Dry Coal, B. t. u. | Equivalent Evaporation, lb. |  | Efficiency of the Boiler Including the Grate, per cent |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  |  | Total | Per sq. ft. of Grate |  |  | Total per Hour | Total in Per cent of the Dry Coal Fired |  | Per Hour per sq. ft. of Heating Surface | Per lb. of Dry Coal |  |
|  | Code Item☞ | 380 | 626 | 627 |  | 407 | 424 & 345 | 426 | 458 | 648 | 658 | 666 |
| 2024 | 55–24–F | 196.3 | 1814 | 36.6 | 120 | .9950 | 68 | 3.7 | 12 712 | 5.56 | 10.07 | 76.87 |
| 2017 | 83–16–F | 193.4 | 1957 | 39.5 | 180 | .9910 | 66 | 3.4 | 12 422 | 5.33 | 8.96 | 69.88 |
| 2021 | 83–16–F | 193.7 | 2211 | 44.6 | 120 | .9945 | 133 | 6.0 | 12 274 | 5.16 | 7.66 | 60.56 |
| 2026 | 110–16–F | 196.9 | 2293 | 46.3 | 130 | .9895 | 140 | 6.1 | 12 309 | 6.00 | 8.59 | 67.75 |
| 2028 | 55–32–F | 198.1 | 2406 | 48.6 | 140 | .9912 | 166 | 6.9 | 12 653 | 6.14 | 8.37 | 64.21 |
| 2020 | 83–24–F | 190.7 | 2472 | 49.9 | 120 | .9929 | 151 | 6.1 | 12 302 | 6.42 | 8.53 | 67.33 |
| 2018 | 83–24–F | 194.2 | 2537 | 51.2 | 160 | .9930 | 153 | 6.1 | 12 265 | 6.50 | 8.41 | 66.49 |
| 2009 | 138–16–F | 190.5 | 2647 | 53.4 | 150 | .9900 | 176 | 6.6 | 12 553 | 6.60 | 8.19 | 63.30 |
| 2019 | 83–32–F | 193.8 | 3215 | 64.9 | 130 | .9910 | 277 | 8.6 | 12 523 | 8.17 | 8.35 | 64.68 |
| 2016 | 193–16–F | 193.9 | 3255 | 65.7 | 140 | .9890 | 297 | 9.1 | 11 992 | 7.56 | 7.62 | 61.68 |
| 2027 | 110–24–F | 196.8 | 3256 | 65.7 | 150 | .9914 | 306 | 9.4 | 12 688 | 7.68 | 7.74 | 59.22 |
| 2022 | 83–32–F | 189.9 | 3673 | 74.1 | 120 | .9956 | 453 | 12.3 | 11 875 | 8.02 | 7.17 | 58.54 |
| 2012 | 138–24–F | 191.8 | 3707 | 74.8 | 110 | .9890 | 359 | 9.7 | 12 280 | 8.69 | 7.69 | 60.80 |
| 2010 | 193–20–F | 192.0 | 3834 | 77.4 | 70 | .9870 | 438 | 11.4 | 12 433 | 8.71 | 7.45 | 57.63 |
| 2030 | 165–24–F | 196.5 | 4013 | 81.0 | 100 | .9894 | 519 | 13.0 | 12 757 | 9.60 | 7.85 | 59.71 |
| 2029 | 110–32–F | 197.1 | 4242 | 85.6 | 90 | .9902 | 589 | 13.9 | 12 486 | 9.78 | 7.57 | 58.85 |
| 2031 | 83–40–F | 196.4 | 4244 | 85.6 | 90 | .9896 | 638 | 15.0 | 11 989 | 10.14 | 7.84 | 63.43 |
| 2013 | 138–32–F | 190.1 | 4749 | 95.8 | 90 | .9850 | 623 | 13.1 | 12 242 | 10.54 | 7.29 | 57.78 |
| 2015 | 193–24–F | 192.1 | 4927 | 99.5 | 90 | .9870 | 736 | 14.9 | 12 307 | 10.04 | 6.69 | 52.84 |
| 2033 | 110–48–F | 196.0 | 5126 | 103.5 | 80 | .9867 | 924 | 18.0 | 12 243 | 11.77 | 7.54 | 59.76 |
| 2032 | 165–32–F | 196.8 | 5352 | 108.0 | 40 | .9862 | 1003 | 18.8 | 12 242 | 11.26 | 6.91 | 54.79 |
| 2035 | 110–40–F | 194.3 | 5565 | 112.3 | 70 | .9833 | 803 | 14.5 | 12 329 | 12.06 | 7.11 | 55.94 |
| 2014 | 193–32–F | 191.5 | 6199 | 125.1 | 70 | .9840 | 901 | 14.6 | 12 184 | 11.83 | 6.27 | 49.92 |
| 2037 | 165–40–F | 196.1 | 6554 | 132.3 | 60 | .9860 | 1217 | 18.6 | 12 441 | 13.97 | 7.00 | 54.54 |
| 2023 | 138–40–F | 190.8 | 6687 | 135.0 | 90 | .9930 | 1388 | 20.8 | 12 311 | 12.89 | 6.33 | 49.86 |
| 2034 | 193–40–F | 192.1 | 7767 | 156.8 | 60 | .9857 | 1509 | 19.4 | 11 835 | 14.13 | 5.97 | 48.95 |

is however so divergent from the other values for similar rates of combustion as to raise doubt of its validity, although errors can not be found in the data. The next highest evaporation per pound of dry coal is 8.96 pounds. The rate of decrease in the evaporation per pound of coal with respect to increase in rate of combustion is shown in Fig. 32. The rate of this decrease with respect to increase in rate of evaporation is shown in Fig. 33. These two figures are comparable with Fig. 8 and 9 of Series 2.

28. *Boiler Efficiency.*—As previously explained, efficiency in this connection means the ratio of the heat absorbed by the steam generated, to the heat contained in the coal as it was fired. If, for the reason above suggested, we exclude test 2024, the highest efficiency obtained during Series 1 was 69.88 per cent, which occurred during test 2017 with a rate of combustion of 39.5 pounds of dry coal per square foot of grate per hour. The lowest efficiency, 48.95 per cent, prevailed during test 2034 in which the rate of combustion was 156.8 pounds of dry coal per square foot of grate per hour. The relation between efficiency and rate of combustion is shown in Fig. 34.

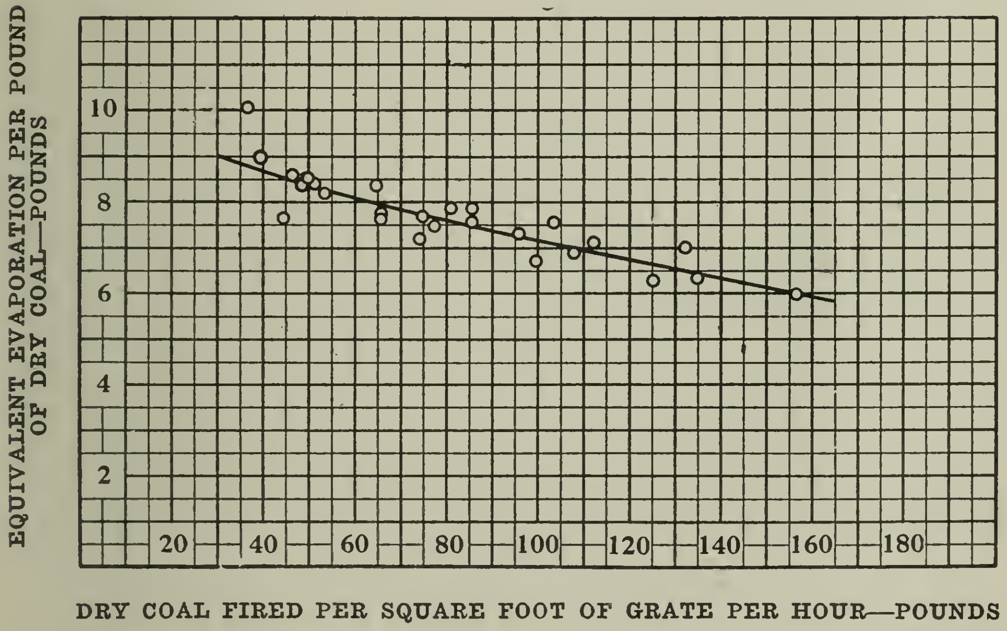

Fig. 32. The Relation Between Evaporation per Pound of Coal and Rate of Combustion.

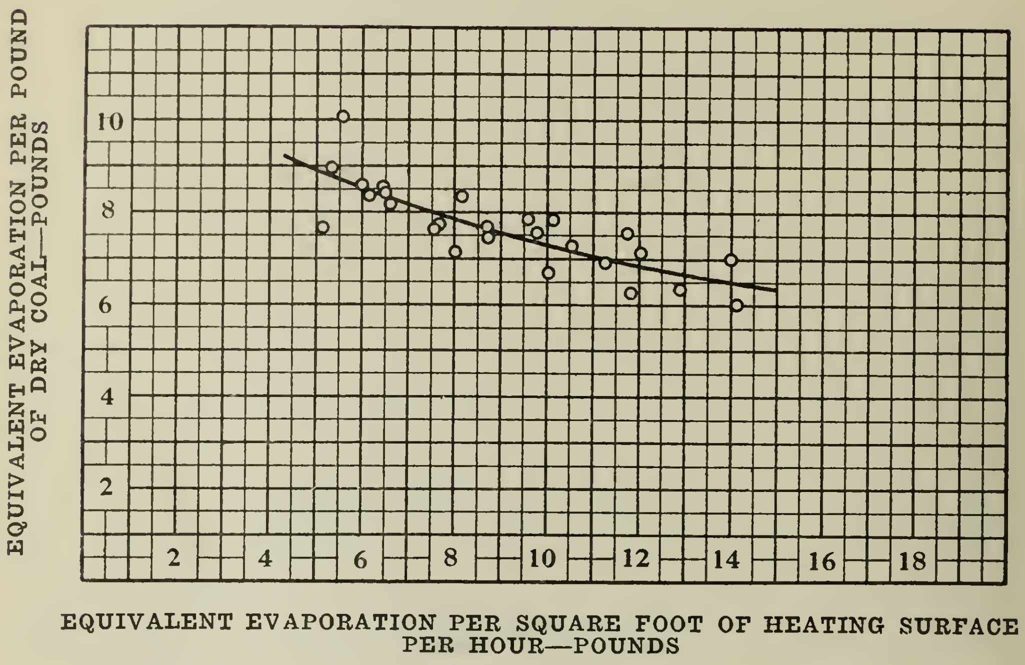

FIG. 33. THE RELATION BETWEEN EVAPORATION PER POUND OF COAL AND RATE OF EVAPORTION.

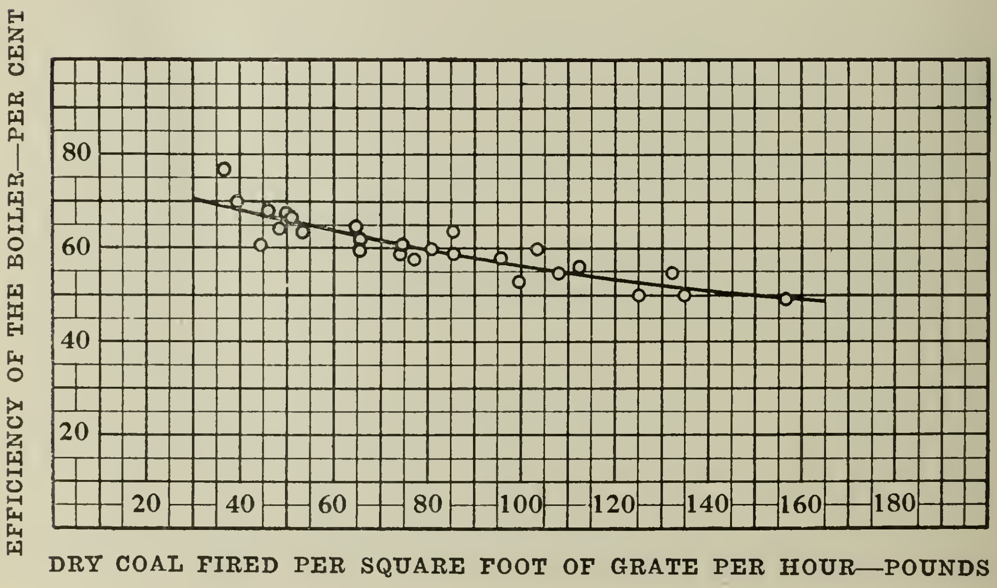

FIG. 34. THE RELATION BETWEEN BOILER EFFICIENCY AND RATE OF COMBUSTION.

## E. Engine Performance and General Performance.

The more important data and results pertaining to the performance of the engines and to the general performance of the locomotive during Series 1 are assembled in Table 10. The remaining data and results appear in Appendix 4. In the table all tests run at like speed are grouped and, within these groups, the tests are arranged in the order of the values of cut-off. The tests of Series 1 were made at speeds varying from 10 to 35 miles per hour, and at cut-offs of 16, 20, 24, 32, 40, and 48 per cent. Only one test however was made at 20 per cent cut-off and one at 48 per cent cut-off, and these are omitted from the figures here included.

29. *Dry Steam per Indicated Horse Power Hour.*—In Fig. 35 is shown the relation between steam consumption and speed for each of the four cut-offs. The curves here drawn show the minimum steam consumption to have occurred in each case at a speed of from 20 to 25

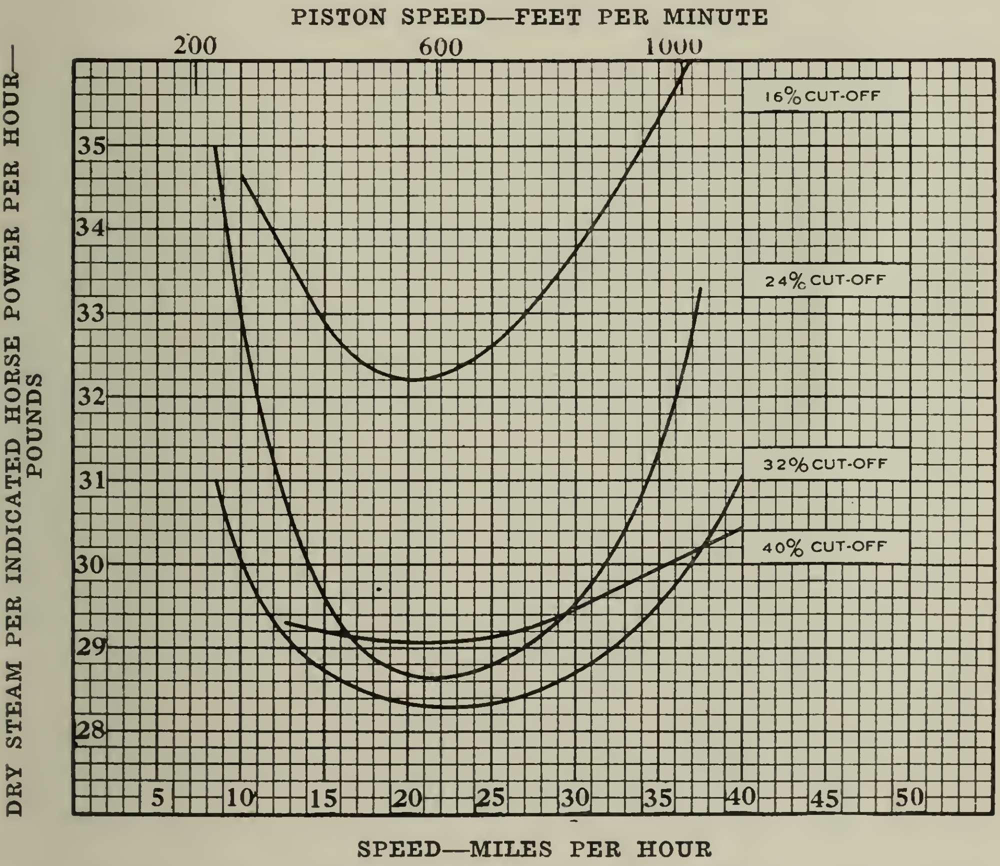

FIG. 35. THE RELATION BETWEEN STEAM CONSUMPTION AND SPEED, AT VARIOUS CUT-OFFS.

## TABLE 10.

## ENGINE AND GENERAL PERFORMANCE—SERIES 1.

| Test No. | Laboratory Designation | Duration of Test, Minutes | Speed in Miles per Hour | Piston Speed in Feet per Minute | Average Cut-off, Per cent of Stroke | Average Least Back Pressure, lb. per sq. in. | Indicated Horse Power, Total | Dry Steam Consumed per I.H.P. per Hour, lb. | Drawbar Horse Power | Machine Efficiency of Locomotive, per cent | Efficiency of Locomotive, per cent |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Code Item☞ | | 353 | 354 | 499 | 615 | 711 | 736 | 743 | 778 | 779 |
| 2025 | 55–16–F | 120 | 9.4 | 257.0 | | | 301.6 | 47.16 | 231.7 | 76.8 | 3.11 |
| 2024 | 55–24–F | 120 | 9.2 | 253.0 | | | 431.0 | 34.74 | 355.6 | 82.5 | 3.97 |
| 2028 | 55–32–F | 140 | 9.1 | 251.0 | 31.7 | | 548.7 | 30.57 | 488.1 | 89.0 | 4.10 |
| 2017 | 83–16–F | 180 | 14.5 | 399.2 | 16.9 | 2.6 | 428.1 | 34.08 | 357.1 | 83.4 | 3.76 |
| 2021 | 83–16–F | 120 | 14.5 | 399.7 | 14.9 | 4.4 | 428.6 | 32.10 | 346.3 | 80.8 | 3.31 |
| 2020 | 83–24–F | 120 | 14.6 | 401.7 | 22.4 | 5.1 | 584.4 | 29.99 | 508.9 | 87.1 | 4.27 |
| 2018 | 83–24–F | 160 | 14.5 | 399.2 | 22.6 | 4.0 | 593.8 | 29.71 | 511.7 | 86.2 | 4.21 |
| 2019 | 83–32–F | 130 | 14.6 | 402.0 | 30.6 | 5.2 | 765.7 | 28.96 | 683.1 | 89.2 | 4.35 |
| 2022 | 83–32–F | 120 | 14.6 | 402.2 | 30.3 | 7.2 | 773.9 | 28.14 | 674.6 | 87.2 | 3.94 |
| 2031 | 83–40–F | 90 | 15.6 | 428.1 | 39.5 | 10.3 | 953.9 | 28.75 | 869.7 | 91.2 | 4.38 |
| 2026 | 110–16–F | 130 | 19.9 | 545.9 | 19.1 | 3.1 | 515.0 | 31.67 | 415.1 | 80.6 | 3.75 |
| 2027 | 110–24–F | 150 | 20.0 | 548.9 | 24.3 | 6.1 | 749.1 | 27.84 | 633.6 | 84.6 | 3.91 |
| 2029 | 110–32–F | 90 | 19.9 | 547.9 | 30.7 | 10.5 | 968.6 | 27.51 | 820.8 | 84.7 | 3.94 |
| 2035 | 110–40–F | 70 | 20.3 | 557.9 | 39.9 | 14.7 | 1119.1 | 29.43 | 942.9 | 84.3 | 3.49 |
| 2033 | 110–48–F | 80 | 20.0 | 548.4 | 40.7 | 15.2 | 1142.3 | 27.83 | 1007.9 | 88.2 | 4.12 |
| 2009 | 138–16–F | 150 | 25.2 | 694.6 | 17.1 | 5.3 | 545.5 | 33.06 | | | |
| 2012 | 138–24–F | 110 | 25.3 | 696.3 | 23.3 | 7.7 | 802.8 | 29.44 | 684.9 | 85.3 | 3.84 |
| 2013 | 138–32–F | 90 | 25.4 | 697.2 | 30.8 | 11.6 | 986.3 | 29.14 | 863.7 | 87.6 | 3.78 |
| 2023 | 138–40–F | 90 | 25.2 | 693.6 | 39.4 | 19.5 | 1188.7 | 29.10 | 1070.5 | 90.1 | 3.36 |
| 2030 | 165–24–F | 100 | 30.8 | 845.3 | 23.4 | 11.5 | 899.6 | 28.99 | 725.8 | 80.7 | 3.62 |
| 2032 | 165–32–F | 40 | 30.5 | 838.8 | 30.2 | | 1094.6 | 27.84 | 922.8 | 84.3 | 3.61 |
| 2037 | 165–40–F | 60 | 30.7 | 844.8 | 40.1 | 18.1 | 1277.7 | 29.56 | 1045.3 | 81.8 | 3.29 |
| 2016 | 193–16–F | 140 | 36.3 | 998.0 | 16.4 | 6.4 | 583.8 | 35.18 | 418.2 | 71.6 | 2.74 |
| 2010 | 193–20–F | 71 | 35.7 | 981.8 | 19.2 | 8.2 | 737.0 | 32.07 | | | |
| 2015 | 193–24–F | 90 | 36.3 | 998.0 | 22.7 | 12.1 | 845.6 | 32.32 | 626.2 | 74.1 | 2.63 |
| 2014 | 193–32–F | 70 | 36.3 | 996.4 | 31.4 | 17.1 | 1079.0 | 29.82 | 853.1 | 79.1 | 2.88 |
| 2034 | 193–40–F | 60 | 36.0 | 990.5 | 41.4 | 24.1 | 1276.7 | 30.02 | 961.7 | 75.3 | 2.67 |

miles per hour. The best performance was obtained during the tests made at 32 per cent cut-off. The minimum steam consumption was 27.51 pounds of dry steam per indicated horse power per hour, and the maximum 35.18 pounds. The difference between these quantities, 7.67 pounds, represents a comparatively small variation in water rate, considering the variety and range of the test conditions. Fig. 35 is comparable with Fig. 17, and a comparison of these figures shows the steam consumption during Series 1 to have been considerably greater than in Series 2, at all cut-offs.

30. *Indicated Horse Power.*—The relation of indicated horse power to speed is shown in Fig. 36, in which each of the curves represents all the tests made at a particular cut-off. This figure shows also the range of the test conditions for Series 1. The maximum load, 1278 indicated horse power, was developed during test 2037 when the speed was 30 miles per hour and the cut-off 40 per cent. The minimum average load, 428 indicated horse power, occurred in test 2017 at a speed of 15 miles per hour and 16 per cent cut-off. The work performed at the locomotive drawbar varied from 346 to 1071 horse power.

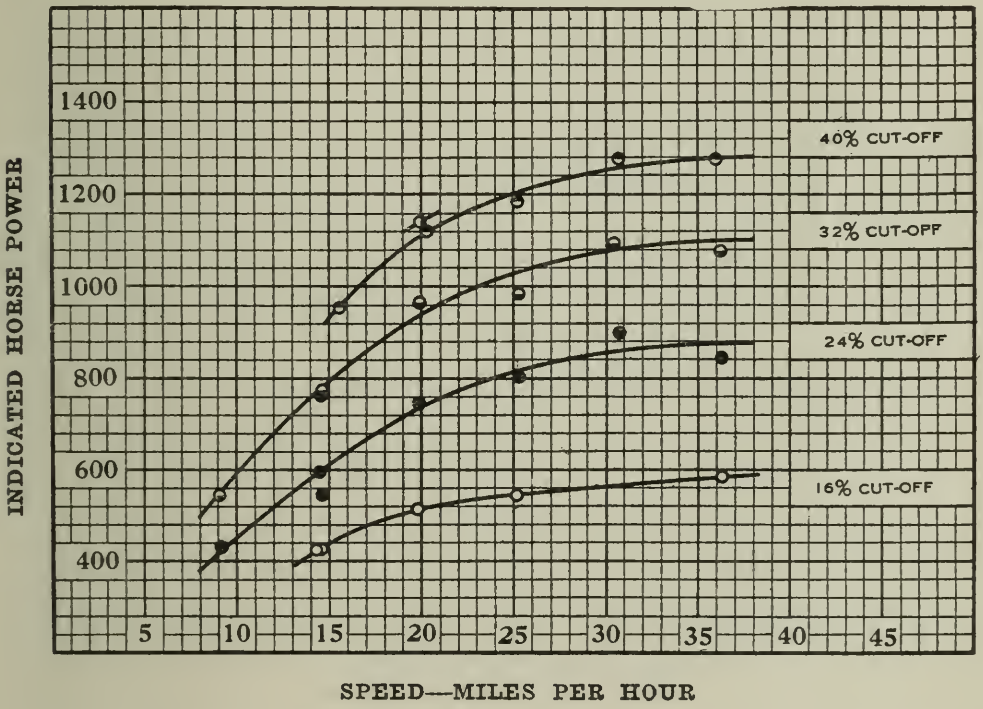

FIG. 36. THE RELATION BETWEEN INDICATED HORSE POWER AND SPEED, AT VARIOUS CUT-OFFS.

31. *Machine Friction.*—Machine friction horse power during Series 1 and its relation to speed are shown in Fig. 37. This figure indicates also the influence of cut-off on machine friction, and is comparable with Fig. 27 which presents the same relations for Series 2.

An analysis of machine friction similar to that developed in section VI for Series 2 has been made for Series 1, although the figures are not here included. This analysis shows that during Series 1 the machine friction was somewhat greater than during Series 2—a result which was expected, in view of the work done upon the machinery between these two groups of tests. This analysis indicates also that for constant cut-off the machine friction increased more rapidly with increasing load during Series 1 than during Series 2. With these two exceptions, the relations shown in the discussion of machine friction for Series 2, and the conclusions there drawn, remain substantially the same for Series 1.

32. *Coal Consumption per Indicated Horse Power.*—The dry coal consumed per indicated horse power per hour, and the relation of this coal consumption to speed are shown in the curves of Fig. 38. The curves show this coal rate to vary in general between 4 and 6 pounds per hour for the range of conditions which prevailed in Series 1. Again excluding test 2024, the lowest coal rate, 4.18 pounds per hour, was obtained during test 2019 at 15 miles per hour and 32 per cent cut-off. The highest coal rate, 6.07 pounds per hour, was obtained during test 2034, at 35 miles per hour and 40 per cent cut-off. Test 2034 is the one during which the lowest boiler efficiency prevailed.

33. *General Efficiency.*—The general efficiency of the locomotive and the relation of this efficiency to speed are shown in Fig. 39, which is comparable with Fig. 31 of Series 2. The greatest efficiency was 4.38 per cent, and the lowest 2.63 per cent. This is practically the same range in efficiency as is represented in Fig. 31.

## VIII. Comparison of the Results of Series 1 and 2.

A few of the differences in the performance of the locomotive during the tests of Series 1 and Series 2 have already been referred to incidentally in the preceding discussion. It is the purpose to discuss in this section the effects on the general performance of the boiler and engines caused by the repairs and changes which were made between these two groups of tests. While these changes are elsewhere enumer-

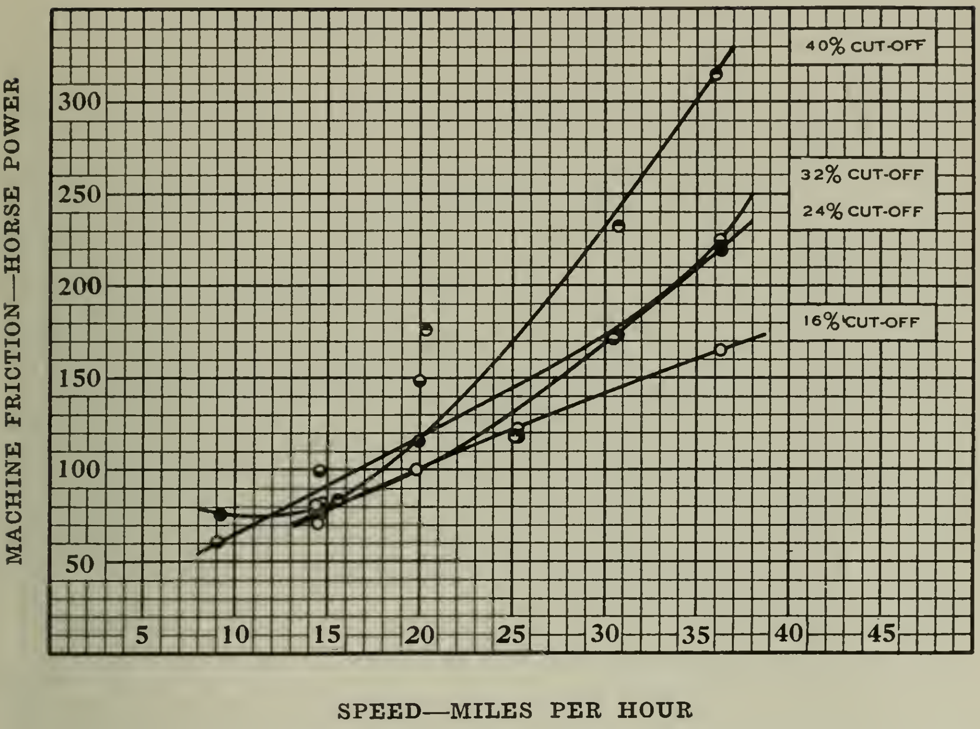

Fig. 37. The Relation Between Machine Friction Power and Speed, at Various Cut-offs.

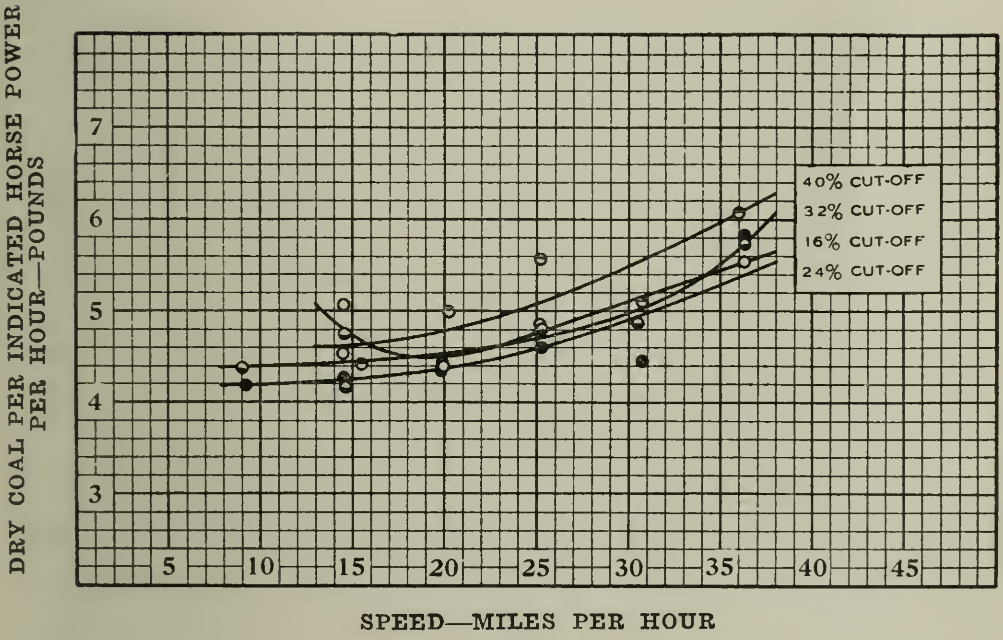

Fig. 38. The Relation Between Coal Consumed per Indicated Horse Power per Hour and Speed, at Various Cut-offs.

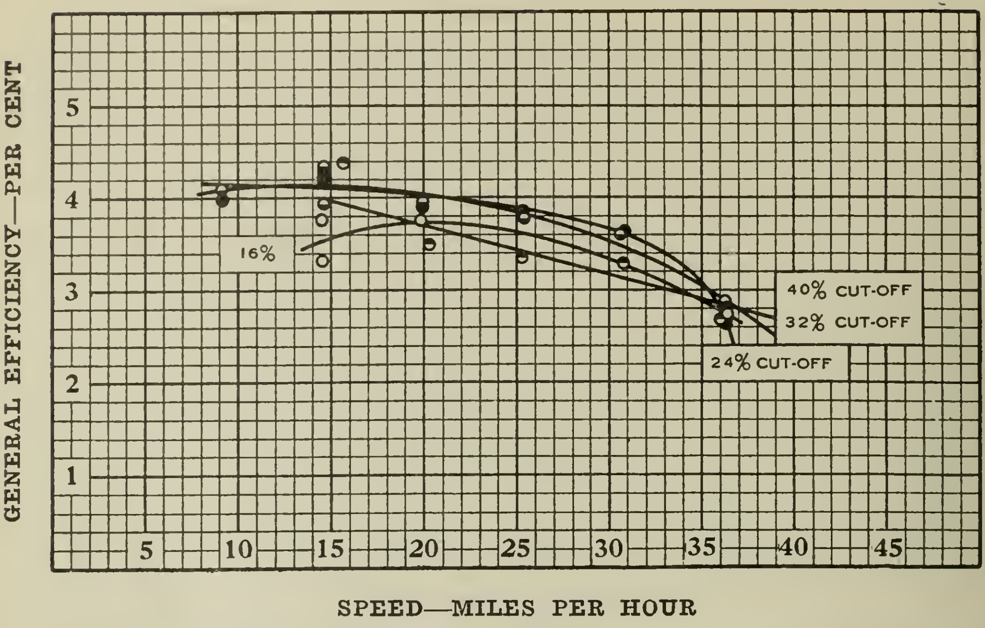

FIG. 39. THE RELATION BETWEEN GENERAL EFFICIENCY AND SPEED, AT VARIOUS CUT-OFFS.

ated, it will be convenient to have restated at this point those which might have affected the general performance. They were:

1. The nozzle tip, 5¼ in. in diameter, used during Series 1 was replaced by a 5⅞-in. tip.
2. A small leak in one of the branch-pipe joints was stopped.
3. The cylinders and valve chambers were re-bored.
4. New pistons and new piston rings were applied.
5. New valve bull-rings and valve packing rings were applied.
6. Lost motion in the eccentric straps was taken up.
7. The valves were reset.
8. The piston rods were trued and the rod packing renewed on both sides.
9. The side-rod bushings were renewed.
10. Three new driving-wheel tires were applied.

34. *Comparison of the Boiler Performance.*—Perhaps the best basis for a comparison of the boiler performance during the two series is to be found in the curves of Fig. 8 and 32, which present the average values of the equivalent evaporation per pound of dry coal per hour for Series 2 and Series 1 respectively. These two curves, when plotted on the same diagram, almost coincide. The curve for Series 1 lies below that for Series 2 by an amount which at no rate of combus-

tion exceeds one-fifth of a pound of equivalent evaporation per pound of coal. At no point throughout the range of the rate of combustion is the performance of the boiler in Series 2 better than in Series 1 by more than 3 per cent. A comparison of boiler efficiency based on Fig. 10 and 34 shows an even closer agreement in the boiler performance of the two series. Of the various changes cited above only item 1, the change in nozzle tip, could have affected boiler performance, and the facts just stated seem to warrant the conclusion that this had no substantial effect upon the general performance of the boiler and furnace.

35. *Comparison of the Cylinder Performance.*—Any of the first seven items in the list of changes given might conceivably have affected the cylinder performance. The steam leak referred to in item 2, however, was proved at the time to have been inconsiderable in amount. The combined effect of these seven items should be disclosed by a comparison of the steam consumption per indicated horse power hour for all tests of both series which are comparable as regards speed and cut-off. The values of steam consumption for such tests are brought together in the following table. In four instances in this table a pair of tests from Series 2 is compared with a single test from Series 1, and in these cases the water rate presented for Series 2 is the average rate for the pair.

TABLE 11.
STEAM CONSUMPTION FOR COMPARABLE TESTS OF SERIES 1 AND 2.

| Speed, m. p. h. | Cut-off, per cent | Test Numbers | | Steam Consumption, lb. per i. h. p. hour | | Difference in Steam Consumption, Percentage of Consumption for Series 2. |
|---|---|---|---|---|---|---|
| | | Series 1 | Series 2 | Series 1 | Series 2 | |
| 10 | 24 | 2024 | 2081<br>2086 | 34.74 | 31.35 | 10.8 |
| ,, | 32 | 2028 | 2075<br>2097 | 30.57 | 28.40 | 7.6 |
| 20 | 16 | 2026 | 2080<br>2087 | 31.67 | 30.76 | 2.9 |
| ,, | 24 | 2027 | 2077 | 27.84 | 27.36 | 1.8 |
| ,, | 32 | 2029 | 2073 | 27.51 | 27.20 | 1.1 |
| ,, | 40 | 2035 | 2072 | 29.43 | 27.19 | 8.2 |
| ,, | 48 | 2033 | 2084 | 27.83 | 28.69 | –3.0 |
| 30 | 24 | 2030 | 2078 | 28.99 | 28.09 | 3.2 |
| ,, | 32 | 2032 | 2074<br>2092 | 27.84 | 27.25 | 2.2 |
| ,, | 40 | 2037 | 2082 | 29.56 | 27.88 | 6.0 |
| | | | | | Average ...... | 4.1 |

Except for the two tests run at 20 miles per hour and 48 per cent cut-off, all the tests of Series 2 show a lower steam consumption than the corresponding tests of Series 1. The differences in steam consumption are shown in the last column of the table, where they are expressed in percentages of the water rate for Series 2. With the one exception cited, the improvement in water rate ranged from 1.1 to 10.8 per cent and the average improvement for all tests amounted to 4.1 per cent. Neither this average nor the range is very great, and these facts emphasize the statement previously made that all the repairs and changes were such as would have been regarded as unnecessary under ordinary conditions, and that they were resorted to only that nothing which would probably improve performance should be left undone.

In the data for tests 2090 and 2091 means are at hand for roughly differentiating the effect of the changes in nozzle tip from the effect of the other changes. It will be recalled that during these two tests the conditions were identical with those prevailing during the tests of Series 2, except that the 5⅞-in. nozzle tip used in Series 2 was replaced by the 5¼-in. tip which had been used in Series 1. A comparison of the water rates for these two tests with the water rates for the comparable tests of Series 2 is exhibited below.

TABLE 12.
COMPARISON OF WATER RATE WITH 5⅞-IN. NOZZLE TIP AND 5¼-IN. NOZZLE TIP.

| Speed, m. p. h. | Cut-off, per cent | Test Numbers | | Steam Consumption, lb. per i. h. p. hour | | Difference in Steam Consumption, Percentage of Consumption for Series 2 |
|---|---|---|---|---|---|---|
| | | Tests with 5¼-in. Tip | Tests with 5⅞-in. Tip, Series 2 | Tests with 5¼-in. Tip | Tests with 5⅞-in. Tip, Series 2 | |
| 20 | 24 | 2090 | 2077 | 28.99 | 27.36 | 6.0 |
| 30 | 32 | 2091 | 2074<br>2092 | 29.10 | 27.25 | 6.8 |

It is apparent that in both tests 2090 and 2091, with the smaller nozzle tip, the water rate was higher than in the corresponding tests of Series 2 in which the larger tip was used. The average difference based on the tests of Series 2 is 6.4 per cent. In view of the range in the differences in water rate for all tests of the two series, which is exhibited in the first table in this section, it is doubtless unsafe to draw too sweeping a conclusion from a showing which rests on a comparison of two pairs of tests only. Since, however, the average difference in steam consumption for all tests of both series was only 4.1 per cent, and since such information as is available concerning the effect of the change in nozzle tip shows that it made an average change in steam consumption of 6.4 per cent; the inference is perhaps warranted, that practically all the improvement effected by the changes and repairs was accomplished by the increase in the size of the exhaust nozzle tip, through its influence on back pressure.

# PART II.

## APPENDIX I.

### THE LOCOMOTIVE.

Illinois Central Railroad locomotive 958 is of the consolidation type. It was built by the Baldwin Locomotive Works in December, 1909, and in the classification of the Associated Lines is designated as C — 63 — $\frac{22}{30}$ — 39.2. The locomotive uses saturated steam in simple cylinders twenty-two inches in diameter by thirty inches stroke, weighs 223 000 pounds, and has a rated tractive effort of 39 180 pounds. This tractive effort assumes a driving wheel diameter of 63 inches. The drivers however had been turned to 61 inches for which the rated tractive effort would be 40 470 pounds. The general design of the locomotive is shown in Fig. 40 and 42, and its appearance in service is shown in Fig. 1. The period of its service and its mileage have been stated in section II.

*Repairs and Changes.*—The repairs and changes which have been referred to in Part I were as follows.

After test No. 2037. The valves were reset; the valve and cylinder packing-rings were renewed; lost motion in the eccentric straps was taken up; the piston-rod packing was renewed; cylinder cocks were repaired or renewed; side-rod bushings were renewed; a new injector was applied; the boiler seams were caulked; three new tires were applied; and certain minor adjustments were made in order to take up lost motion.

After test No. 2045. The cylinders and the valve chambers were rebored; the pistons and piston packing-rings were renewed; the valve bull-rings and packing-rings were renewed; the piston rods were trued; a small leak in one of the branch pipe joints was stopped; and the $5\frac{1}{4}$-in. nozzle tip previously in use was replaced by a $5\frac{7}{8}$-in. tip. The $5\frac{1}{4}$-in. tip was again used during tests 2090 and 2091.

*The Boiler.*—The boiler, which carried a working pressure of 200 pounds, was of the crown-bar type, with straight top and wide firebox. Its general design appears in Fig. 41, 43, and 44. The principal dimensions of the boiler are given in the following list.

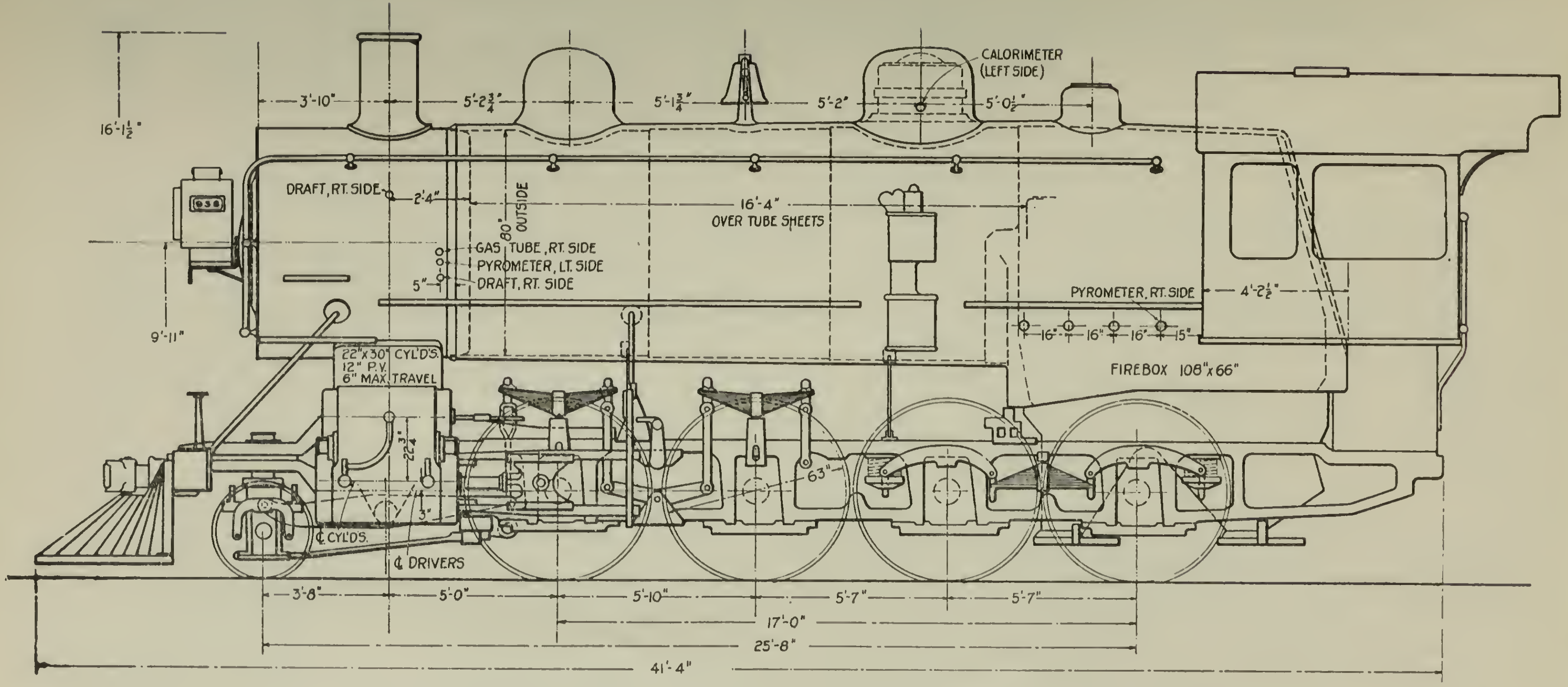

Fig. 40. Side Elevation of Illinois Central Railroad Locomotive 958.

| | | |
|---|---|---|
| Outside diameter of first ring *(205)** | 80 | inches |
| Thickness of sheets: | | |
| Cylindrical courses | ¾ | ” |
| Wrapper sheet | ⅝ | ” |
| Flue sheets | ½ | ” |
| Firebox sheets | ⅜ | ” |
| Number of tubes *(211)* | 413 | |
| Outside diameter of tubes *(212)* | 2 | inches |
| Thickness of tubes *(213)* | 0.133 | ” |
| Length between tube sheets *(214)* | 195.2 | ” |
| Length of firebox, inside *(234)* | 108.25 | ” |
| Width of firebox, inside *(235)* | 66.13 | ” |
| Volume of firebox *(238)* | 244 | cu. ft. |
| Heating surface of the tubes, fire side *(272)* | 3094 | sq. ft. |
| Heating surface of front tube sheet *(277)* | 21 | ” ” |
| Heating surface of the firebox, fire side *(273)* | 168 | ” ” |
| Total heating surface, fire side *(275)* | 3283 | ” ” |
| Water space in the boiler *(282)* | 429 | cu. ft. |
| Steam space in the boiler *(283)* | 107 | ” ” |
| Grates, of the interlocking finger type, area *(252)* | 49.55 | sq. ft. |

*The Cylinders and Valves.*—The cylinders and valve chambers were of cast-iron with cast iron bushings. The pistons were of the box type, cast in one piece. The steam distribution was controlled by piston valves with inside admission. The nominal piston travel was 6 inches, and the actual travel 5-55/64 inches on the right side, and 5-27/32 on the left side. The valves were set with 1/32 inch lead in full gear and were actuated by an indirect shifting link motion.

The principal cylinder and valve dimensions during the various tests included in this report were as follows:

| | During Tests 2009-2045 | During Tests 2072-2098 |
|---|---|---|
| Cylinder Diameter | | |
| Right side *(68)* | 22.071 inches | 22.107 inches |
| Left side *(69)* | 22.282 ” | 22.314 ” |
| Valve Chamber Diameter | | |
| Right side | 12.031 ” | 12.102 ” |
| Left side | 12.020 ” | 12.078 ” |

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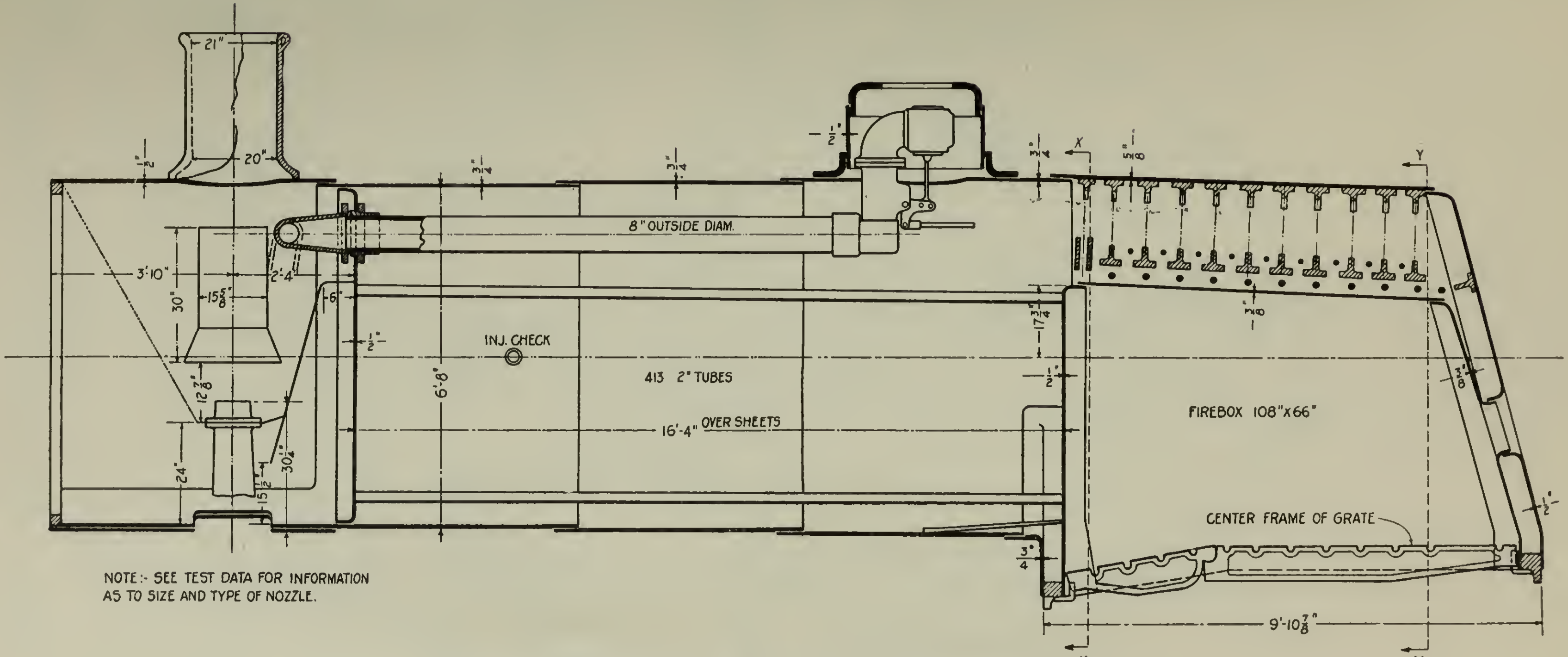
*Code item number.



NOTE:- SEE TEST DATA FOR INFORMATION AS TO SIZE AND TYPE OF NOZZLE.

FIG. 41. LONGITUDINAL SECTION THROUGH THE BOILER.

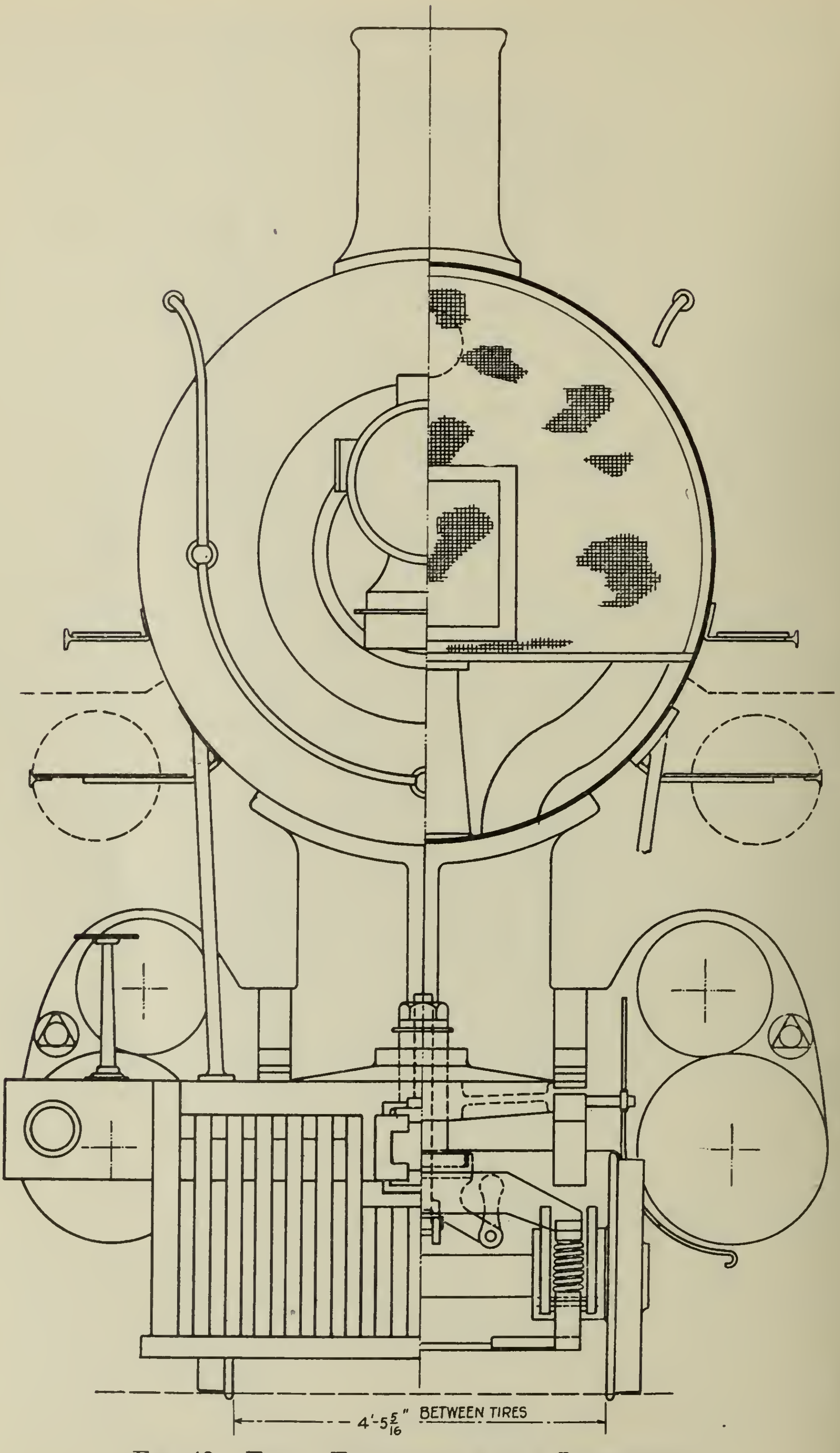


Fig. 42. Front Elevation of the Locomotive.

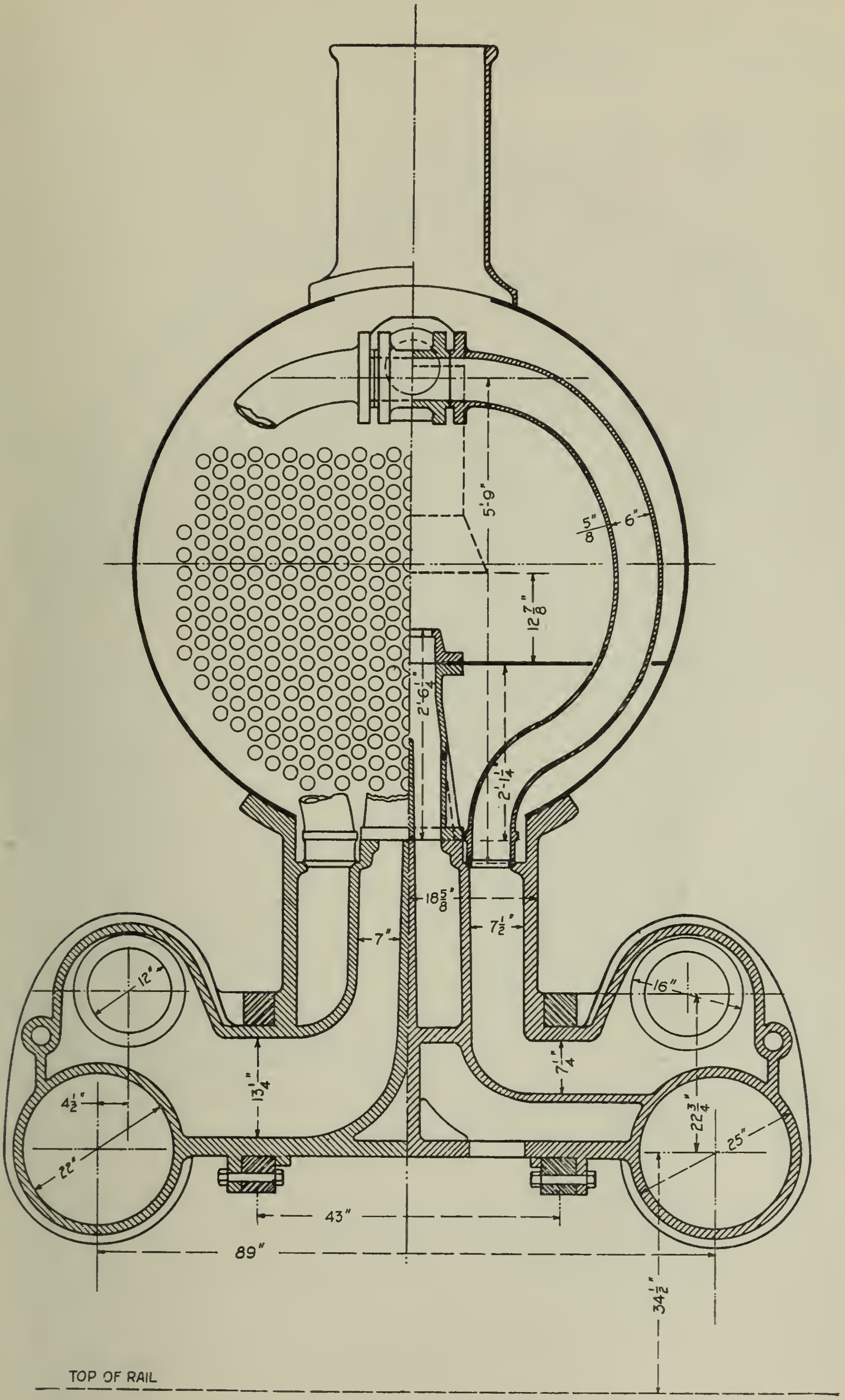
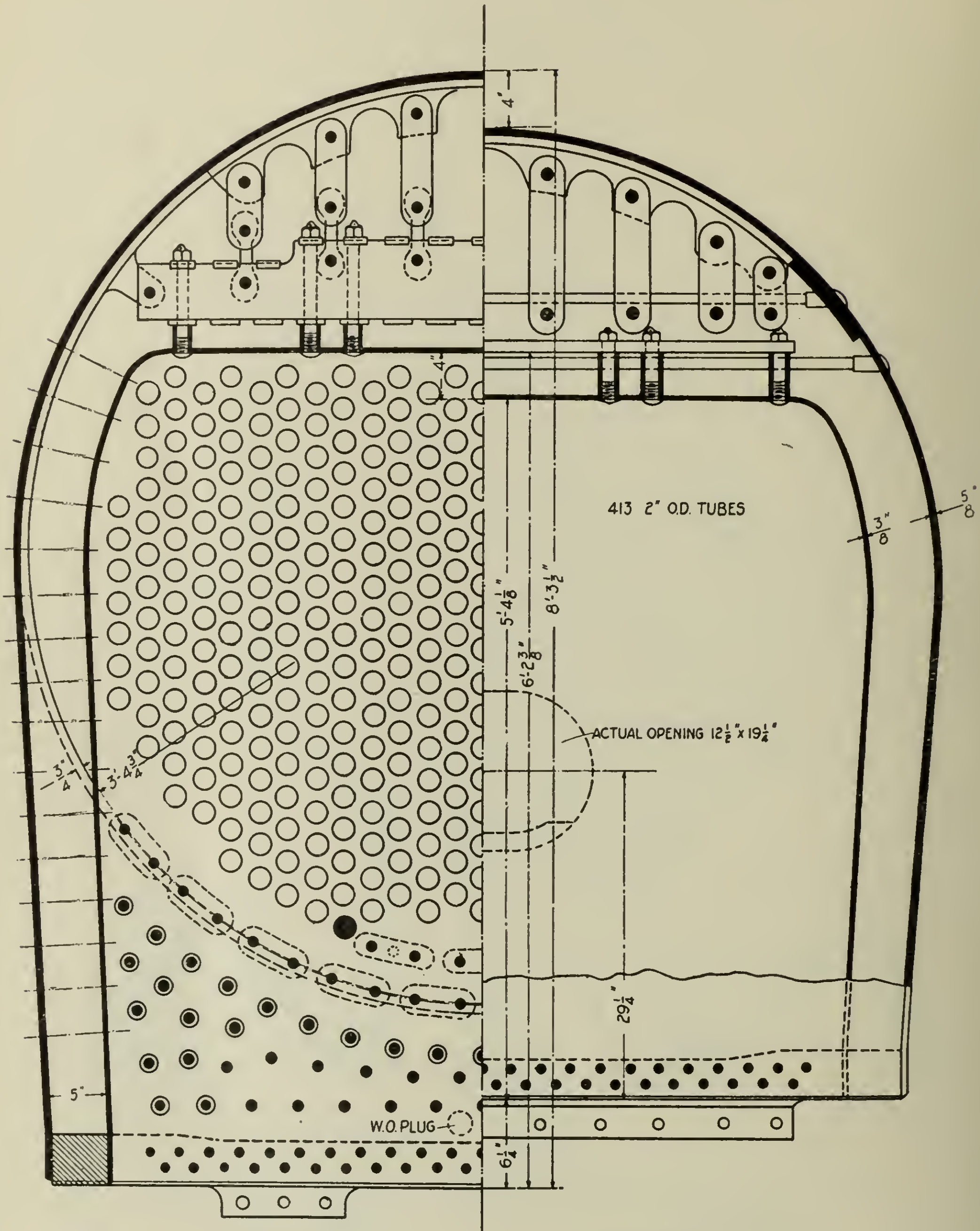


Fig. 43. Cross Section Through the Front-end and Cylinders.



SECTION AT X-X

SECTION AT Y-Y

FIG. 44. CROSS SECTIONS THROUGH THE FIREBOX

| | During Tests 2009-2045 | | During Tests 2072-2098 | |
|---|---|---|---|---|
| Piston Stroke | | | | |
| Right side *(77)** | 29.92 | inches | 29.94 | inches |
| Left side *(78)* | 29.94 | ” | 29.94 | ” |
| Piston Rod Diameter | | | | |
| Right side *(135)* | 3.917 | ” | 3.907 | ” |
| Left side *(136)* | 3.928 | ” | 3.907 | ” |
| Cylinder Volume (both sides) | 13.199 | cu. ft. | 13.199 | cu. ft. |

| | During Tests 2009-2037 | | During Tests 2038-2098 | |
|---|---|---|---|---|
| Clearance Volume | | | | |
| Ride side, head end *(86)* | 9.89 | per cent | 11.42 | per cent |
| Right side, crank end *(87)* | 9.74 | ” | 11.01 | ” |
| Left side, head end *(88)* | 9.73 | ” | 10.60 | ” |
| Left side, crank end *(89)* | 10.06 | ” | 11.01 | ” |

*General Dimensions.*—The principal general dimensions not already cited are shown in the following list.

| | |
|---|---|
| Total weight in working order *(63)* | 223 000 lb. |
| Weight on drivers *(64)* | 200 900 ” |
| Weight on leading truck *(48)* | 22 100 ” |
| Weight of tender, loaded | 135 000 ” |
| Weight of locomotive and tender, in working order | 358 000 ” |
| Driving wheel base *(39)* | 17 ft. – 0 in. |
| Total wheel base *(41)* | 25 ft. – 8 in. |
| Driving wheel diameter, over tires (nominal) *(2)* | 63 in. |
| Truck wheel diameter *(27)* | 33½ in. |
| Driving journal, main | 10 in. x 12 in. |
| Driving journals, other | 9 in. x 12 in. |
| Truck journals | 6 in. x 10 in. |

The actual average driving wheel diameter was 61.01 inches during tests 2009–2037, and 61.03 inches during tests 2038–2098. The corresponding actual average circumferences *(code No. 19)* were 15.972 and 15.978 feet respectively. The principal ratios are given below. Where two values of the ratio appear, the first is based on nominal dimensions, the second on actual dimensions.

---

*Code item number.

$$\frac{\text{Weight on drivers}}{\text{Tractive effort}} = \frac{200\,900}{39\,180} = 5.12$$

$$\frac{\text{Weight on drivers}}{\text{Tractive effort}} = \frac{200\,900}{40\,470} = 4.96$$

$$\frac{\text{Total weight}}{\text{Tractive effort}} = \frac{223\,000}{39\,180} = 5.69$$

$$\frac{\text{Total weight}}{\text{Tractive effort}} = \frac{223\,000}{40\,470} = 5.51$$

$$\frac{\text{Tractive effort} \times \text{diameter of drivers}}{\text{Total heating surface}} = \frac{39\,180 \times 63}{3283} = 751.8$$

$$\frac{\text{Tractive effort} \times \text{diameter of drivers}}{\text{Total heating surface}} = \frac{40\,470 \times 61}{3283} = 751.8$$

$$\frac{\text{Firebox heating surface}}{\text{Total heating surface}} = \frac{168}{3283} = .0513$$

$$\frac{\text{Weight on drivers}}{\text{Total heating surface}} = \frac{200\,900}{3283} = 61.19$$

$$\frac{\text{Total weight}}{\text{Total heating surface}} = \frac{223\,000}{3283} = 67.92$$

$$\frac{\text{Heating surface}}{\text{Grate area}} = \frac{3283}{49.55} = 66.26$$

$$\frac{\text{Tube surface}}{\text{Firebox heating surface}} = \frac{3094}{168} = 18.41$$

$$\frac{\text{Total heating surface}}{\text{Cylinder volume}} = \frac{3283}{13.199} = 248.8$$

*Horse Power Constants.*—The constants used in computing the test results are as follows:

For dynamometer horse power (power developed when the speed is one revolution per minute and the pull is one pound) the constants are

For tests 2009 to 2037 *(318)**........................ .0004840
For tests 2038 to 2098 *(318)*........................ .0004842

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*Code item number.

For indicated horse power (power developed at one revolution per minute and one pound mean effective pressure) the constants are

| | Tests 2009-2045 | 2072-2098 |
|---|---|---|
| For right cylinder, head end *(319)** | .02893 | .02902 |
| For right cylinder, crank end *(320)* | .02802 | .02811 |
| For left cylinder, head end *(321)* | .02948 | .02957 |
| For left cylinder, crank end *(322)* | .02857 | .02866 |

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*Code item number.

# APPENDIX 2.

## THE LABORATORY.

The general purpose underlying the design of this and of all other locomotive laboratories is to provide means whereby the locomotive machinery may be run and the locomotive worked throughout its range of capacity, while the locomotive as a whole remains stationary; thus permitting all test measurements to be made with the degree of accuracy possible in a stationary power plant test.

The laboratory equipment consists of, first, a means for so supporting the locomotive that its driving wheels may be rotated and that the power developed may be absorbed and dissipated; second, a means for anchoring the locomotive when so mounted and for measuring the tractive effort developed; third, means for supplying and measuring coal and water; and finally, means for disposing of the waste gases and exhaust steam.

*The Building.*—The building in which the plant is housed is shown in Fig. 45. It is 40 feet wide and 115 feet long, with a height of 22 feet under the roof trusses. At the rear end of the building is a coal room, above which are a platform for the exhaust fan and a wash-room. A basement extends under all of the main floor, except under the space occupied by the coal room. The walls are laid up both inside and out with faced brick, the floors are of reinforced concrete, and the roof is of the same material covered with slate. With the exception of the coal room all portions of the building are served by a ten-ton traveling crane.

*Supporting Wheels and Axles.*—The supporting mechanism consists primarily of pairs of wheels, whose location may be adjusted to suit the wheel base of any locomotive. Fig. 51 shows the general design of wheels, axles, bearings, and bed-plates. The supporting element for each pair of locomotive drivers consists of an axle, two wheels, and two bearings. The supporting wheels are 52 inches in diameter, have plain 5-inch tires, and are pressed on 11½-inch axles.

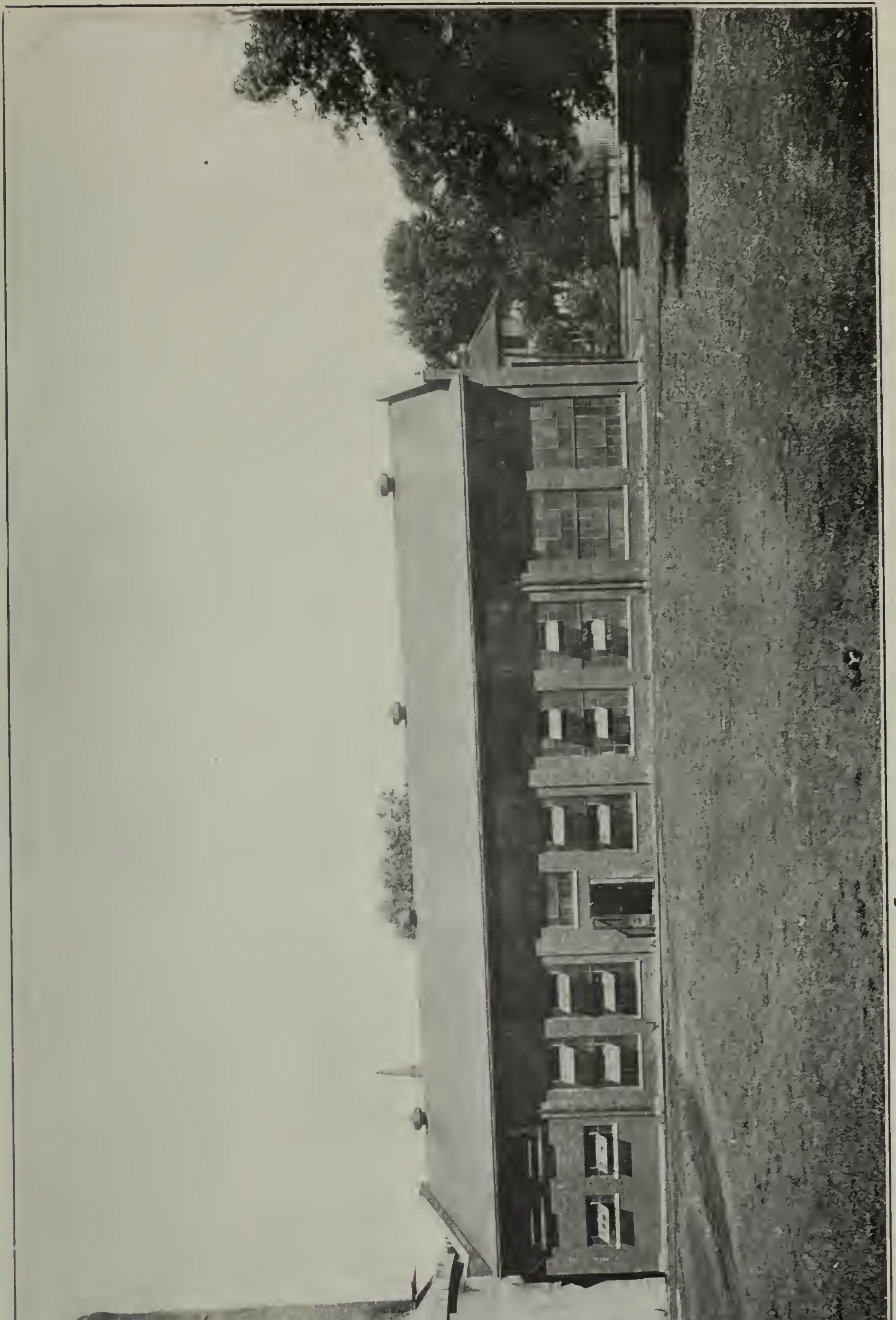


Fig. 45. The Locomotive Laboratory.

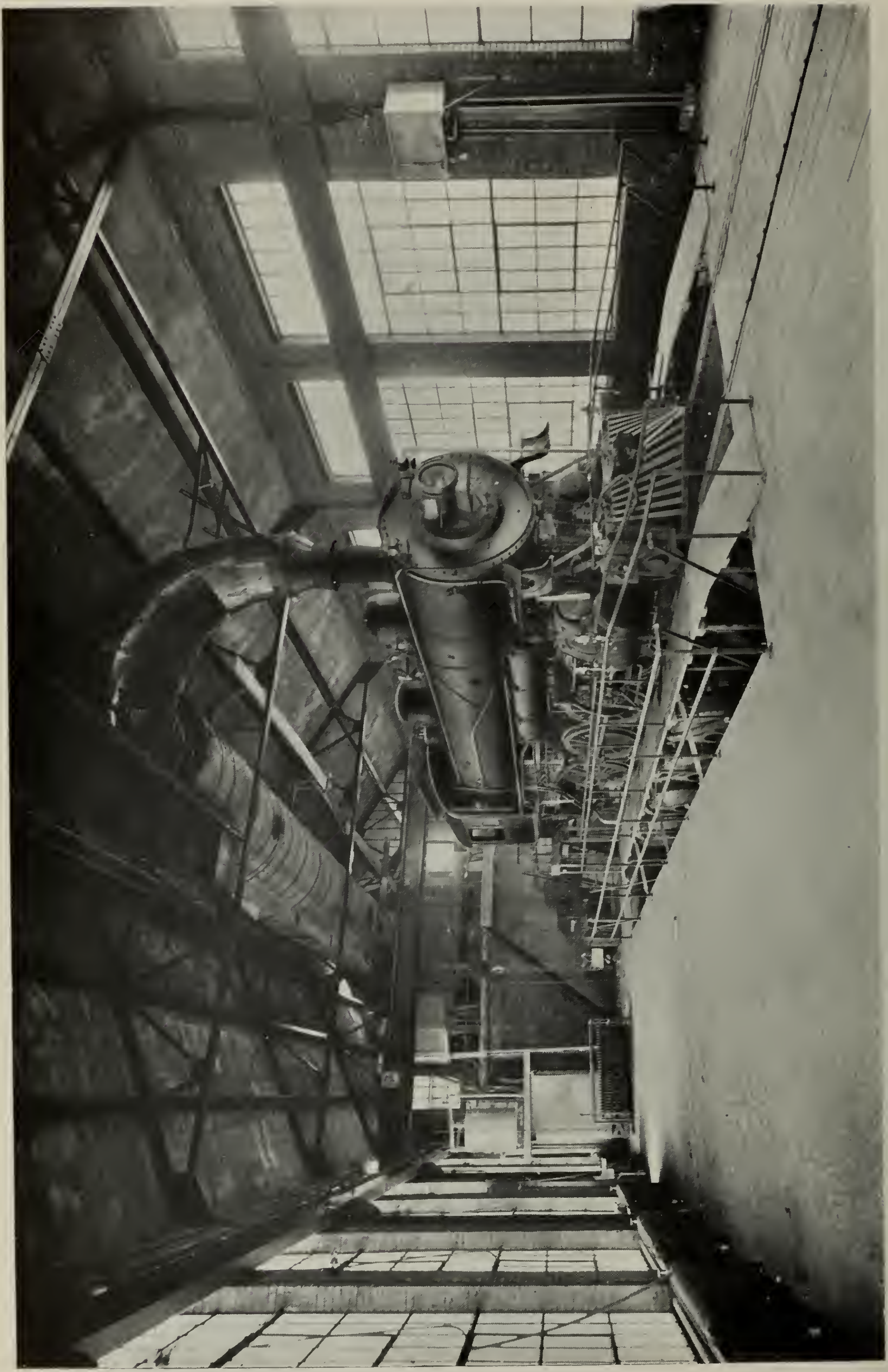


Fig. 46. An Interior View of the Laboratory with Locomotive 958 In Position.

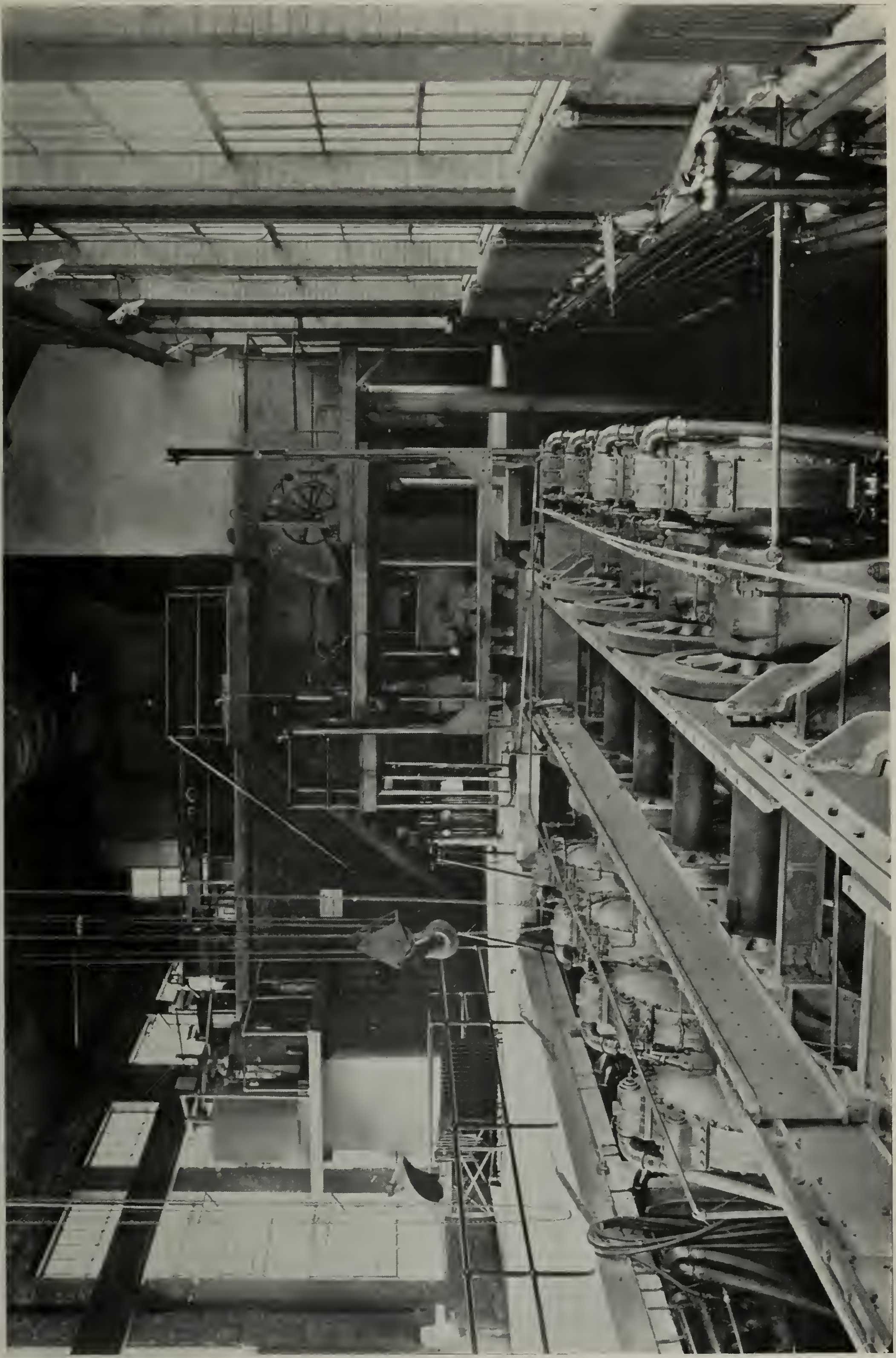


FIG. 47. THE REAR END OF THE TESTING PIT. SHOWING THE REMOVABLE TRACK, THE SUPPORTING WHEELS AND THEIR BEARINGS, AND THE BRAKES.

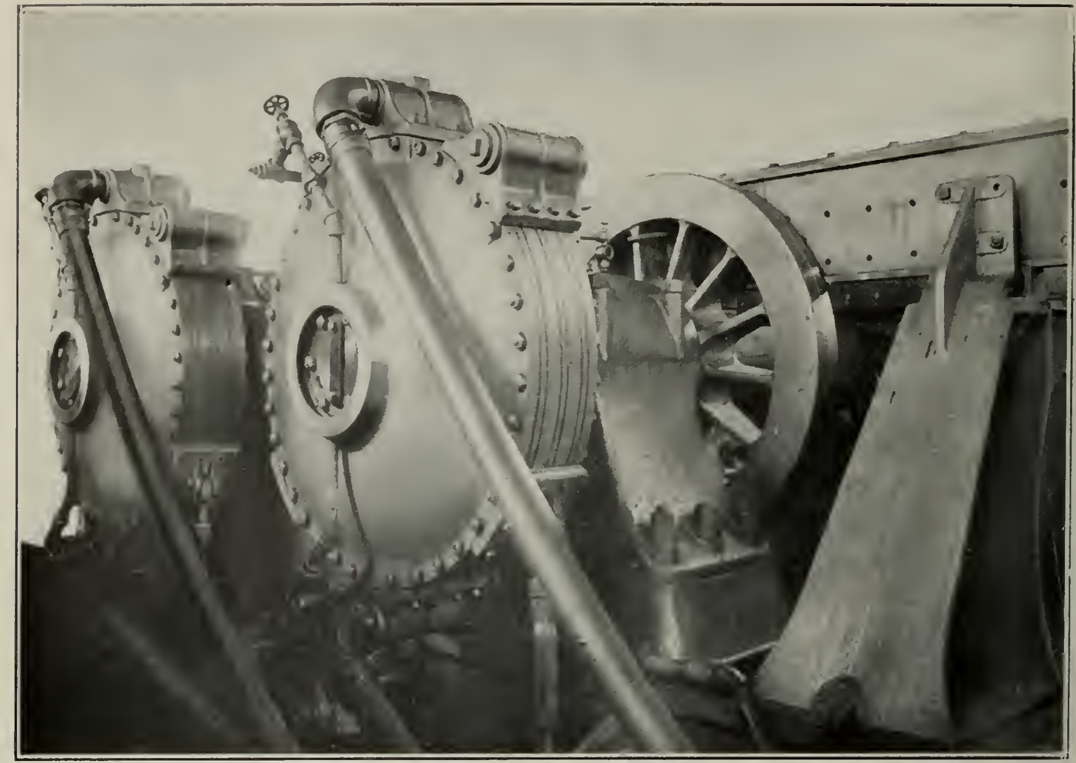


FIG. 48. AN EXTERIOR VIEW OF THE BRAKES.

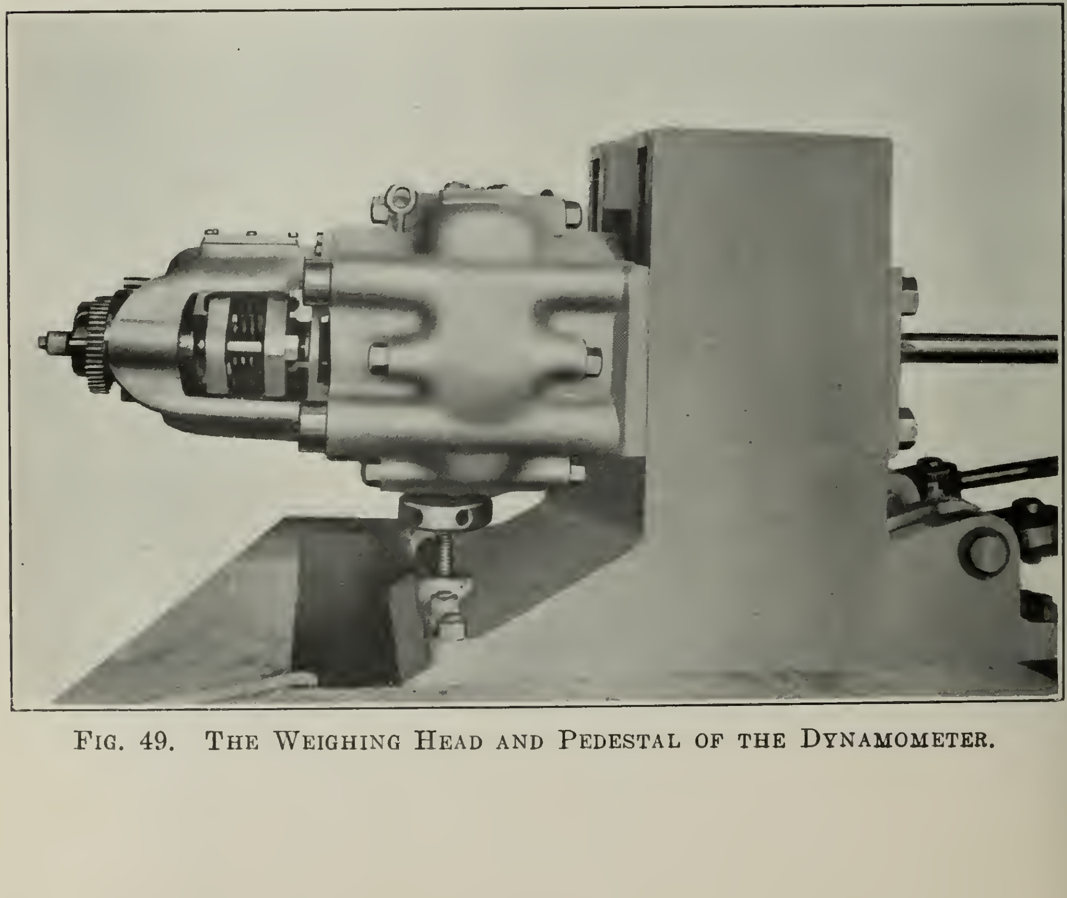


FIG. 49. THE WEIGHING HEAD AND PEDESTAL OF THE DYNAMOMETER.

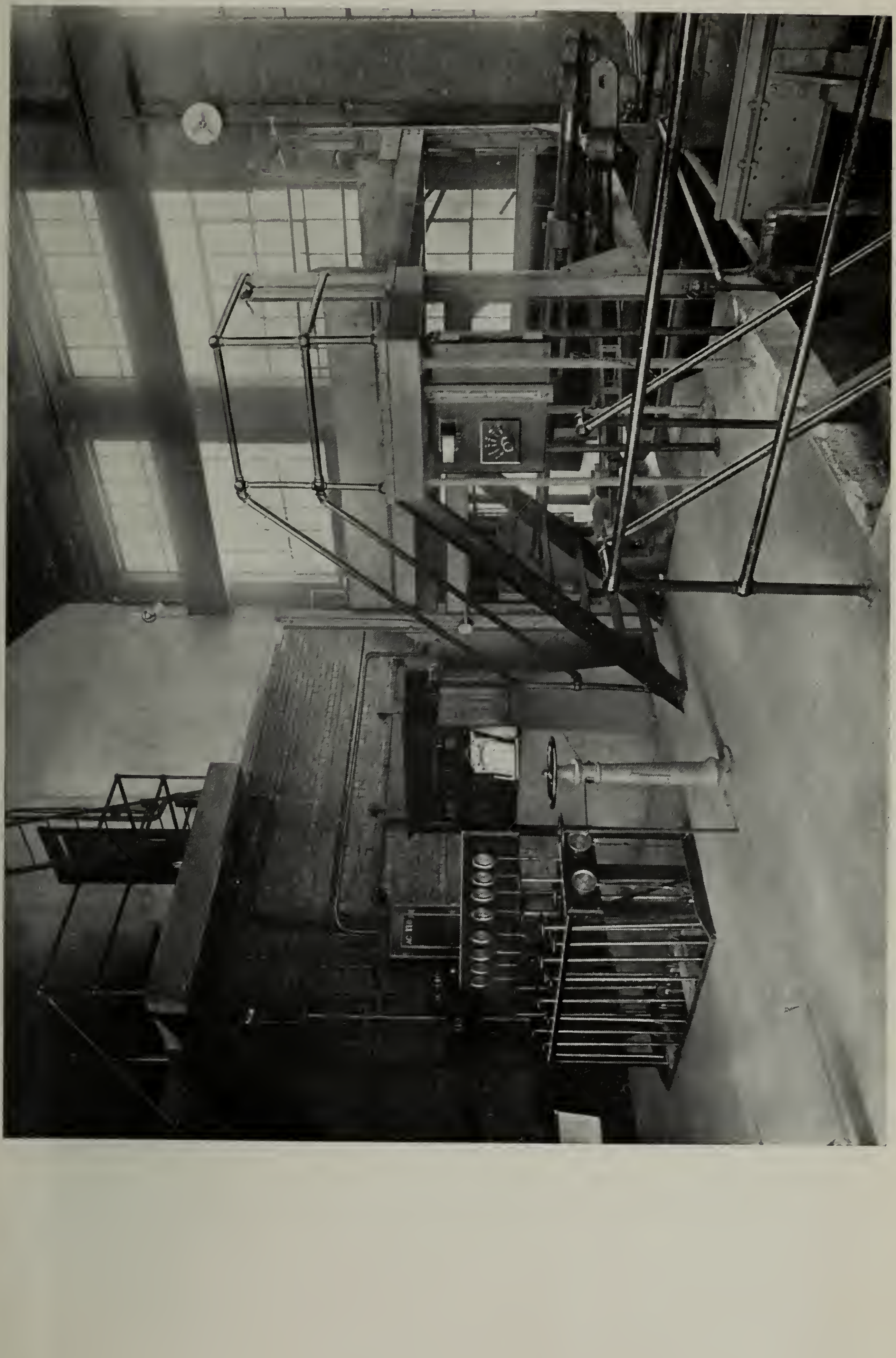


FIG. 50. THE BRAKE CONTROL VALVES, THE DYNAMOMETER SCALE, THE DRAWBAR AND SAFETY LINKS, AND THE FIRING PLATFORM.

The axles and tires are of the highest grade of heat-treated carbon steel and were donated by the Midvale Steel Company. Provision has been made for replacing the 52-inch wheels by 72-inch wheels for testing high speed locomotives, where the use of the smaller wheels would involve rotating speeds as high as 530 revolutions per minute.

*Bearings.*—The bearings for the supporting-wheel axles are self-aligning, their shells being carried in spherical sockets which form the upper part of the pedestal. The journals are 9½ inches by 20 inches, and the axles bear on the underside only. Oil for lubrication enters the bearing cap at two points and is supplied under head from an elevated tank. The pedestal is made in two parts, so that by removing the lower section, its height may be adjusted to provide for the 72-inch supporting wheels. This arrangement will continue to bring the top edge of the larger supporting wheels level with the outside track. The base of the pedestal is secured to a massive cast-iron bed-plate by T-bolts held in slots running the entire length of the bed. Each bed-plate consists of three sections placed end to end, 18 inches in height and 36 inches wide over all. The length of the present bed-plate is 42 feet, which provides for a maximum driving-wheel base of 36 feet, and the foundation is built to receive two more 14-foot sections of bed-plate. The supporting machinery rests on a concrete foundation 12 feet wide and 93 feet long, which varies in thickness from 3½ feet at the front to 5 feet at the rear end. The rear end is surmounted by a pyramidal base of reinforced concrete, to which the dynamometer is bolted.

*Hydraulic Brakes.*—Supported in this way the driving wheels are free to turn and the power produced at the driving wheel rim is absorbed by means of the brakes shown in Fig. 47, 48, and 52. One of these brakes is mounted on each end of each supporting-wheel axle. Each brake consists essentially of three cast iron discs (C, Fig. 52) which, bolted to the cast iron hub (F), are keyed to the supporting axle and form with it an integral revolving element. These three discs rotate between 1/16-inch copper diaphragms (D), bolted to the rim of a stationary housing (H), and flanged over the edges of the floating rings (E) and of the housing, to which they are secured by means of the expanding rings (G). The housing is prevented from turning by means of the links (L) attached to the bed-plate. The rubbing surfaces of the discs and diaphragms are lubricated with a medium grade of cylinder oil which enters the brake under pressure through the oil-

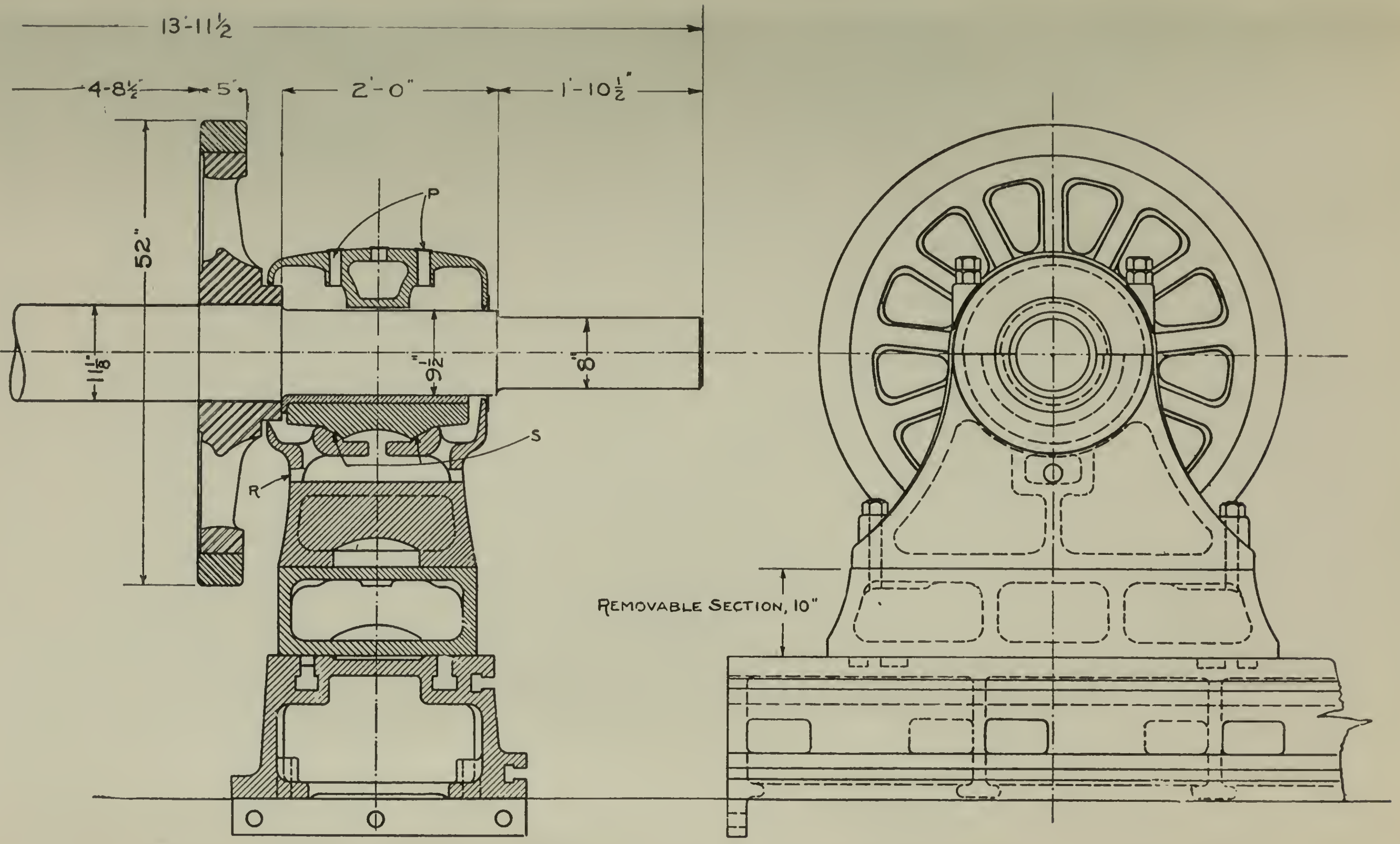


FIG. 51. ONE OF THE SUPPORTING WHEELS, WITH ITS AXLE, BEARING, PEDESTAL, AND BED PLATE.

header (N) at the periphery of the discs, and through the oil-duct (K). The oil is taken off at M as it oozes out around the disc hub. The diaphragms form also within the casing four water compartments which have no communication whatever with the compartments in which the discs rotate. Water at about 60 deg. F. is forced through 3-inch hose connections into the brake at the lower header (B) and leaves through the upper header (A). The amount of water passing through any individual brake and the water pressure within the brake may be regulated at will by means of suitable valves in the piping. The brake load is controlled by thus modifying the water pressure. This is accomplished simultaneously for all of the brakes by means of the large control-valve in the brake supply main, shown in Fig. 50. The auxiliary brake-valves and gages shown at the left in this same figure permit the separate adjustment of load on each brake. Each of these brakes has the capacity of absorbing 450 horse power, having been designed to develop a resisting torque of 18 000 pounds-feet at speeds up to 130 revolutions per minute. This capacity allows for a considerable increase in wheel loads above that which could be imposed by the most heavily loaded driver of the present day.

*Placing the Locomotive.*—Fig. 47 shows the mounting machinery arranged to receive an eight-driver locomotive. The top of the supporting-wheels is level with the main floor of the building and one-fourth inch higher than the outside track. Before the locomotive to be tested is placed upon the plant, its wheel-spacing is determined and the supporting-wheel centers spaced accordingly. The tender having been removed, the locomotive is backed into the laboratory and onto the temporary track shown in place between the supporting-wheels. The drivers run on their flanges over the temporary track, which leaves their treads free to engage the supporting-wheels, so that when the locomotive is properly placed the supporting-wheels carry all of the weight except, of course, that borne by leading or trailing trucks. The temporary track being relieved, may be removed. Mounted in this way, the locomotive is held in place and prevented from moving forward or backward by means of the dynamometer draw-bar, which is supplemented by two safety-bars that come into play in case of failure of the draw-bar. These three bars are shown in Fig. 50. Forward and trailing-truck wheels are carried on track sections which are level with the supporting wheels.

*The Dynamometer.*—The dynamometer, the chief function of which is to permit the tractive force of the locomotive to be measured, is

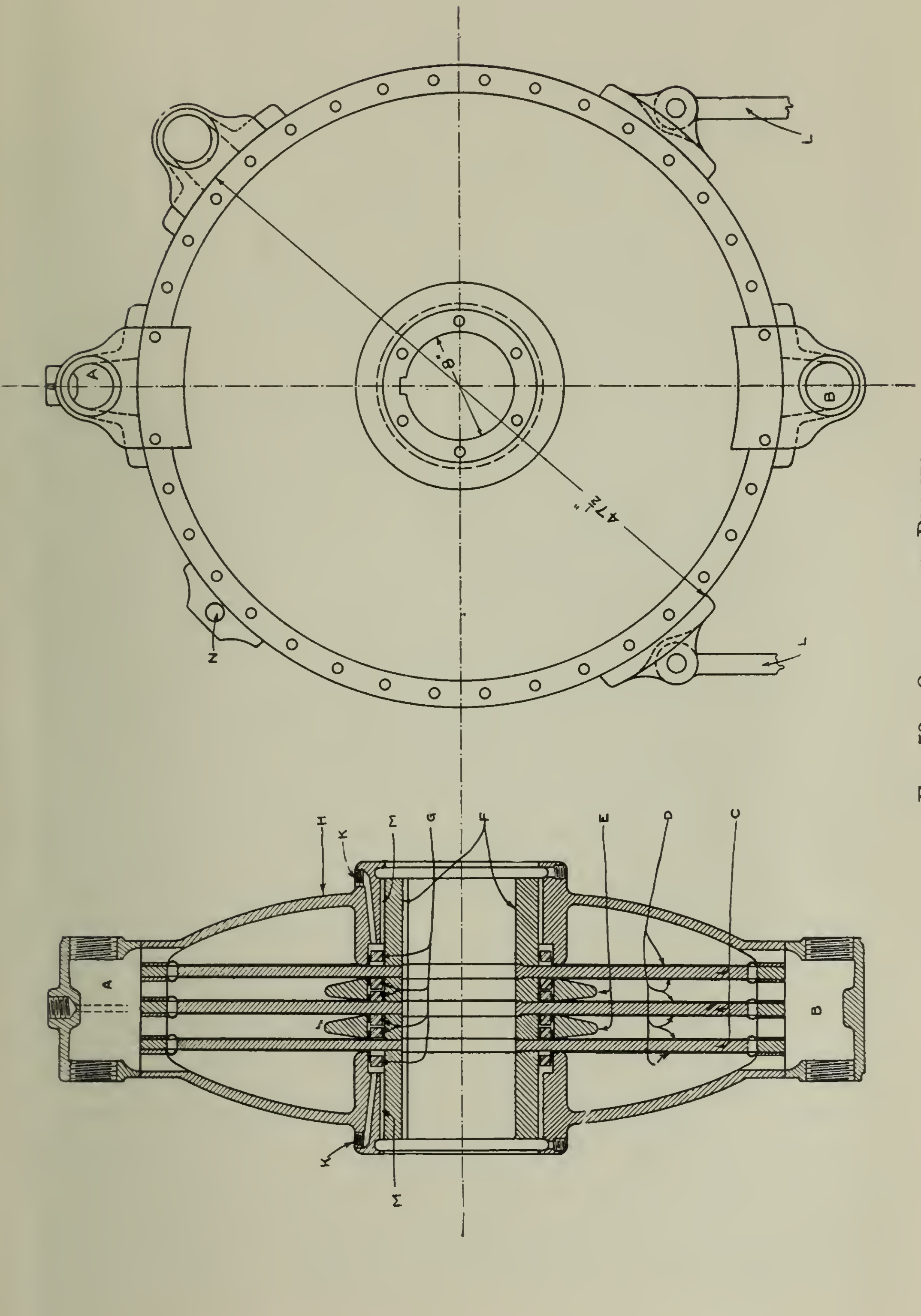


Fig. 52. One of the Brakes.

shown in Fig. 47, 49, and 50. It is of the Emery type and consists essentially of two parts: the weighing head, carried on a pedestal and shown in Fig. 49, and the measuring and recording scale shown in Fig. 50. The weighing head may be raised or lowered to suit the height of the drawbar of any locomotive. Within the weighing head is an enclosed oil-chamber with a flexible wall or diaphragm, which receives and balances any force transmitted through the drawbar of the locomotive. The pressure within this oil-chamber varies with the load, and is transmitted through a copper tube of small bore to a smaller oil-chamber known as the reducing chamber, located in the case with the measuring apparatus. The pressure thus produced in the reducing chamber moves the beam of a substantial but sensitive scale which measures the tractive force of the locomotive.

In order to prevent undue shocks from taking place within the weighing head of the dynamometer on account of variations in the force in the drawbar, an initial load of 50 000 pounds is imposed upon the oil behind the diaphragm by means of the capstan and springs located at the rear of the weighing head and shown at the left in Fig. 49. The weighing head of the dynamometer is so designed that by an adjustment of the capstan the tractive effort may be measured whether the locomotive drivers are turning forward or backward. For the sake of accuracy in determining the drawbar pull it is essential that the locomotive drivers be placed and maintained with their centers precisely above the centers of the supporting-wheels. To satisfy this requirement the longitudinal travel of the dynamometer drawbar from no load to full load must be reduced to a minimum. In this instrument the range of movement is only three one-thousandths of an inch. The scale beam reads directly to 20 000 pounds in 100-pound divisions, and a vernier gives readings to ten pounds. For drawbar pulls of more than 20 000 pounds, weights may be added as required. The dynamometer will measure drawbar pulls as high as 125 000 pounds.

A feature of interest in the design of the scale lies in the fact that the adjustment of the poise weight on the scale beam may be made automatically. This is accomplished by means of a small motor which is mounted on the scale beam and geared to a screw which passes through the poise weight. Attached to the scale beam is a contact arm, and any movement of the beam in either direction causes a series of mercury-cup contacts; the number of contacts depending on the amount of deflection of the beam, which in turn is caused by a change in the load. When contact is made, an electrical circuit is closed and the motor runs in the direction required to bring the poise weight to a position of equilibrium. As soon as the load is balanced, the circuit is broken and the motor stops. This operation is repeated as often as the load changes, and is practically continuous. The action of the poise weight may also be controlled by a hand switch.

*Water and Coal Supply.*—The general water supply of the University is from driven wells, the demand upon which at times approaches their full capacity. In order therefore that the water which passes through the brakes shall not be wasted, provision has been made for collecting, cooling, and recirculating it. For this purpose a 100 000-gallon concrete storage reservoir (see Fig. 45) has been built in the ground outside of the building. A supply pump for the brakes draws water from this reservoir through a 6-inch main and pumps it through the main control valve to a header, whence it is distributed through auxiliary supply control valves to the several brakes; after which it flows back through another set of auxiliary back-pressure control valves to a sump located in the basement of the laboratory. (See Fig. 53). The water is drawn from the sump by another pump and forced through five 2-inch whirling-spray nozzles above the surface of the water in the reservoir. Water direct from the University mains may also be used in the brakes when desirable.

Water for the locomotive boiler may be drawn from the reservoir or direct from the University mains, and forced by a separate pump to two elevated tanks which are shown in Fig. 47 and 54. Each of these tanks has a capacity of 2000 pounds and rests permanently on a platform scale. At a supply pressure of 45 pounds, each tank can be filled, weighed, and emptied in two and one-half minutes. From the weighing tanks, the water falls into the 18 000-pound capacity feed tank below, and thence passes through two 4-inch supply pipes to the locomotive injectors. Water for the hydraulic elevator used in raising coal from the main floor to the firing platform may be taken from the University main or from the storage tank. In either case the pressure is maintained at 60 pounds by a throttle-control valve on the supply pump. By these provisions in the piping, reservoir water alone may be used for feed-water, brakes, and elevator.

The coal-room shown in Fig. 53 occupies the rear end of the building. It is 22 feet wide and 40 feet long, and has a storage capacity of 300 tons. Coal for the tests is loaded into 1000-pound capacity wagons, run out onto the scales. raised by the elevator to the firing plat-

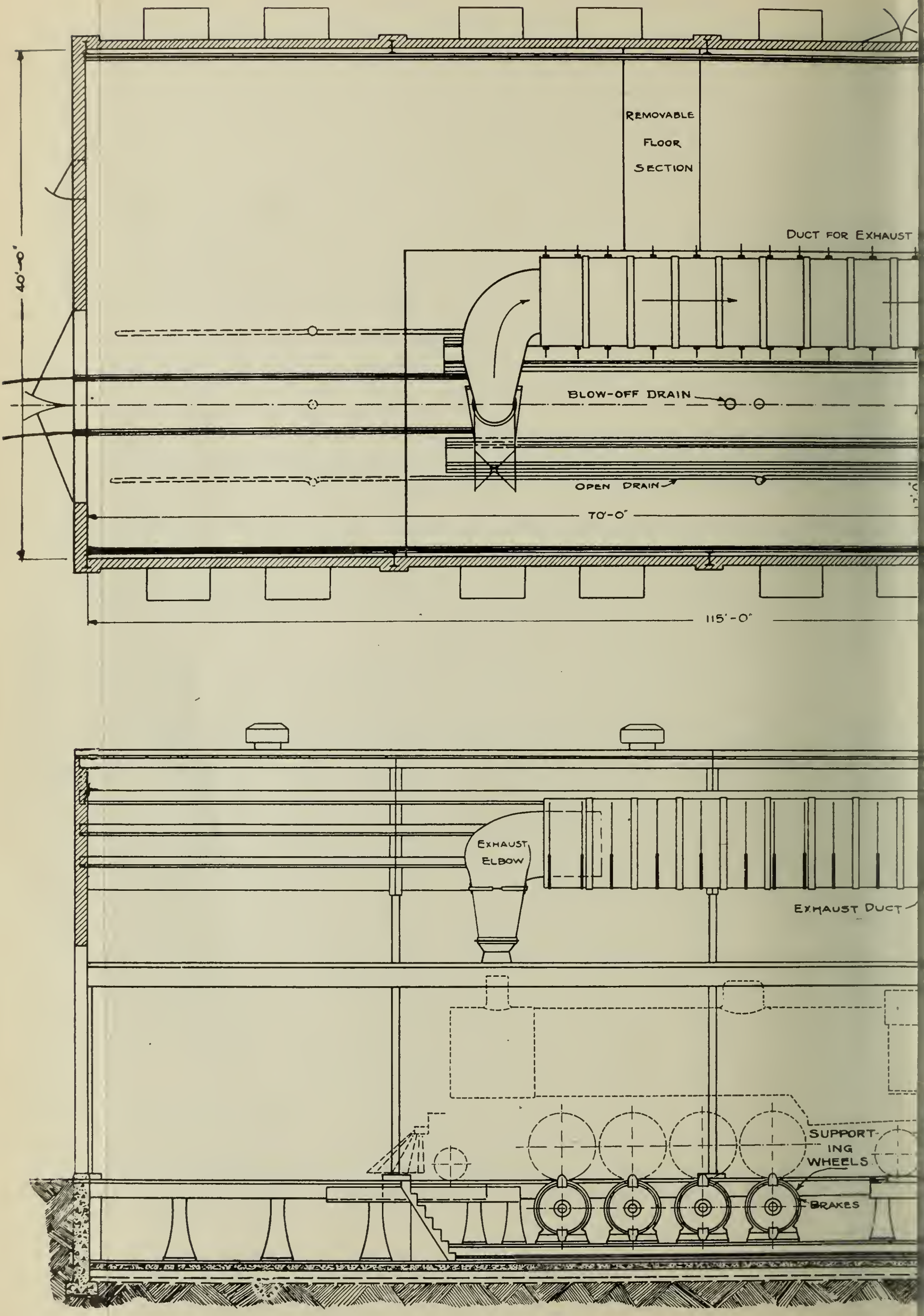
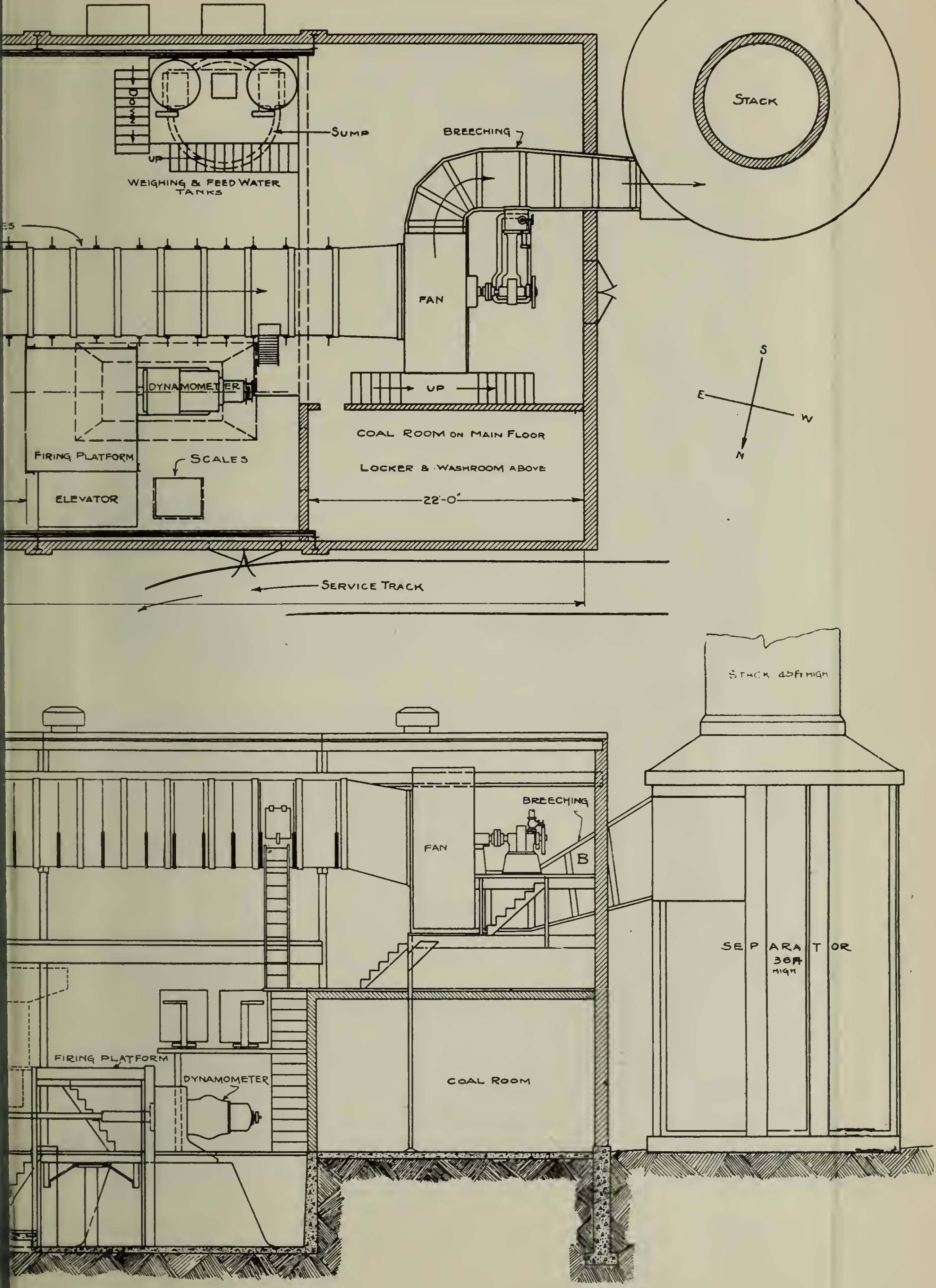


Fig. 53. Sectional Plan a



LEVATION OF THE LABORATORY.

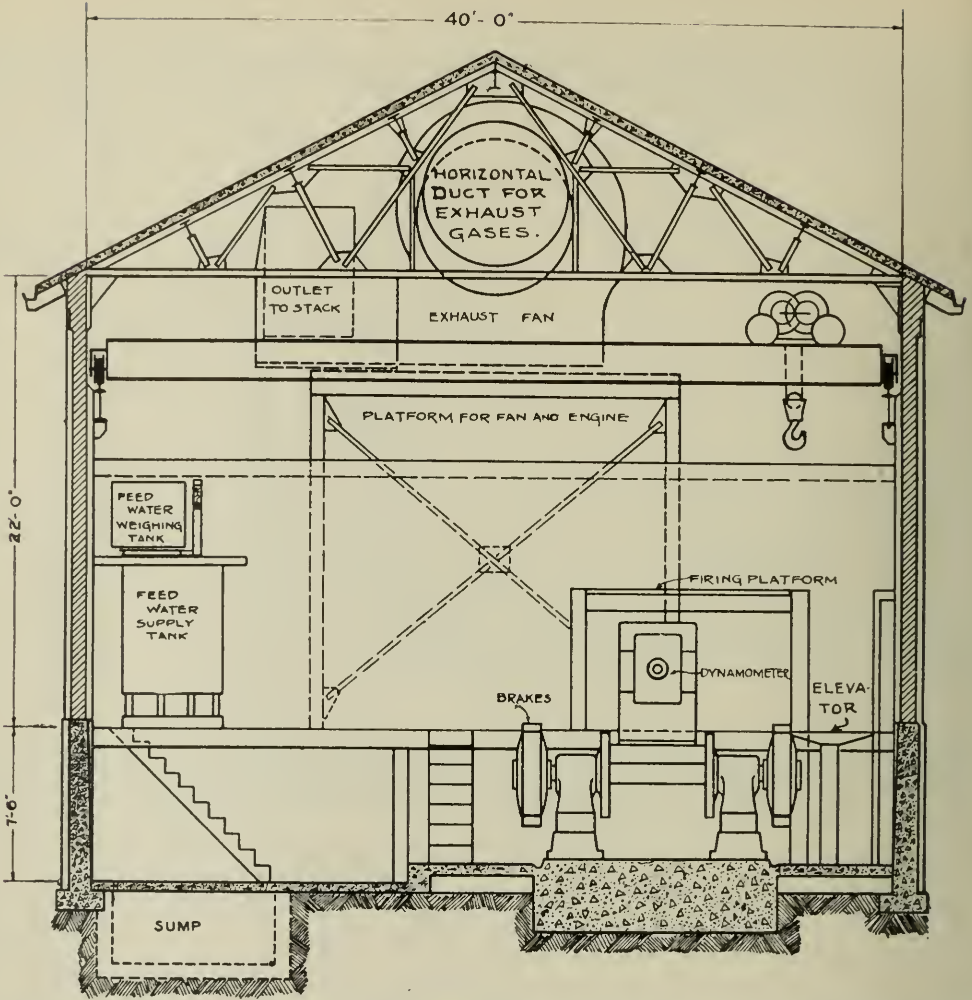


FIG. 54. A CROSS SECTION THROUGH THE MIDDLE OF THE LABORATORY.

form, and there dumped. The firing platform is adjustable in height so as to suit the deck of any locomotive cab. The elevator has a capacity of 2000 pounds. It is also used to raise ashes from the level of the basement.

*The Exhaust System.*—Recognizing the value of accurate determininations of the total amount of cinders lost through the stack of the locomotive, it was early decided that if possible some means should be incorporated in this plant to collect all of the solid matter which passes through the locomotive front end. Preliminary designs of a cinder catcher which should have sufficient capacity to pass the total volume of waste gas, exhaust steam, and entrained air, and at the same time collect all the cinders from the largest modern locomotive working at high power, made it clear that such a collector would be too

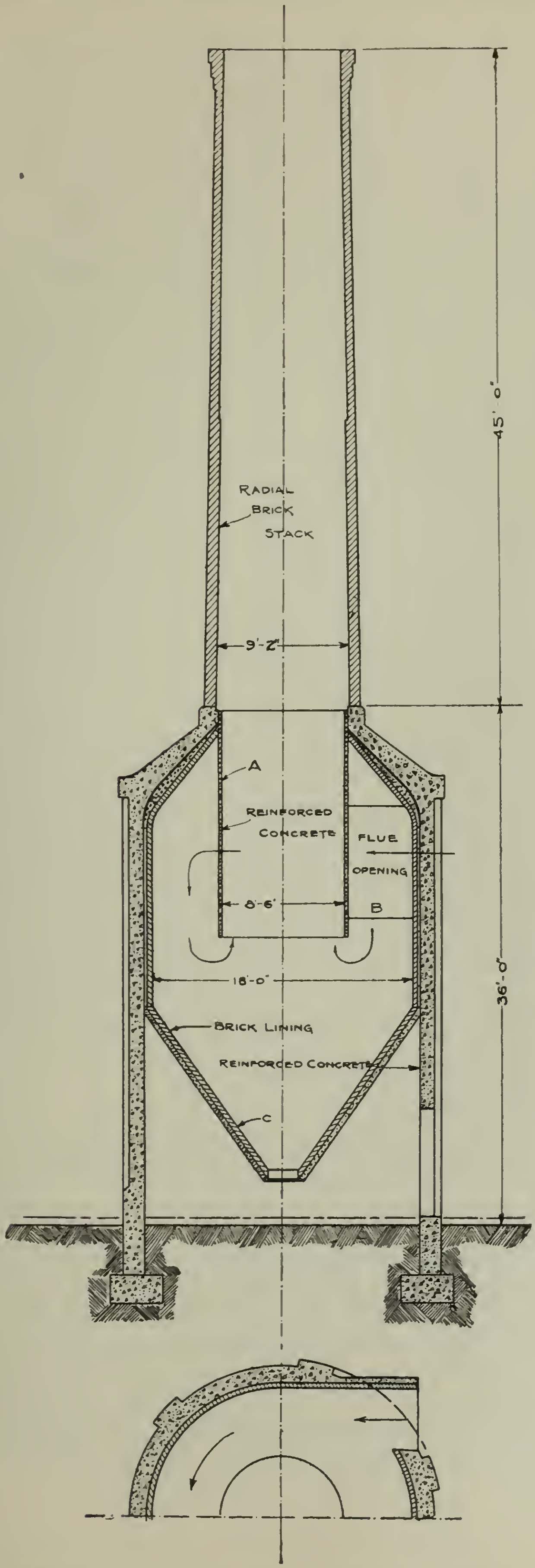


Fig. 55. Cross Section Through Cinder Collector and Stack.

large to be located conveniently within the building. Another point considered in the design of the exhaust system was the necessity of a stack of sufficient height to insure that the exhaust gases would be discharged far enough above ground to prove inoffensive to occupants of neighboring residences and University buildings. For this purpose it was decided that a stack 8 feet in diameter and 80 feet high would be required. Further study made it apparent that these two decisions could be embodied in one structure combining the cinder separator and the stack. This has been accomplished in the construction represented in cross-section by Fig. 55, and which is located outside and at the rear of the laboratory.

The system will be most easily understood by following the course of the exhaust gases after they leave the locomotive stack. (Fig. 53 and 54.) The gases and exhaust steam are discharged across an open space above the locomotive stack into a steel exhaust elbow, which carries them up and over to a horizontal duct running through the center of the roof-trusses of the building. The gases, exhaust steam, and solid matter are drawn through this elbow and duct by the exhaust fan, located near the roof at the rear end of the building. They are then passed through a breeching or flue to the separator previously referred to, the action within which may be best explained by reference to Fig. 55. The cinder-laden gases enter the separator at B and in order to leave, they must pass downward and around the sleeve A. This operation gives them a whirling motion, which causes the cinders by centrifugal force to move toward the outside wall, along which they drop to the hopper below, while the gases pass downward and out through the mouth of the sleeve. The cinders collecting at the bottom of the hopper are drawn off, weighed, and analysed between tests. The separator is surmounted by a 45-foot radial brick stack, through which the gases and steam are finally discharged.

On account of the corrosive nature of the mixture of exhaust gases and steam, it was necessary to avoid the use of metal throughout the exhaust system, as far as it was possible to do so. The exhaust elbow which receives the gases from the locomotive stack is necessarily made of steel and needs occasionally to be renewed. The horizontal duct, running through the center of the roof-trusses, is made of a hard and tough asbestos board known as "Transite," which is proof against corrosion. This duct is seven feet in diameter, and is built up in sections so that its length may be varied to suit the position of the locomotive stack. The final adjustment of the elbow above the stack of the locomotive is obtained through the medium of a telescopic connection between the elbow and the duct. The fan has a runner six feet in diameter, and at a maximum speed of 300 revolutions per minute, will pass 140 000 cubic feet of gas per minute. The breeching between the fan and separator is also built of transite, and has a minimum cross-sectional area of about 24 square feet. The outer shell of the separator is built of reinforced concrete, and it is lined throughout with a course of hard red brick as protection from the corroding action of the gases. Between the lining and the shell is a 2-inch air space which acts as an insulator to protect the shell from overheating. Any leakage of gas through the lining into the air space is vented to the outside air through openings which are provided in the shell, and which serve also to circulate cool air through the air-space. Both the inside sleeve and the hopper are built of reinforced concrete. The stack itself is unlined, but is laid up with acid-proof cement. Provision was made in the design for traps in the bottom of the horizontal duct, whereby any solid matter that should accumulate within the duct could be removed and weighed. Experience has proved this to be unnecessary, as all portions of the duct and breeching have been self-cleaning under all test conditions thus far encountered.

## APPENDIX 3.

### TEST METHODS.

The test methods employed were, in general, those outlined in the "Method of Conducting Locomotive and Road Tests" published in the Proceedings of the American Railway Master Mechanics' Association, volume 47, page 538.

Each test was made under predetermined conditions of speed and cut-off. Throughout each test all conditions subject to control were maintained as nearly constant as possible. Variations between different tests, or groups of tests, relative to engine conditions and fuel used have been recorded and discussed.

The test methods employed were, with minor exceptions, the same throughout all tests. All instruments were known to be correct within reasonable limits or were calibrated at intervals and suitable corrections applied to the observed data. Observations were, in general, taken every ten minutes. Indicator diagrams, particularly on comparatively short tests, were often taken at five minute intervals. The locations of the more important instruments and apparatus are indicated in Fig. 40 in Appendix 1, and in the figures of Appendix 2. The methods of applying the load to the engine, of measuring the drawbar-pull, and of collecting the stack cinders are made clear by the description of the Locomotive Laboratory in Appendix 2.

*Duration of Tests.*—The tests reported in Appendix 4 varied in length from 30 minutes to 3 hours. Tests were in general of such duration that from 120 to 180 pounds of coal would be burned per square foot of grate during the test. This is equivalent to a coal consumption of approximately 6000 to 9000 pounds per test. An examination of the data shows that for 39 tests the coal consumption was within this range; for 15 tests it was less than 6000 pounds; and for 4 tests it was more than 9000 pounds per test.

*Starting and Closing a Test.*—In general, fires were built upon a clean grate for each test. With sufficient steam pressure the locomotive was started and gradually brought to the required conditions of speed and cut-off. The locomotive was operated for a short time under the required conditions and until a satisfactory fire and satisfactory boiler pressure were being maintained. On signal, the ash pan and cinder separator were closed, observations of water levels and steam pressure were made, and the test thereby started.

In closing a test, simultaneous observations were made upon water levels, steam pressure, and condition of fire. The locomotive was then stopped as quickly as conditions permitted. As soon as possible after stopping the locomotive, ashes were removed from the ash pan and cinders were removed from the front-end and from the cinder separator. In closing tests, it was sometimes advisable to remove some of the ash from the fire previous to the close of the test in order to bring the fire to the desired condition, and it was also occasionally advisable to remove some ash immediately after the close of the test. In all cases it was endeavored to have the same amount of combustible matter upon the grate at the close of the test as at the start. The removal of ash from the fire in connection with the closing of the test was primarily for the purpose of judging the amount of combustible upon the grate, not for the purpose of collecting the ash. The endeavor was made to have the boiler pressure and water level in the boiler substantially the same at the close as at the beginning of the test. Corrections were made for such irregularities as occurred.

*Temperatures, Pressures, Gas Samples, Etc.*—Temperatures in the fire-box were observed by means of a radiation pyrometer and temperatures in the front-end by means of a thermo-couple. Mercury thermometers were used at other points where temperature observations were made. Boiler pressure observations were taken from a gage located in the engine cab. Draft pressures were measured by means of U–tubes with water. Quality of steam was determined by means of a throttling calorimeter fitted with a suitable sampling tube. Front-end gas samples were collected through a sampling pipe provided with numerous small holes along the pipe, through which the gas was drawn. The time during which a single sample was collected varied from 20 to 60 minutes, depending mainly upon the total length of the test. The taking of samples usually covered the entire test period. Gas samples were analysed immediately after collection by means of the Orsat apparatus. Speed was measured by means of a stroke counter operated through the indicator reducing-motion.

*Indicator Diagrams.*—Four indicators were used, one at each end of each cylinder. During a majority of the tests, indicators were used which, through the operation of electrical attachments, took the four diagrams simultaneously. On account of minor accidents, it was sometimes necessary to use indicators where the pencil applications on some or all of the indicators were made by hand. In all cases, however, the applications were practically simultaneous.

*Coal and Water.*—Coal was delivered to the firing platform in lots of approximately 1000 pounds each. The time of firing the last shovelful of each lot was recorded. Water was weighed by means of two tanks upon platform scales. Each tank holds approximately 2000 pounds of water. The weighing tanks emptied into the feed-water supply-tank which has a capacity of 18 000 pounds. The height of water in the feed-water tank was maintained at an approximately constant level throughout a test. Observations were so taken that the amount of water furnished to the boiler could be calculated for intervals determined by the emptying of each weighing tank. Fig. 47 and 53 in Appendix 2 show the arrangement of the coal and water weighing apparatus.

*Firing.*—The locomotive was hand fired during all tests. The method of level firing was used, single shovelfuls of coal at a fairly constant rate being distributed uniformly over the grate. All large pieces of coal were broken, before firing, to lumps whose greatest dimension was from 3 to 4 inches. Two experienced firemen were employed. One of these men, however, was held entirely responsible and did practically all of the firing.

*Samples of Coal, Ash, and Cinders.*—Following the close of a test the ashes collected in the ash pan, the cinders collected in the front-end, and the cinders collected in the cinder separator were weighed and sampled.

A coal sample weighing from 200 to 500 lb. was collected during each test. This sample was collected while loading the cars taking coal to the firing platform, by placing about 50 lb. in the sampling can for each 1000-lb. car loaded. Care was exercised to make the sample representative.

The front-end cinders after being weighed were thoroughly mixed and about two pounds of cinders set aside as a sample. A sample of the stack cinders, weighing from 25 to 50 lb., was collected as the cinders were being weighed, a small amount being taken from each barrow load after passing over the scales. A sample of ash, weighing from 50 to 100 lb. was collected as the ash was being weighed, representative portions of the ash being selected.

The large samples of coal, ash, and stack cinders were ground or crushed as necessary to reduce them to a maximum size of ¼ inch, then thoroughly mixed and reduced by "quartering" to samples weighing about two pounds. The two pound samples of coal, ash, front-end and stack cinders were submitted to the Chemical Laboratory for analysis.

*Chemical Analysis.*—The chemical analyses and heat determinations were made by the Department of Chemistry of the University of Illinois. The methods employed were substantially those which have been recommended in the preliminary reports of the Joint Committee on Coal Analysis, of the American Society for Testing Materials and the American Chemical Society.

Proximate analyses and B.t.u. determinations were made for the coal sample for each test. Four ultimate analyses of coal were made —one for tests 2009 to 2019 inclusive, one for tests 2020 to 2045, one for tests 2046 to 2071, and one for tests 2072 to 2095. The ultimate analyses were made from composite samples. Each composite sample was made by combining from each air-dried sample of the tests to be represented, an amount proportional to the coal burned during the test. The ultimate analyses for the individual tests which appear in the tabulated results are based upon the percentages of moisture, ash, and sulphur as determined by the proximate analysis and upon the assumption that the carbon, hydrogen, oxygen, and nitrogen are proportional to the percentages as determined for the composite sample by ultimate analysis.

Each ash and cinder sample was subjected to analysis to determine carbon, earthy matter, and moisture. B.t.u. determinations for ash, front-end and stack cinders were estimated in the following manner. B.t.u. determinations were made upon ten ash samples of Series 1, upon three samples of Series 2, and upon one composite sample representing all tests 2072 to 2095 inclusive. Upon the assumptions that the heat content of the ash was entirely due to its carbon content, and that the heat content of the carbon was uniform in all of the ash, an average value for the heat content of one pound of carbon in the ash was computed. This value was determined as 14 672 B.t.u. per pound of carbon contained in the ash. Using this value as a basis the heat content per pound of ash was calculated for each test.

B.t.u. determinations were made for 10 front-end cinder samples of Series 1 and for one composite sample representing tests 2072 to 2095 inclusive. In a manner similar to that outlined in the case of the ash, it was computed that the average value for the heat content of one pound of carbon contained in the front-end cinders was 14 336 B.t.u. and with this value as a basis the heat content per pound of front-end cinders was calculated for each test.

B.t.u. determinations were made for 10 stack cinder samples of Series 1, for a composite sample representing 10 tests during which the draft ranged from 2.2 to 4.5 inches of water, for a composite sample representing 9 tests during which the draft ranged from 5.3 to 9.2 inches of water, for a composite sample representing 5 tests during which the draft ranged from 10 to 12.8 inches of water, and for 3 individual tests during which the drafts were respectively 2.9, 7.0, and 11.9 inches of water. The 24 tests represented by the composite samples were tests 2072 to 2095 inclusive. The 3 individual tests mentioned were 2087, 2079, and 2089. With these heat determinations and in a manner involving the same assumptions as were made in the case of the ash and front-end cinders, it was computed that the heat content of one pound of carbon contained in the stack cinders was equivalent to $(14\,932 - 44D)$ B.t.u. In this expression $D$ is the draft in front of the diaphragm, expressed as inches of water. With the values determined by means of this formula the heat content per pound of stack cinders was calculated for each test.

# APPENDIX 4.

## Detailed Data and Results.

The purpose of this appendix is to present in detail the data and results of all the tests. It consists of 24 tables and 3 figures. Tables 13 to 35 inclusive contain the data and results for 61 tests, arranged in four groups. Table 36 contains information relating to the representative indicator diagrams which are shown in Fig. 56, 57, and 58 at the end of the appendix.

The first of the four groups of tests comprises tests 2009 to 2037 and has been designated as Series 1. The third group, designated as Series 2, comprises tests 2072 to 2098 (excepting No. 2090 and 2091). The results of Series 1 and 2 have been presented and discussed in sections VI to VIII in Part I. The data and results of the two remaining groups appear only in this appendix, and are elsewhere merely referred to incidentally.

Section IV of Part I defines these four groups of tests, and states the difference between them as regards the condition of the locomotive. The locomotive's condition has also been explained in Section II and in Appendix 1. The differences in condition as regards fuel are stated in Section V of Part I. The evaporative efficiencies recorded for tests 2024 and 2038 are enough higher than the corresponding results recorded for other tests to raise some question as to the correctness of their results. The conditions, however, under which they were made were such that high boiler efficiency was to be expected. The data and results for these tests have been included in the tables, and have been used throughout the discussion except as may have been specifically indicated.

The data and results are presented under 182 headings. The column-heading numbers are included between the numbers 344 and 900 and are arranged consecutively in the tables. Tables throughout the bulletin carry corresponding column-headings and numbers wherever the same data are presented. In general the column-headings and column-heading numbers are the same as used in the Code for Testing Locomotives published in the Proceedings of the American Railway Master Mechanics' Association, volume 47, page 538. The methods of calculation, unless entirely obvious, are given in detail in Appendix 5.

## TABLE 13.
## GENERAL CONDITIONS

| Test No. | Laboratory Designation | Duration of Test, Hours | Speed: Revolutions: Total | Speed: Revolutions: Average per Minute | Speed: Equivalent: Speed in Miles per Hour | Speed: Equivalent: Piston Speed in Feet per Minute | Position of Levers: Reverse Lever, Notches from Center | Position of Levers: Throttle |
|---|---|---|---|---|---|---|---|---|
| | Code Item☞ | 345 | 351 | 352 | 353 | 354 | 360 | 363 |
| 2009 | 138–16–F | 2.50 | 20 878 | 139.2 | 25.2 | 694.6 | 4 | Full |
| 2010 | 193–20–F | 1.18 | 13 969 | 196.8 | 35.7 | 981.8 | 5 | Full |
| 2012 | 138–24–F | 1.83 | 15 349 | 139.5 | 25.3 | 696.3 | 6 | Full |
| 2013 | 138–32–F | 1.50 | 12 575 | 139.7 | 25.4 | 697.2 | 8 | Full |
| 2014 | 193–32–F | 1.17 | 13 981 | 199.7 | 36.3 | 996.4 | 8 | Full |
| 2015 | 193–24–F | 1.50 | 18 000 | 200.0 | 36.3 | 998.0 | 6 | Full |
| 2016 | 193–16–F | 2.33 | 27 998 | 200.0 | 36.3 | 998.0 | 4 | Full |
| 2017 | 83–16–F | 3.00 | 14 398 | 80.0 | 14.5 | 399.2 | 4 | Full |
| 2018 | 83–24–F | 2.67 | 12 802 | 80.0 | 14.5 | 399.2 | 6 | Full |
| 2019 | 83–32–F | 2.17 | 10 474 | 80.6 | 14.6 | 402.0 | 8 | Full |
| 2020 | 83–24–F | 2.00 | 9 656 | 80.5 | 14.6 | 401.7 | 6 | Full |
| 2021 | 83–16–F | 2.00 | 9 615 | 80.1 | 14.5 | 399.7 | 4 | Full |
| 2022 | 83–32–F | 2.00 | 9 670 | 80.6 | 14.6 | 402.2 | 8 | Full |
| 2023 | 138–40–F | 1.50 | 12 515 | 139.0 | 25.2 | 693.6 | 10 | Full |
| 2024 | 55–24–F | 2.00 | 6 080 | 50.7 | 9.2 | 253.0 | 6 | Full |
| 2026 | 110–16–F | 2.17 | 14 220 | 109.4 | 19.9 | 545.9 | 4 | Full |
| 2027 | 110–24–F | 2.50 | 16 500 | 110.0 | 20.0 | 548.9 | 6 | Full |
| 2028 | 55–32–F | 2.33 | 7 050 | 50.3 | 9.1 | 251.0 | 8 | Full |
| 2029 | 110–32–F | 1.50 | 9 880 | 109.8 | 19.9 | 547.9 | 8 | Full |
| 2030 | 165–24–F | 1.67 | 16 935 | 169.4 | 30.8 | 845.3 | 6 | Full |
| 2031 | 83–40–F | 1.50 | 7 723 | 85.8 | 15.6 | 428.1 | 10 | Full |
| 2032 | 165–32–F | 0.67 | 6 726 | 168.1 | 30.5 | 838.8 | 8 | Full |
| 2033 | 110–48–F | 1.33 | 8 793 | 109.9 | 20.0 | 548.4 | 12 | Full |
| 2034 | 193–40–F | 1.00 | 11 910 | 198.5 | 36.0 | 990.5 | 10 | Full |
| 2035 | 110–40–F | 1.17 | 7 830 | 111.8 | 20.3 | 557.9 | 10 | Full |
| 2037 | 165–40–F | 1.00 | 10 160 | 169.3 | 30.7 | 844.8 | 10 | Full |
| 2038 | 55–24–F | 1.83 | 5 586 | 50.8 | 9.2 | 253.4 | 6 | Full |
| 2039 | 110–32–F | 1.50 | 9 896 | 110.0 | 20.0 | 548.7 | 8 | Full |
| 2040 | 165–40–F | 1.00 | 10 127 | 168.8 | 30.7 | 842.2 | 10 | Full |
| 2041 | 110–40–F | 1.17 | 7 709 | 110.1 | 20.0 | 549.5 | 10 | Full |
| 2042 | 110–24–F | 2.00 | 13 190 | 109.9 | 20.0 | 548.5 | 6 | Full |
| 2043 | 110–48–F | 1.17 | 7 681 | 109.7 | 19.9 | 547.6 | 12 | Full |
| 2044 | 110–56–F | 1.00 | 6 585 | 109.8 | 19.9 | 547.7 | 14 | Full |
| 2045 | 110–16–F | 2.17 | 14 277 | 109.8 | 19.9 | 548.0 | 4 | Full |
| 2072 | 110–40–F | 1.00 | 6 575 | 109.6 | 19.9 | 546.8 | 10 | Full |
| 2073 | 110–32–F | 1.33 | 8 751 | 109.4 | 19.9 | 545.9 | 8 | Full |
| 2074 | 165–32–F | 1.00 | 10 178 | 169.6 | 30.8 | 846.3 | 8 | Full |
| 2075 | 55–32–F | 2.33 | 7 081 | 50.6 | 9.2 | 252.3 | 8 | Full |
| 2076 | 220–32–F | 0.58 | 8 047 | 229.9 | 41.7 | 1147.2 | 8 | Full |
| 2077 | 110–24–F | 1.83 | 12 174 | 110.7 | 20.1 | 552.2 | 6 | Full |
| 2078 | 165–24–F | 1.17 | 11 830 | 169.0 | 30.7 | 843.3 | 6 | Full |
| 2079 | 220–24–F | 1.00 | 13 914 | 231.9 | 42.1 | 1157.2 | 6 | Full |
| 2080 | 110–16–F | 2.17 | 14 346 | 110.4 | 20.0 | 550.7 | 4 | Full |
| 2081 | 55–24–F | 2.50 | 7 593 | 50.6 | 9.2 | 252.5 | 6 | Full |
| 2082 | 165–40–F | 0.83 | 8 484 | 169.7 | 30.8 | 846.7 | 10 | Full |
| 2083 | 165–16–F | 1.67 | 17 031 | 170.3 | 30.9 | 849.8 | 4 | Full |
| 2084 | 110–48–F | 0.83 | 5 519 | 110.4 | 20.0 | 550.8 | 12 | Full |
| 2085 | 55–40–F | 2.00 | 6 136 | 51.1 | 9.3 | 255.1 | 10 | Full |
| 2086 | 55–24–F | 2.83 | 8 725 | 51.3 | 9.3 | 256.1 | 6 | Full |
| 2087 | 110–16–F | 2.50 | 16 660 | 111.1 | 20.2 | 554.4 | 4 | Full |
| 2088 | 220–16–F | 1.67 | 23 418 | 234.2 | 42.5 | 1168.6 | 4 | Full |
| 2089 | 220–40–F | 0.58 | 8 075 | 230.7 | 41.9 | 1151.2 | 10 | Full |
| 2092 | 165–32–F | 0.83 | 8 425 | 168.5 | 30.6 | 840.8 | 8 | Full |
| 2093 | 165–48–F | 0.50 | 5 023 | 167.4 | 30.4 | 835.5 | 12 | Full |
| 2094 | 110–56–F | 0.42 | 2 772 | 110.9 | 20.1 | 553.3 | 14 | Full |
| 2095 | 55–48–F | 1.00 | 3 077 | 51.3 | 9.3 | 255.9 | 12 | Full |
| 2096 | 55–40–F | 1.50 | 4 636 | 51.5 | 9.4 | 257.0 | 10 | Full |
| 2097 | 55–32–F | 1.83 | 5 732 | 52.1 | 9.5 | 260.0 | 8 | Full |
| 2098 | 55–48–F | 0.83 | 2 587 | 51.7 | 9.4 | 258.2 | 12 | Full |
| 2090 | 110–24–F | 1.00 | 6 640 | 110.7 | 20.1 | 552.2 | 6 | Full |
| 2091 | 165–32–F | 0.50 | 5 079 | 169.3 | 30.7 | 844.8 | 8 | Full |

## TABLE 14.
## TEMPERATURE AND PRESSURE

| Test No. | Laboratory Designation | Temperature, Deg. Fahr. | | | | | Pressure, lb. per sq. in. | |
|---|---|---|---|---|---|---|---|---|
| | | Front-end | Laboratory | | Feed Water | Fire Box | Boiler, Average | Laboratory Barometric |
| | | | Dry Bulb | Wet Bulb | | | | |
| | Code Item☞ | 367 | 368 | 369 | 373 | 374 | 380 | 388 |
| 2009 | 138–16–F | | 93 | 75 | 64.1 | 1950 | 190.5 | 14.3 |
| 2010 | 193–20–F | 761 | 62 | 57 | 59.3 | 2081 | 192.0 | 14.4 |
| 2012 | 138–24–F | 712 | 86 | 69 | 61.7 | 1959 | 191.8 | 14.4 |
| 2013 | 138–32–F | 754 | 87 | 70 | 62.2 | 1957 | 190.1 | 14.4 |
| 2014 | 193–32–F | 751 | 94 | 75 | 62.9 | | 191.5 | 14.4 |
| 2015 | 193–24–F | 702 | 97 | 75 | 61.7 | | 192.1 | 14.4 |
| 2016 | 193–16–F | 671 | 94 | 75 | 63.4 | | 193.9 | 14.4 |
| 2017 | 83–16–F | 619 | 93 | 76 | 72.2 | | 193.4 | 14.4 |
| 2018 | 83–24–F | 649 | 92 | 71 | 64.0 | | 194.2 | 14.3 |
| 2019 | 83–32–F | 684 | 86 | 73 | 62.1 | | 193.8 | 14.3 |
| 2020 | 83–24–F | 499 | 70 | 61 | 64.9 | | 190.7 | 14.4 |
| 2021 | 83–16–F | 494 | 63 | 53 | 57.8 | 1552 | 193.7 | 14.5 |
| 2022 | 83–32–F | 554 | 66 | 56 | 58.3 | 1808 | 189.9 | 14.5 |
| 2023 | 138–40–F | 639 | 64 | 55 | 57.7 | 1898 | 190.8 | 14.4 |
| 2024 | 55–24–F | 517 | 66 | 51 | 59.3 | 1829 | 196.3 | 14.5 |
| 2026 | 110–16–F | 531 | 58 | 52 | 61.0 | 1748 | 196.9 | 14.5 |
| 2027 | 110–24–F | 552 | 64 | 56 | 60.2 | 1677 | 196.8 | 14.4 |
| 2028 | 55–32–F | 515 | 66 | 58 | 70.4 | 1700 | 198.1 | 14.4 |
| 2029 | 110–32–F | 560 | 73 | 65 | 60.9 | 1690 | 197.1 | 14.4 |
| 2030 | 165–24–F | 565 | 76 | 68 | 60.2 | 1636 | 196.5 | 14.3 |
| 2031 | 83–40–F | 570 | 72 | 66 | 60.6 | 1811 | 196.4 | 14.3 |
| 2032 | 165–32–F | 613 | 71 | 66 | 59.6 | 1630 | 196.8 | 14.3 |
| 2033 | 110–48–F | 603 | 73 | 67 | 60.6 | 1806 | 196.0 | 14.3 |
| 2034 | 193–40–F | 632 | 76 | 69 | 59.8 | 1879 | 192.1 | 14.3 |
| 2035 | 110–40–F | 589 | 73 | 65 | 61.7 | 1663 | 194.3 | 14.3 |
| 2037 | 165–40–F | 651 | 75 | 69 | 60.8 | 1800 | 196.1 | 14.4 |
| 2038 | 55–24–F | 510 | 64 | 52 | 58.2 | 1544 | 198.3 | 14.5 |
| 2039 | 110–32–F | 578 | 62 | 55 | 59.1 | 1815 | 197.1 | 14.5 |
| 2040 | 165–40–F | 640 | 64 | 55 | 57.9 | 1828 | 190.3 | 14.5 |
| 2041 | 110–40–F | 621 | 58 | 52 | 55.0 | 1800 | 192.3 | 14.5 |
| 2042 | 110–24–F | 557 | 59 | 53 | 57.1 | 1758 | 196.8 | 14.5 |
| 2043 | 110–48–F | 646 | 55 | 50 | 58.4 | 1856 | 194.4 | 14.5 |
| 2044 | 110–56–F | 686 | 54 | 48 | 56.8 | 1871 | 190.1 | 14.4 |
| 2045 | 110–16–F | 551 | 61 | 54 | 59.6 | 1775 | 197.1 | 14.4 |
| 2072 | 110–40–F | 620 | 59 | 54 | 59.5 | 1643 | 196.7 | 14.4 |
| 2073 | 110–32–F | 595 | 53 | 48 | 56.9 | 1662 | 197.6 | 14.5 |
| 2074 | 165–32–F | 637 | 59 | 52 | 59.7 | 1662 | 197.1 | 14.4 |
| 2075 | 55–32–F | 543 | 58 | 52 | 58.1 | 1661 | 198.1 | 14.4 |
| 2076 | 220–32–F | 675 | 63 | 54 | 60.1 | 1785 | 196.0 | 14.4 |
| 2077 | 110–24–F | 565 | 58 | 51 | 58.7 | 1570 | 196.0 | 14.3 |
| 2078 | 165–24–F | 595 | 62 | 53 | 58.4 | 1597 | 196.4 | 14.5 |
| 2079 | 220–24–F | 614 | 64 | 54 | 58.8 | 1688 | 197.4 | 14.5 |
| 2080 | 110–16–F | 534 | 60 | 53 | 59.7 | 1418 | 198.8 | 14.6 |
| 2081 | 55–24–F | 507 | 63 | 54 | 57.9 | 1407 | 198.2 | 14.6 |
| 2082 | 165–40–F | 673 | 55 | 51 | 63.6 | 1458 | 195.2 | 14.3 |
| 2083 | 165–16–F | 563 | 62 | 54 | 60.5 | 1267 | 198.7 | 14.4 |
| 2084 | 110–48–F | 653 | 60 | 56 | 52.6 | | 194.0 | 14.5 |
| 2085 | 55–40–F | 545 | 62 | 55 | 44.7 | | 197.9 | 14.4 |
| 2086 | 55–24–F | 506 | 67 | 57 | 56.2 | | 199.1 | 14.4 |
| 2087 | 110–16–F | 524 | 69 | 59 | 59.5 | | 199.2 | 14.3 |
| 2088 | 220–16–F | 563 | 73 | 61 | 58.1 | | 197.8 | 14.2 |
| 2089 | 220–40–F | 703 | 59 | 50 | 58.4 | | 194.9 | 14.4 |
| 2092 | 165–32–F | 643 | 52 | 49 | 59.4 | | 198.4 | 14.4 |
| 2093 | 165–48–F | 702 | 60 | 52 | 59.3 | | 191.5 | 14.3 |
| 2094 | 110–56–F | 679 | 65 | 58 | 59.0 | | 196.3 | 14.2 |
| 2095 | 55–48–F | 567 | 60 | 53 | 60.0 | | 198.1 | 14.2 |
| 2096 | 55–40–F | 584 | | | 61.4 | | 196.1 | 14.4 |
| 2097 | 55–32–F | 573 | | | 58.6 | | 198.8 | 14.4 |
| 2098 | 55–48–F | 611 | | | 59.2 | | 198.2 | 14.4 |
| 2090 | 110–24–F | 552 | 67 | 58 | 60.1 | | 199.0 | 14.4 |
| 2091 | 165–32–F | 626 | 64 | 57 | 59.5 | | 198.6 | 14.4 |

## TABLE 15.

## DRAFT, INJECTORS, QUALITY OF STEAM.

| Test No. | Laboratory Designation | Draft, in. of Water | | | | Injectors in Action | | | | Quality of Steam in Dome | Factor of Correction for Quality of Steam |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Front-end | | Fire Box | Ash Pan | Right, Total Hours | Left, Total Hours | Right, No. of Times | Left, No. of Times | | |
| | | Front of Diaphragm | Back of Diaphragm | | | | | | | | |
| | Code Item☞ | 394 | 395 | 396 | 397 | 403 | 404 | 405 | 406 | 407 | 412 |
| 2009 | 138–16–F | 3.7 | 2.6 | 1.6 | | | | | | 0.990 | 0.9919 |
| 2010 | 193–20–F | 5.5 | 3.9 | 2.1 | | | | | | 0.987 | 0.9908 |
| 2012 | 138–24–F | 5.8 | 4.0 | 2.1 | | | | | | 0.989 | 0.9918 |
| 2013 | 138–32–F | 7.6 | 5.1 | 2.7 | | | | | | 0.985 | 0.9888 |
| 2014 | 193–32–F | 9.1 | 6.0 | 3.2 | | | | | | 0.984 | 0.9883 |
| 2015 | 193–24–F | 7.3 | 4.9 | 2.8 | | | | | | 0.987 | 0.9908 |
| 2016 | 193–16–F | 5.0 | 3.5 | 2.0 | | | | | | 0.989 | 0.9919 |
| 2017 | 83–16–F | 2.7 | 1.9 | 0.9 | | | | | | 0.991 | 0.9935 |
| 2018 | 83–24–F | 3.9 | 2.7 | 1.5 | | | | | | 0.993 | 0.9949 |
| 2019 | 83–32–F | 5.4 | 3.7 | 2.0 | | | | | | 0.991 | 0.9935 |
| 2020 | 83–24–F | 3.4 | 2.3 | 1.2 | | 1.6 | 0.0 | 19 | 0 | 0.9929 | 0.9949 |
| 2021 | 83–16–F | 2.2 | 1.4 | 0.7 | | 1.3 | 0.0 | 29 | 0 | 0.9945 | 0.9961 |
| 2022 | 83–32–F | 4.8 | 3.0 | 1.6 | | 2.0 | 0.0 | 3 | 0 | 0.9956 | 0.9968 |
| 2023 | 138–40–F | 9.0 | 5.6 | 2.1 | | 0.5 | 1.5 | 26 | 1 | 0.9930 | 0.9950 |
| 2024 | 55–24–F | 2.6 | 1.6 | 1.0 | | 1.2 | 0.0 | 37 | 0 | 0.9950 | 0.9964 |
| 2026 | 110–16–F | 2.9 | 1.9 | 1.0 | | 1.4 | 0.0 | 50 | 0 | 0.9895 | 0.9925 |
| 2027 | 110–24–F | 4.7 | 2.7 | 1.5 | | 2.1 | 0.0 | 33 | 0 | 0.9914 | 0.9938 |
| 2028 | 55–32–F | 3.2 | 2.2 | 1.1 | | 2.1 | 0.0 | 32 | 0 | 0.9912 | 0.9936 |
| 2029 | 110–32–F | 6.1 | 3.9 | 1.8 | | 1.5 | 0.0 | 3 | 0 | 0.9902 | 0.9930 |
| 2030 | 165–24–F | 6.2 | 3.9 | 2.0 | | 1.7 | 0.0 | 1 | 0 | 0.9894 | 0.9924 |
| 2031 | 83–40–F | 6.8 | 4.3 | 2.2 | | 1.5 | 0.1 | 1 | 4 | 0.9896 | 0.9926 |
| 2032 | 165–32–F | 8.4 | 4.9 | 2.3 | | 0.7 | 0.1 | 1 | 5 | 0.9862 | 0.9901 |
| 2033 | 110–48–F | 8.2 | 5.1 | 2.2 | | 0.3 | 1.3 | 16 | 1 | 0.9867 | 0.9904 |
| 2034 | 193–40–F | 10.7 | 6.6 | 2.3 | | 0.6 | 1.0 | 17 | 1 | 0.9857 | 0.9898 |
| 2035 | 110–40–F | 8.4 | 5.2 | 2.7 | | 0.5 | 1.2 | 13 | 1 | 0.9833 | 0.9879 |
| 2037 | 165–40–F | 10.2 | 6.2 | 3.2 | | 0.5 | 0.0 | 24 | 0 | 0.9860 | 0.9900 |
| 2038 | 55–24–F | 2.6 | 1.7 | 1.1 | 0.3 | 1.8 | 0.0 | 1 | 0 | 0.9934 | 0.9953 |
| 2039 | 110–32–F | 6.9 | 4.5 | 2.4 | 0.6 | 1.5 | 0.0 | 1 | 0 | 0.9917 | 0.9941 |
| 2040 | 165–40–F | 9.9 | 6.8 | 3.2 | 0.6 | 0.6 | 1.0 | 16 | 1 | 0.9917 | 0.9940 |
| 2041 | 110–40–F | 8.7 | 5.5 | 2.2 | 0.5 | 0.3 | 1.2 | 20 | 1 | 0.9900 | 0.9852 |
| 2042 | 110–24–F | 4.9 | 3.2 | 1.7 | 0.4 | 2.0 | 0.0 | 1 | 0 | 0.9932 | 0.9952 |
| 2043 | 110–48–F | 9.7 | 5.9 | 2.4 | 0.7 | 0.6 | 1.2 | 24 | 1 | 0.9866 | 0.9904 |
| 2044 | 110–56–F | 11.8 | 7.5 | 3.2 | 0.7 | 0.8 | 1.0 | 5 | 1 | 0.9900 | 0.9928 |
| 2045 | 110–16–F | 3.2 | 2.2 | 1.1 | 0.3 | 2.2 | 0.0 | 1 | 0 | 0.9934 | 0.9953 |
| 2072 | 110–40–F | 8.0 | 4.9 | 2.0 | 0.7 | 0.3 | 1.0 | 20 | 1 | 0.9895 | 0.9925 |
| 2073 | 110–32–F | 5.7 | 3.5 | 1.6 | 0.5 | 1.3 | 0.0 | 1 | 0 | 0.9919 | 0.9942 |
| 2074 | 165–32–F | 8.5 | 5.3 | 2.4 | 0.5 | 0.5 | 0.9 | 21 | 1 | 0.9861 | 0.9901 |
| 2075 | 55–32–F | 2.8 | 1.8 | 0.9 | 0.3 | 2.3 | 0.0 | 1 | 0 | 0.9952 | 0.9970 |
| 2076 | 220–32–F | 9.2 | 5.7 | 2.8 | 0.5 | 3.8 | 0.6 | 10 | 1 | 0.9844 | 0.9888 |
| 2077 | 110–24–F | 4.1 | 2.5 | 1.3 | 0.4 | 1.8 | 0.0 | 3 | 0 | 0.9947 | 0.9963 |
| 2078 | 165–24–F | 6.0 | 3.9 | 1.9 | 0.5 | 1.2 | 0.1 | 3 | 3 | 0.9915 | 0.9939 |
| 2079 | 220–24–F | 7.0 | 4.8 | 2.3 | 0.4 | 0.1 | 1.0 | 13 | 1 | 0.9889 | 0.9921 |
| 2080 | 110–16–F | 2.9 | 2.0 | 1.0 | 0.2 | 2.2 | 0.0 | 1 | 0 | 0.9956 | 0.9968 |
| 2081 | 55–24–F | 2.2 | 1.5 | 0.7 | 0.2 | 2.5 | 0.0 | 4 | 0 | 0.9963 | 0.9974 |
| 2082 | 165–40–F | 11.2 | 7.3 | 3.4 | 0.6 | 0.4 | 0.8 | 18 | 1 | 0.9886 | 0.9917 |
| 2083 | 165–16–F | 4.3 | 2.7 | 1.6 | 0.4 | 1.7 | 0.0 | 1 | 0 | 0.9934 | 0.9953 |
| 2084 | 110–48–F | 10.0 | 6.3 | 2.9 | 0.8 | 0.8 | 0.4 | 21 | 1 | 0.9896 | 0.9926 |
| 2085 | 55–40–F | 4.0 | 2.5 | 1.1 | 0.3 | 2.0 | 0.0 | 1 | 0 | 0.9962 | 0.9973 |
| 2086 | 55–24–F | 2.2 | 1.4 | 0.7 | 0.3 | 2.8 | 0.0 | 1 | 0 | 0.9963 | 0.9974 |
| 2087 | 110–16–F | 2.9 | 1.9 | 0.9 | 0.2 | 2.5 | 0.0 | 1 | 0 | 0.9956 | 0.9968 |
| 2088 | 220–16–F | 4.5 | 2.9 | 1.2 | 0.3 | 1.7 | 0.0 | 1 | 0 | 0.9919 | 0.9942 |
| 2089 | 220–40–F | 11.9 | 7.1 | 3.5 | 0.7 | 0.6 | 0.6 | 3 | 1 | 0.9884 | 0.9917 |
| 2092 | 165–32–F | 8.2 | 5.1 | 2.2 | 0.6 | 0.3 | 0.8 | 16 | 1 | 0.9894 | 0.9924 |
| 2093 | 165–48–F | 12.8 | 7.8 | 3.4 | 0.9 | 0.3 | 0.5 | 3 | 1 | 0.9879 | 0.9913 |
| 2094 | 110–56–F | 12.1 | 7.3 | 3.0 | 0.8 | 0.4 | 0.4 | 2 | 1 | 0.9887 | 0.9919 |
| 2095 | 55–48–F | 5.3 | 3.0 | 1.5 | 0.4 | 1.0 | 0.0 | 1 | 0 | 0.9943 | 0.9959 |
| 2096 | 55–40–F | 4.2 | 2.6 | 1.2 | 0.4 | 1.5 | 0.1 | 1 | 1 | 0.9891 | 0.9922 |
| 2097 | 55–32–F | 3.2 | 2.3 | 1.0 | 0.5 | 1.8 | 0.0 | 1 | 0 | 0.9853 | 0.9895 |
| 2098 | 55–48–F | 5.4 | 3.7 | 1.9 | 0.8 | 0.8 | 0.0 | 1 | 0 | 0.9840 | 0.9885 |
| 2090 | 110–24–F | 4.2 | 2.9 | 1.1 | 0.3 | 1.0 | 0.0 | 1 | 0 | 0.9952 | 0.9966 |
| 2091 | 165–32–F | 8.2 | 4.9 | 2.1 | 0.6 | 0.2 | 0.5 | 8 | 1 | 0.9911 | 0.9936 |

## TABLE 16.

## COAL, CINDERS, ASH, SMOKE, AND HUMIDITY.

| Test No. | Laboratory Designation | Coal Fired Total, lb. | Dry Coal Fired Total, lb. | Combustible by Analysis Total, lb. | Ash by Analysis Total, lb. | Front-end Cinders Total, lb. | Stack Cinders Total, lb. | Front End and Stack Cinders Total, lb. |
|---|---|---|---|---|---|---|---|---|
| | Code Item☞ | 418 | 419 | 420 | 421 | 422 | 423 | 424 |
| 2009 | 138–16–F | 7497 | 6618 | 5786 | 832 | 26 | 413 | 439 |
| 2010 | 193–20–F | 5147 | 4537 | 3943 | 593 | 10 | 507 | 517 |
| 2012 | 138–24–F | 7657 | 6797 | 5835 | 962 | 9 | 648 | 657 |
| 2013 | 138–32–F | 7832 | 7124 | 6119 | 1005 | 10 | 925 | 935 |
| 2014 | 193–32–F | 7978 | 7230 | 6153 | 1077 | 9 | 1045 | 1054 |
| 2015 | 193–24–F | 8298 | 7391 | 6365 | 1026 | 12 | 1092 | 1104 |
| 2016 | 193–16–F | 8603 | 7595 | 6368 | 1228 | 11 | 681 | 692 |
| 2017 | 83–16–F | 6535 | 5872 | 5079 | 793 | 12 | 187 | 199 |
| 2018 | 83–24–F | 7589 | 6765 | 5795 | 970 | 29 | 380 | 409 |
| 2019 | 83–32–F | 7793 | 6965 | 6096 | 865 | 14 | 586 | 600 |
| 2020 | 83–24–F | 5416 | 4943 | 4318 | 625 | 16 | 286 | 302 |
| 2021 | 83–16–F | 5040 | 4422 | 3837 | 585 | 22 | 243 | 265 |
| 2022 | 83–32–F | 8198 | 7346 | 6222 | 1124 | 8 | 898 | 906 |
| 2023 | 138–40–F | 11556 | 10031 | 8637 | 1394 | 7 | 2075 | 2082 |
| 2024 | 55–24–F | 4104 | 3628 | 3171 | 456 | 20 | 115 | 135 |
| 2026 | 110–16–F | 5693 | 4969 | 4265 | 704 | 25 | 279 | 304 |
| 2027 | 110–24–F | 9322 | 8139 | 7075 | 1064 | 20 | 745 | 765 |
| 2028 | 55–32–F | 6414 | 5615 | 4887 | 729 | 14 | 372 | 386 |
| 2029 | 110–32–F | 7257 | 6363 | 5445 | 918 | 15 | 869 | 884 |
| 2030 | 165–24–F | 7501 | 6689 | 5888 | 801 | 11 | 856 | 867 |
| 2031 | 83–40–F | 7686 | 6366 | 5414 | 952 | 13 | 944 | 957 |
| 2032 | 165–32–F | 4104 | 3568 | 3037 | 532 | 10 | 662 | 672 |
| 2033 | 110–48–F | 7940 | 6835 | 5839 | 996 | 10 | 1219 | 1229 |
| 2034 | 193–40–F | 8916 | 7767 | 6501 | 1266 | 10 | 1499 | 1509 |
| 2035 | 110–40–F | 7590 | 6493 | 5575 | 918 | 6 | 934 | 940 |
| 2037 | 165–40–F | 7625 | 6554 | 5631 | 923 | 11 | 1206 | 1217 |
| 2038 | 55–24–F | 4202 | 3688 | 2985 | 703 | 19 | 126 | 145 |
| 2039 | 110–32–F | 7678 | 6775 | 5639 | 1136 | 17 | 808 | 825 |
| 2040 | 165–40–F | 8515 | 7482 | 6325 | 1157 | 10 | 1617 | 1627 |
| 2041 | 110–40–F | 7957 | 6838 | 5851 | 987 | 6 | 1218 | 1224 |
| 2042 | 110–24–F | 7771 | 6713 | 5720 | 992 | 6 | 663 | 669 |
| 2043 | 110–48–F | 9979 | 8637 | 7431 | 1205 | 17 | 1829 | 1846 |
| 2044 | 110–56–F | 9623 | 8361 | 7080 | 1282 | 10 | 1771 | 1781 |
| 2045 | 110–16–F | 6061 | 5258 | 4369 | 889 | 23 | 453 | 476 |
| 2072 | 110–40–F | 6802 | 5927 | 5103 | 824 | | 1140 | |
| 2073 | 110–32–F | 6703 | 5812 | 5116 | 694 | 12 | 736 | 748 |
| 2074 | 165–32–F | 6766 | 6015 | 5206 | 805 | 3 | 949 | 952 |
| 2075 | 55–32–F | 6236 | 5430 | 4791 | 639 | 13 | 385 | 398 |
| 2076 | 220–32–F | 5394 | 4568 | 3983 | 585 | 14 | 1057 | 1071 |
| 2077 | 110–24–F | 6896 | 6015 | 5240 | 775 | 14 | 449 | 463 |
| 2078 | 165–24–F | 6332 | 5492 | 4786 | 706 | 16 | 758 | 774 |
| 2079 | 220–24–F | 6332 | 5783 | 5121 | 662 | 14 | 873 | 887 |
| 2080 | 110–16–F | 6031 | 5248 | 4666 | 581 | 15 | 210 | 225 |
| 2081 | 55–24–F | 5591 | 4937 | 4355 | 582 | 18 | 155 | 173 |
| 2082 | 165–40–F | 8506 | 7495 | 6624 | 871 | 19 | 1640 | 1659 |
| 2083 | 165–16–F | 6453 | 5564 | 4847 | 717 | 21 | 432 | 453 |
| 2084 | 110–48–F | 7592 | 6595 | 5633 | 963 | 20 | 1244 | 1264 |
| 2085 | 55–40–F | 6991 | 6116 | 5353 | 763 | 17 | 434 | 451 |
| 2086 | 55–24–F | 6660 | 5860 | 5082 | 778 | 17 | 310 | 327 |
| 2087 | 110–16–F | 7004 | 6185 | 5337 | 847 | 18 | 296 | 314 |
| 2088 | 220–16–F | 6445 | 5588 | 4688 | 900 | 17 | 456 | 473 |
| 2089 | 220–40–F | 7401 | 6491 | 5558 | 933 | 13 | 1728 | 1741 |
| 2092 | 165–32–F | 5491 | 4700 | 4128 | 572 | 12 | 698 | 710 |
| 2093 | 165–48–F | 5933 | 5108 | 4444 | 664 | 10 | 1391 | 1401 |
| 2094 | 110–56–F | 4095 | 3514 | 3055 | 459 | 13 | 811 | 824 |
| 2095 | 55–48–F | 3799 | 3334 | 2854 | 480 | 10 | 250 | 260 |
| 2096 | 55–40–F | | | | | | | |
| 2097 | 55–32–F | | | | | | | |
| 2098 | 55–48–F | | | | | | | |
| 2090 | 110–24–F | 3605 | 3176 | 2772 | 404 | 14 | 312 | 326 |
| 2091 | 165–32–F | 3028 | 2626 | 2302 | 324 | 12 | 431 | 443 |

## TABLE 17.

## COAL, CINDERS, ASH, SMOKE, AND HUMIDITY.

| Test No. | Laboratory Designation | Cinder Loss, Per cent of Total Dry Coal Fired | Stack Cinder Loss, Per cent of Total Dry Coal Fired | Ash from Ash Pan | | | Smoke | Humidity |
|---|---|---|---|---|---|---|---|---|
| | | | | Total, lb. | Per cent of Total Dry Coal Fired | Per cent of Ash by Analysis | Per cent of Blackness by Ringlemann Chart | Moisture per lb. of Dry Air, lb. |
| | Code Item ☞ | 426 | 427 | 428 | 429 | 430 | 431 | 435 |
| 2009 | 138–16–F | 6.6 | 6.2 | 172 | 2.6 | 20.7 | | .014 |
| 2010 | 193–20–F | 11.4 | 11.2 | 69 | 1.5 | 11.6 | | .008 |
| 2012 | 138–24–F | 9.7 | 9.5 | 318 | 4.7 | 33.0 | | .011 |
| 2013 | 138–32–F | 13.1 | 13.0 | 301 | 4.2 | 30.0 | | .011 |
| 2014 | 193–32–F | 14.6 | 14.5 | 159 | 2.2 | 14.7 | | .014 |
| 2015 | 193–24–F | 14.9 | 14.8 | 557 | 7.5 | 54.2 | | .014 |
| 2016 | 193–16–F | 9.1 | 9.0 | 445 | 5.9 | 36.2 | | .014 |
| 2017 | 83–16–F | 3.4 | 3.2 | 295 | 5.0 | 37.2 | | .015 |
| 2018 | 83–24–F | 6.1 | 5.6 | 309 | 4.6 | 31.9 | | .011 |
| 2019 | 83–32–F | 8.6 | 8.4 | 387 | 5.6 | 44.5 | | .014 |
| 2020 | 83–24–F | 6.1 | 5.8 | 151 | 3.1 | 24.2 | | .010 |
| 2021 | 83–16–F | 6.0 | 5.5 | 455 | 10.3 | 77.8 | | .006 |
| 2022 | 83–32–F | 12.3 | 12.2 | 568 | 7.7 | 50.5 | | .010 |
| 2023 | 138–40–F | 20.8 | 20.7 | 631 | 6.3 | 45.3 | | .007 |
| 2024 | 55–24–F | 3.7 | 3.2 | 202 | 5.6 | 44.3 | | .005 |
| 2026 | 110–16–F | 6.1 | 5.6 | 390 | 7.9 | 55.4 | 45 | .007 |
| 2027 | 110–24–F | 9.4 | 9.2 | 513 | 6.3 | 48.2 | 43 | .008 |
| 2028 | 55–32–F | 6.9 | 6.6 | 429 | 7.6 | 58.9 | 20 | .009 |
| 2029 | 110–32–F | 13.9 | 13.7 | 503 | 7.9 | 54.8 | 44 | .011 |
| 2030 | 165–24–F | 13.0 | 12.8 | 374 | 5.6 | 46.7 | 55 | .013 |
| 2031 | 83–40–F | 15.0 | 14.8 | 434 | 6.8 | 45.6 | | .012 |
| 2032 | 165–32–F | 18.8 | 18.6 | 222 | 6.2 | 41.7 | 51 | .013 |
| 2033 | 110–48–F | 18.0 | 17.8 | 409 | 6.0 | 41.1 | | .013 |
| 2034 | 193–40–F | 19.4 | 19.3 | 645 | 8.3 | 51.0 | | .014 |
| 2035 | 110–40–F | 14.5 | 14.4 | 518 | 8.0 | 56.4 | | .011 |
| 2037 | 165–40–F | 18.6 | 18.4 | 445 | 6.8 | 48.2 | | .014 |
| 2038 | 55–24–F | 3.9 | 3.4 | 280 | 7.6 | 39.8 | | .006 |
| 2039 | 110–32–F | 12.2 | 11.9 | 476 | 7.0 | 41.9 | | .008 |
| 2040 | 165–40–F | 21.8 | 21.6 | 393 | 5.3 | 34.0 | | .007 |
| 2041 | 110–40–F | 17.9 | 17.8 | 400 | 5.9 | 40.5 | | .007 |
| 2042 | 110–24–F | 10.0 | 9.9 | 410 | 6.1 | 41.3 | | .007 |
| 2043 | 110–48–F | 21.4 | 21.2 | 463 | 5.4 | 38.4 | | .006 |
| 2044 | 110–56–F | 21.3 | 21.2 | 410 | 4.9 | 32.0 | | .006 |
| 2045 | 110–16–F | 9.0 | 8.6 | 400 | 7.6 | 45.0 | 32 | .007 |
| 2072 | 110–40–F | | 19.2 | 273 | 4.6 | 33.1 | 42 | .008 |
| 2073 | 110–32–F | 12.9 | 12.7 | 336 | 5.8 | 48.4 | 35 | .006 |
| 2074 | 165–32–F | 15.8 | 15.8 | 416 | 6.9 | 51.7 | 45 | .007 |
| 2075 | 55–32–F | 7.3 | 7.1 | 216 | 4.0 | 33.8 | 20 | .007 |
| 2076 | 220–32–F | 23.4 | 23.1 | 313 | 6.9 | 53.5 | | .007 |
| 2077 | 110–24–F | 7.7 | 7.5 | 200 | 3.3 | 25.8 | | .006 |
| 2078 | 165–24–F | 14.1 | 13.8 | 282 | 5.1 | 39.9 | | .007 |
| 2079 | 220–24–F | 15.3 | 15.1 | 569 | 9.8 | 86.0 | | .007 |
| 2080 | 110–16–F | 4.3 | 4.0 | 428 | 8.2 | 73.7 | | .007 |
| 2081 | 55–24–F | 3.5 | 3.1 | 427 | 8.7 | 73.4 | | .007 |
| 2082 | 165–40–F | 22.1 | 21.9 | 576 | 7.7 | 66.1 | | .007 |
| 2083 | 165–16–F | 8.2 | 7.8 | 564 | 10.1 | 78.7 | | .007 |
| 2084 | 110–48–F | 19.2 | 18.9 | 501 | 7.6 | 52.0 | | .008 |
| 2085 | 55–40–F | 7.4 | 7.1 | 530 | 8.7 | 69.5 | | .008 |
| 2086 | 55–24–F | 5.6 | 5.3 | 555 | 9.5 | 71.3 | | .008 |
| 2087 | 110–16–F | 5.1 | 4.8 | 604 | 9.8 | 71.3 | | .008 |
| 2088 | 220–16–F | 8.5 | 8.2 | 523 | 9.4 | 58.1 | | .008 |
| 2089 | 220–40–F | 26.8 | 26.6 | 429 | 6.6 | 46.0 | | .006 |
| 2092 | 165–32–F | 15.1 | 14.9 | 357 | 7.6 | 62.4 | | .006 |
| 2093 | 165–48–F | 27.4 | 27.2 | 495 | 9.7 | 74.5 | | .006 |
| 2094 | 110–56–F | 23.5 | 23.1 | 234 | 6.7 | 51.0 | | .008 |
| 2095 | 55–48–F | 7.8 | 7.5 | 256 | 7.7 | 53.3 | | .007 |
| 2096 | 55–40–F | | | | | | | |
| 2097 | 55–32–F | | | | | | | |
| 2098 | 55–48–F | | | | | | | |
| 2090 | 110–24–F | 10.3 | 9.8 | 315 | 9.9 | 78.0 | | .008 |
| 2091 | 165–32–F | 16.9 | 16.4 | 260 | 9.9 | 80.3 | | .008 |

## TABLE 18.

## COAL ANALYSIS.

| Test No. | Laboratory Designation | Proximate Analysis Coal as Fired | | | | | Calorific Value per lb. of Coal as Fired, B.t.u. | Ultimate Analysis Coal as Fired | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Fixed Carbon, per cent | Volatile Matter, per cent | Moisture, per cent | Ash, per cent | Sulphur Separately Determined, per cent | | Carbon, per cent | Hydrogen, per cent | Nitrogen, per cent | Oxygen, per cent |
| | CodeItem☞ | 437 | 438 | 440 | 441 | 442 | 443 | 449 | 450 | 451 | 452 |
| 2009 | 138–16–F | 38.21 | 38.97 | 11.72 | 11.10 | 2.33 | 11 083 | 61.68 | 4.48 | 0.85 | 7.84 |
| 2010 | 193–20–F | 37.71 | 38.90 | 11.86 | 11.53 | 3.54 | 10 959 | 60.22 | 4.37 | 0.83 | 7.65 |
| 2012 | 138–24–F | 38.08 | 38.12 | 11.23 | 12.57 | 3.43 | 10 901 | 59.97 | 4.36 | 0.83 | 7.62 |
| 2013 | 138–32–F | 38.92 | 39.21 | 9.04 | 12.83 | 3.56 | 11 135 | 61.45 | 4.46 | 0.85 | 7.81 |
| 2014 | 193–32–F | 38.14 | 38.99 | 9.37 | 13.50 | 3.43 | 11 042 | 60.74 | 4.41 | 0.84 | 7.72 |
| 2015 | 193–24–F | 38.16 | 38.55 | 10.92 | 12.37 | 3.51 | 10 963 | 60.32 | 4.38 | 0.83 | 7.66 |
| 2016 | 193–16–F | 36.86 | 37.16 | 11.71 | 14.27 | 4.16 | 10 588 | 57.57 | 4.18 | 0.79 | 7.31 |
| 2017 | 83–16–F | 38.80 | 38.92 | 10.15 | 12.13 | 3.50 | 11 179 | 61.17 | 4.44 | 0.84 | 7.72 |
| 2018 | 83–24–F | 38.25 | 38.11 | 10.86 | 12.78 | 3.12 | 10 932 | 60.36 | 4.38 | 0.83 | 7.67 |
| 2019 | 83–32–F | 37.68 | 40.55 | 10.62 | 11.15 | 3.13 | 11 192 | 61.89 | 4.43 | 0.85 | 7.86 |
| 2020 | 83–24–F | 39.40 | 40.33 | 8.73 | 11.54 | 3.86 | 11 228 | 62.42 | 3.80 | 1.61 | 8.03 |
| 2021 | 83–16–F | 37.59 | 38.54 | 12.27 | 11.60 | 3.69 | 10 768 | 59.60 | 3.63 | 1.54 | 7.67 |
| 2022 | 83–32–F | 37.57 | 38.33 | 10.39 | 13.71 | 4.36 | 10 642 | 58.86 | 3.59 | 1.52 | 7.57 |
| 2023 | 138–40–F | 36.41 | 38.33 | 13.20 | 12.06 | 4.38 | 10 686 | 57.89 | 3.53 | 1.50 | 7.45 |
| 2024 | 55–24–F | 37.02 | 40.25 | 11.61 | 11.12 | 3.41 | 11 236 | 60.77 | 3.70 | 1.57 | 7.82 |
| 2026 | 110–16–F | 35.94 | 38.98 | 12.72 | 12.36 | 3.68 | 10 743 | 58.61 | 3.57 | 1.51 | 7.54 |
| 2027 | 110–24–F | 37.38 | 38.52 | 12.69 | 11.41 | 3.42 | 11 078 | 59.64 | 3.63 | 1.54 | 7.67 |
| 2028 | 55–32–F | 37.11 | 39.08 | 12.45 | 11.36 | 3.21 | 11 077 | 60.05 | 3.66 | 1.55 | 7.72 |
| 2029 | 110–32–F | 35.06 | 39.97 | 12.32 | 12.65 | 3.67 | 10 948 | 58.71 | 3.58 | 1.52 | 7.55 |
| 2030 | 165–24–F | 38.03 | 40.46 | 10.83 | 10.68 | 3.61 | 11 376 | 61.61 | 3.75 | 1.59 | 7.92 |
| 2031 | 83–40–F | 34.11 | 36.33 | 17.18 | 12.38 | 3.15 | 9 929 | 55.36 | 3.37 | 1.43 | 7.12 |
| 2032 | 165–32–F | 35.55 | 38.44 | 13.05 | 12.96 | 4.36 | 10 644 | 57.29 | 3.49 | 1.48 | 7.37 |
| 2033 | 110–48–F | 35.99 | 37.55 | 13.92 | 12.54 | 4.16 | 10 539 | 57.08 | 3.48 | 1.48 | 7.34 |
| 2034 | 193–40–F | 34.86 | 38.05 | 12.89 | 14.20 | 3.99 | 10 309 | 56.71 | 3.45 | 1.47 | 7.29 |
| 2035 | 110–40–F | 34.41 | 39.04 | 14.46 | 12.09 | 4.09 | 10 547 | 57.07 | 3.48 | 1.48 | 7.34 |
| 2037 | 165–40–F | 35.69 | 38.10 | 14.05 | 12.10 | 4.31 | 10 693 | 57.22 | 3.49 | 1.48 | 7.36 |
| | | | | | | | | | | | |
| 2038 | 55–24–F | 34.50 | 36.54 | 12.24 | 16.72 | 3.63 | 10 041 | 55.46 | 3.38 | 1.43 | 7.13 |
| 2039 | 110–32–F | 35.80 | 37.65 | 11.76 | 14.79 | 4.36 | 10 355 | 56.85 | 3.46 | 1.47 | 7.31 |
| 2040 | 165–40–F | 35.58 | 38.70 | 12.13 | 13.59 | 4.19 | 10 688 | 57.67 | 3.51 | 1.49 | 7.42 |
| 2041 | 110–40–F | 35.64 | 37.89 | 14.06 | 12.41 | 3.55 | 10 550 | 57.58 | 3.51 | 1.49 | 7.41 |
| 2042 | 110–24–F | 36.24 | 37.37 | 13.62 | 12.77 | 3.73 | 10 602 | 57.50 | 3.50 | 1.49 | 7.39 |
| 2043 | 110–48–F | 36.48 | 37.99 | 13.45 | 12.08 | 3.50 | 10 841 | 58.39 | 3.56 | 1.51 | 7.51 |
| 2044 | 110–56–F | 36.25 | 37.32 | 13.11 | 13.32 | 3.56 | 10 594 | 57.60 | 3.51 | 1.49 | 7.41 |
| 2045 | 110–16–F | 35.43 | 36.66 | 13.25 | 14.66 | 4.55 | 10 310 | 55.57 | 3.39 | 1.44 | 7.15 |
| | | | | | | | | | | | |
| 2072 | 110–40–F | 37.21 | 37.81 | 12.87 | 12.11 | 4.28 | 10 857 | 57.75 | 4.27 | 2.10 | 6.62 |
| 2073 | 110–32–F | 38.60 | 37.72 | 13.33 | 10.35 | 3.78 | 11 051 | 59.21 | 4.38 | 2.15 | 6.79 |
| 2074 | 165–32–F | 38.79 | 38.16 | 11.15 | 11.90 | 3.36 | 11 173 | 60.07 | 4.44 | 2.19 | 6.89 |
| 2075 | 55–32–F | 38.85 | 37.98 | 12.93 | 10.24 | 3.59 | 11 074 | 59.79 | 4.42 | 2.18 | 6.86 |
| 2076 | 220–32–F | 35.81 | 38.03 | 15.31 | 10.85 | 2.79 | 10 602 | 58.00 | 4.29 | 2.11 | 6.65 |
| 2077 | 110–24–F | 37.94 | 38.04 | 12.78 | 11.24 | 3.64 | 11 019 | 59.05 | 4.37 | 2.15 | 6.77 |
| 2078 | 165–24–F | 37.88 | 37.71 | 13.26 | 11.15 | 3.66 | 10 858 | 58.72 | 4.34 | 2.14 | 6.73 |
| 2079 | 220–24–F | 40.44 | 40.44 | 8.67 | 10.45 | 3.56 | 11 660 | 63.12 | 4.67 | 2.30 | 7.24 |
| 2080 | 110–16–F | 36.71 | 40.66 | 12.99 | 9.64 | 3.32 | 11 178 | 60.45 | 4.47 | 2.20 | 6.93 |
| 2081 | 55–24–F | 38.17 | 39.72 | 11.70 | 10.41 | 3.29 | 11 214 | 60.90 | 4.51 | 2.22 | 6.98 |
| 2082 | 165–40–F | 37.92 | 39.95 | 11.89 | 10.24 | 3.27 | 11 125 | 60.90 | 4.51 | 2.22 | 6.98 |
| 2083 | 165–16–F | 36.71 | 38.41 | 13.77 | 11.11 | 3.16 | 10 916 | 58.74 | 4.35 | 2.14 | 6.74 |
| 2084 | 110–48–F | 35.11 | 39.08 | 13.13 | 12.68 | 3.51 | 10 689 | 57.70 | 4.27 | 2.10 | 6.62 |
| 2085 | 55–40–F | 37.53 | 39.04 | 12.52 | 10.91 | 3.13 | 11 042 | 59.95 | 4.44 | 2.18 | 6.87 |
| 2086 | 55–24–F | 36.86 | 39.45 | 12.01 | 11.68 | 3.58 | 11 075 | 59.37 | 4.39 | 2.16 | 6.81 |
| 2087 | 110–16–F | 36.77 | 39.43 | 11.70 | 12.10 | 3.68 | 10 836 | 59.20 | 4.38 | 2.15 | 6.79 |
| 2088 | 220–16–F | 35.86 | 36.88 | 13.30 | 13.96 | 4.04 | 10 487 | 56.08 | 4.15 | 2.04 | 6.43 |
| 2089 | 220–40–F | 36.54 | 38.56 | 12.30 | 12.60 | 3.33 | 10 837 | 58.59 | 4.33 | 2.13 | 6.72 |
| 2092 | 165–32–F | 37.35 | 37.83 | 14.40 | 10.42 | 3.02 | 10 802 | 58.90 | 4.36 | 2.14 | 6.75 |
| 2093 | 165–48–F | 37.16 | 37.74 | 13.90 | 11.20 | 2.69 | 10 807 | 58.95 | 4.36 | 2.14 | 6.76 |
| 2094 | 110–56–F | 36.92 | 37.68 | 14.20 | 11.20 | 3.41 | 10 662 | 58.11 | 4.30 | 2.11 | 6.66 |
| 2095 | 55–48–F | 37.26 | 37.87 | 12.24 | 12.63 | 2.89 | 10 808 | 58.97 | 4.36 | 2.15 | 6.76 |
| 2096 | 55–40–F | | | | | | | | | | |
| 2097 | 55–32–F | | | | | | | | | | |
| 2098 | 55–48–F | | | | | | | | | | |
| | | | | | | | | | | | |
| 2090 | 110–24–F | 36.99 | 39.90 | 11.91 | 11.20 | 3.51 | 11 094 | 59.90 | 4.43 | 2.18 | 6.87 |
| 2091 | 165–32–F | 37.15 | 38.87 | 13.28 | 10.70 | 3.99 | 10 965 | 58.80 | 4.35 | 2.14 | 6.74 |

## TABLE 19.

## CALORIFIC VALUE OF COAL AND CINDERS, ANALYSIS OF FRONT-END GASES.

| Test No. | Laboratory Designation | Calorific Value, B.t.u. per lb. | | | | | Analysis of Front-end Gases | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Dry Coal | Combustible | Front-end Cinders | Stack Cinders | Ash | Oxygen $O_2$ | Carbon Monoxide CO | Carbon Dioxide $CO_2$ | Nitrogen $N_2$ |
| | Code Item ☞ | 458 | 459 | 461 | 462 | 463 | 466 | 467 | 468 | 469 |
| 2009 | 138–16–F | 12 553 | 14 360 | 7796 | 6685 | 5069 | 10.9 | 0.0 | 8.1 | 81.0 |
| 2010 | 193–20–F | 12 433 | 14 305 | 5544 | 8947 | 4587 | 13.7 | 0.0 | 6.0 | 80.3 |
| 2012 | 138–24–F | 12 280 | 14 306 | 6242 | 8042 | 4515 | 13.7 | 0.0 | 5.7 | 80.7 |
| 2013 | 138–32–F | 12 242 | 14 252 | 5312 | 8779 | 4785 | 13.5 | 0.1 | 6.0 | 80.6 |
| 2014 | 193–32–F | 12 184 | 14 316 | 5611 | 9704 | 4046 | 11.4 | 0.0 | 7.4 | 81.1 |
| 2015 | 193–24–F | 12 307 | 14 291 | 2586 | 9438 | 5297 | 11.9 | 0.0 | 7.3 | 80.8 |
| 2016 | 193–16–F | 11 992 | 14 304 | 6104 | 8359 | 4844 | 12.1 | 0.0 | 7.1 | 80.7 |
| 2017 | 83–16–F | 12 422 | 14 384 | 4821 | 6112 | 4624 | 11.3 | 0.1 | 8.0 | 80.5 |
| 2018 | 83–24–F | 12 265 | 14 316 | 6155 | 7515 | 4134 | 11.0 | 0.2 | 8.3 | 80.6 |
| 2019 | 83–32–F | 12 523 | 14 307 | 6523 | 8218 | 3272 | 10.3 | 0.2 | 8.9 | 80.6 |
| 2020 | 83–24–F | 12 302 | 14 083 | 5342 | 6850 | 6528 | 10.7 | 0.0 | 8.7 | 80.6 |
| 2021 | 83–16–F | 12 274 | 14 144 | 6573 | 7659 | 5267 | 11.8 | 0.0 | 7.7 | 80.5 |
| 2022 | 83–32–F | 11 875 | 14 021 | 3907 | 9492 | 3850 | 11.8 | 0.0 | 8.0 | 80.2 |
| 2023 | 138–40–F | 12 311 | 14 298 | 7357 | 10341 | 4842 | 6.6 | 0.0 | 11.2 | 82.2 |
| 2024 | 55–24–F | 12 712 | 14 541 | 5657 | 6459 | 4261 | 10.9 | 0.0 | 8.0 | 81.1 |
| 2026 | 110–16–F | 12 309 | 14 339 | 7003 | 8311 | 6064 | 11.6 | 0.0 | 7.7 | 80.7 |
| 2027 | 110–24–F | 12 688 | 14 596 | 6996 | 8274 | 4853 | 10.8 | 0.0 | 8.1 | 81.1 |
| 2028 | 55–32–F | 12 653 | 14 539 | 7109 | 7864 | 4032 | 10.4 | 0.0 | 8.1 | 81.5 |
| 2029 | 110–32–F | 12 486 | 14 591 | 4841 | 8914 | 5429 | 9.5 | 0.0 | 9.4 | 81.1 |
| 2030 | 165–24–F | 12 757 | 14 494 | 7007 | 9867 | 5618 | 8.9 | 0.0 | 9.7 | 81.4 |
| 2031 | 83–40–F | 11 989 | 14 096 | 2985 | 9677 | 4262 | 8.2 | 0.2 | 9.8 | 81.8 |
| 2032 | 165–32–F | 12 242 | 14 386 | 7539 | 4922 | 6021 | | | | |
| 2033 | 110–48–F | 12 243 | 14 331 | 2798 | 9888 | 4685 | 7.6 | 0.0 | 10.2 | 82.2 |
| 2034 | 193–40–F | 11 835 | 14 139 | 6172 | 10324 | 5327 | 7.0 | 0.0 | 10.4 | 82.6 |
| 2035 | 110–40–F | 12 329 | 14 359 | 5839 | 9698 | 5547 | 8.5 | 0.0 | 8.3 | 83.2 |
| 2037 | 165–40–F | 12 441 | 14 479 | 6543 | 10098 | 5942 | 6.0 | 0.0 | 10.8 | 83.2 |
| 2038 | 55–24–F | 11 442 | 14 134 | 6650 | 5772 | 6168 | 10.0 | 0.0 | 8.4 | 81.6 |
| 2039 | 110–32–F | 11 734 | 14 098 | 6127 | 8557 | 4341 | 8.1 | 0.0 | 10.2 | 81.7 |
| 2040 | 165–40–F | 12 164 | 14 389 | 4986 | 10227 | 5659 | 6.7 | 0.1 | 11.5 | 81.7 |
| 2041 | 110–40–F | 12 276 | 14 348 | 6656 | 9634 | 5122 | 8.1 | 0.1 | 10.5 | 81.3 |
| 2042 | 110–24–F | 12 273 | 14 403 | 6850 | 8425 | 5361 | 9.3 | 0.0 | 9.7 | 81.0 |
| 2043 | 110–48–F | 12 523 | 14 558 | 1512 | 10046 | 4840 | 7.0 | 0.4 | 11.7 | 80.9 |
| 2044 | 110–56–F | 12 192 | 14 400 | 3755 | 10654 | 4400 | 5.5 | 0.4 | 11.1 | 83.0 |
| 2045 | 110–16–F | 11 885 | 14 302 | 7518 | 6890 | 4862 | 11.5 | 0.0 | 7.3 | 81.2 |
| 2072 | 110–40–F | 12 460 | 14 472 | 4659 | 9926 | 3920 | 7.4 | 0.3 | 10.4 | 81.9 |
| 2073 | 110–32–F | 12 751 | 14 480 | 4934 | 9485 | 5216 | 7.7 | 0.1 | 9.8 | 82.4 |
| 2074 | 165–32–F | 12 575 | 14 520 | 5873 | 9780 | 4331 | 7.4 | 0.5 | 10.3 | 81.8 |
| 2075 | 55–32–F | 12 718 | 14 414 | 6273 | 6914 | 4450 | 10.7 | 0.1 | 8.0 | 81.2 |
| 2076 | 220–32–F | 12 519 | 14 358 | 6331 | 11014 | 4871 | 6.8 | 0.2 | 11.0 | 82.0 |
| 2077 | 110–24–F | 12 633 | 14 502 | 8064 | 8289 | 7618 | 9.1 | 0.1 | 9.2 | 81.7 |
| 2078 | 165–24–F | 12 517 | 14 364 | 6129 | 9454 | 4168 | 9.2 | 0.0 | 9.1 | 81.7 |
| 2079 | 220–24–F | 12 767 | 14 416 | 6024 | 9867 | 5587 | 8.1 | 0.0 | 9.9 | 82.0 |
| 2080 | 110–16–F | 12 848 | 14 448 | 6337 | 5522 | 4451 | 10.7 | 0.2 | 7.9 | 81.2 |
| 2081 | 55–24–F | 12 700 | 14 398 | 5995 | 6097 | 4697 | 11.5 | 0.0 | 7.1 | 81.3 |
| 2082 | 165–40–F | 12 626 | 14 287 | 6573 | 10548 | 4792 | 6.3 | 0.0 | 11.2 | 82.5 |
| 2083 | 165–16–F | 12 660 | 14 531 | 7740 | 9157 | 5126 | 9.5 | 0.0 | 9.2 | 81.3 |
| 2084 | 110–48–F | 12 305 | 14 408 | 3484 | 10655 | 4604 | 7.0 | 0.0 | 9.8 | 83.1 |
| 2085 | 55–40–F | 12 622 | 14 421 | 7364 | 9496 | 4865 | 9.4 | 0.0 | 8.9 | 81.7 |
| 2086 | 55–24–F | 12 586 | 14 513 | 3244 | 5777 | 4358 | 11.0 | 0.1 | 8.1 | 80.8 |
| 2087 | 110–16–F | 12 272 | 14 220 | 5656 | 6711 | 4393 | 10.1 | 0.0 | 8.6 | 81.3 |
| 2088 | 220–16–F | 12 095 | 14 418 | 2770 | 8456 | 4132 | 8.7 | 0.0 | 9.6 | 81.7 |
| 2089 | 220–40–F | 12 356 | 14 431 | 5914 | 10926 | 3691 | 4.3 | 0.4 | 12.2 | 83.1 |
| 2092 | 165–32–F | 12 620 | 14 368 | 5266 | 10165 | 5022 | 6.0 | 0.1 | 11.5 | 82.5 |
| 2093 | 165–48–F | 12 551 | 14 429 | 5928 | 10295 | 6473 | 4.7 | 0.2 | 12.4 | 82.7 |
| 2094 | 110–56–F | 12 426 | 14 292 | 6159 | 10447 | 5525 | 6.0 | 0.1 | 11.8 | 82.1 |
| 2095 | 55–48–F | 12 315 | 14 385 | 8983 | 8508 | 4670 | 9.4 | 0.0 | 9.7 | 80.9 |
| 2096 | 55–40–F | | | | | | | | | |
| 2097 | 55–32–F | | | | | | | | | |
| 2098 | 55–48–F | | | | | | | | | |
| 2090 | 110–24–F | 12 594 | 14 429 | 7406 | 8440 | 5118 | 8.9 | 0.1 | 9.8 | 81.3 |
| 2091 | 165–32–F | 12 626 | 14 423 | 7885 | 9831 | 5370 | 7.2 | 0.0 | 10.5 | 82.3 |

## TABLE 20.

## WATER AND DRAWBAR PULL.

| Test No. | Laboratory Designation | Water | | | | | | Drawbar Pull, lb. |
|---|---|---|---|---|---|---|---|---|
| | | Delivered to Boiler by Injectors, lb. | Weight of Water in Boiler at Start of Test Minus Weight in Boiler at Close of Test, lb. | Correction for Change of Water Level and Steam Pressure in Boiler, Start to Close, lb. | Loss From Boiler, lb. | Loss From Boiler Corrected, lb. | Presumably Evaporated, lb. | |
| | Code Item ☞ | 476 | 477 | 478 | 479 | 480 | 481 | 487 |
| 2009 | 138–16–F | 45 314 | +294 | +122 | | | 45 436 | |
| 2010 | 193–20–F | 28 122 | +142 | +147 | | | 28 269 | |
| 2012 | 138–24–F | 43 727 | + 47 | + 34 | | | 43 761 | 10 140 |
| 2013 | 138–32–F | 43 335 | +317 | +277 | | | 43 612 | 12 772 |
| 2014 | 193–32–F | 38 210 | –185 | –114 | | | 38 096 | 8 823 |
| 2015 | 193–24–F | 41 286 | +140 | +136 | | | 41 422 | 6 469 |
| 2016 | 193–16–F | 48 590 | – 94 | – 76 | | | 48 514 | 4 320 |
| 2017 | 83–16–F | 44 488 | –182 | –149 | | | 44 339 | 9 222 |
| 2018 | 83–24–F | 47 725 | –279 | –184 | | | 47 542 | 13 215 |
| 2019 | 83–32–F | 48 834 | –313 | –224 | | | 48 610 | 17 522 |
| 2020 | 83–24–F | 35 196 | + 94 | +104 | | | 35 300 | 13 072 |
| 2021 | 83–16–F | 28 176 | – 50 | – 44 | | | 28 132 | 8 931 |
| 2022 | 83–32–F | 43 759 | 0 | – 18 | | | 43 741 | 17 292 |
| 2023 | 138–40–F | 52 634 | +141 | +165 | | | 52 799 | 15 911 |
| 2024 | 55–24–F | 30 466 | –144 | –111 | | | 30 355 | 14 528 |
| 2026 | 110–16–F | 35 634 | + 49 | + 43 | | | 35 677 | 7 839 |
| 2027 | 110–24–F | 52 496 | + 47 | + 60 | | | 52 556 | 11 903 |
| 2028 | 55–32–F | 39 512 | + 51 | + 37 | | | 39 549 | 20 048 |
| 2029 | 110–32–F | 40 044 | +250 | +179 | | | 40 223 | 15 444 |
| 2030 | 165–24–F | 43 818 | + 47 | + 59 | | | 43 876 | 8 852 |
| 2031 | 83–40–F | 41 688 | + 49 | + 17 | | | 41 704 | 20 947 |
| 2032 | 165–32–F | 20 484 | +144 | +138 | | | 20 622 | 11 343 |
| 2033 | 110–48–F | 42 984 | +203 | +137 | | | 43 121 | 18 946 |
| 2034 | 193–40–F | 38 656 | +193 | +186 | | | 38 842 | 10 009 |
| 2035 | 110–40–F | 38 650 | +288 | +152 | | | 38 802 | 17 426 |
| 2037 | 165–40–F | 38 911 | –659 | –471 | | | 38 440 | 12 756 |
| 2038 | 55–24–F | 27 068 | –203 | –129 | | | 26 939 | 14 998 |
| 2039 | 110–32–F | 41 779 | + 51 | + 87 | | | 41 886 | 15 477 |
| 2040 | 165–40–F | 37 933 | +152 | +197 | | | 38 130 | 13 869 |
| 2041 | 110–40–F | 39 277 | –152 | – 5 | | | 39 272 | 18 895 |
| 2042 | 110–24–F | 45 085 | + 50 | – 7 | | | 45 078 | 12 680 |
| 2043 | 110–48–F | 44 973 | +355 | +341 | | | 45 314 | 21 800 |
| 2044 | 110–56–F | 43 838 | +101 | +260 | | | 44 098 | 23 666 |
| 2045 | 110–16–F | 37 376 | – 51 | – 37 | | | 37 339 | 8 212 |
| 2072 | 110–40–F | 33 886 | 0 | + 24 | | | 33 910 | 20 877 |
| 2073 | 110–32–F | 37 169 | +260 | +203 | | | 37 372 | 16 961 |
| 2074 | 165–32–F | 34 914 | 0 | – 18 | | | 34 896 | 13 486 |
| 2075 | 55–32–F | 38 781 | –156 | –111 | | | 38 670 | 20 483 |
| 2076 | 220–32–F | 23 011 | –153 | –109 | | | 22 902 | 10 396 |
| 2077 | 110–24–F | 40 308 | –106 | – 45 | | | 40 263 | 12 512 |
| 2078 | 165–24–F | 33 546 | –102 | – 98 | | | 33 448 | 10 188 |
| 2079 | 220–24–F | 32 074 | –253 | –225 | | | 31 849 | 8 270 |
| 2080 | 110–16–F | 37 696 | 0 | – 10 | | | 37 686 | |
| 2081 | 55–24–F | 35 815 | 0 | 0 | | | 35 815 | |
| 2082 | 165–40–F | 34 198 | 0 | + 34 | | | 34 232 | 14 783 |
| 2083 | 165–16–F | 36 739 | – 50 | – 36 | | | 36 703 | 7 078 |
| 2084 | 110–48–F | 32 266 | +260 | +177 | | | 32 443 | 22 403 |
| 2085 | 55–40–F | 41 387 | +102 | + 72 | | | 41 459 | 24 833 |
| 2086 | 55–24–F | 40 553 | + 51 | + 46 | | | 40 599 | 15 532 |
| 2087 | 110–16–F | 43 365 | –102 | – 79 | | | 43 286 | 8 135 |
| 2088 | 220–16–F | 37 413 | 0 | + 18 | | | 37 431 | 5 568 |
| 2089 | 220–40–F | 26 703 | +102 | + 73 | | | 26 776 | 11 831 |
| 2092 | 165–32–F | 28 808 | +360 | +247 | | | 29 055 | 13 701 |
| 2093 | 165–48–F | 23 767 | +675 | +441 | | | 24 208 | 17 660 |
| 2094 | 110–56–F | 18 810 | + 51 | + 88 | | | 18 898 | 25 225 |
| 2095 | 55–48–F | 23 813 | 0 | – 8 | | | 23 805 | 28 922 |
| 2096 | 55–40–F | 31 887 | –102 | – 55 | | | 31 832 | 24 980 |
| 2097 | 55–32–F | 33 084 | +197 | +140 | | | 33 224 | 20 820 |
| 2098 | 55–48–F | 20 353 | + 60 | + 62 | | | 20 415 | 29 240 |
| 2090 | 110–24–F | 21 688 | 0 | 0 | | | 21 688 | 11 477 |
| 2091 | 165–32–F | 16 786 | – 51 | – 19 | | | 16 767 | 12 024 |

## TABLE 21.

## EVENTS OF STROKE FROM INDICATOR CARDS—CUT-OFF AND RELEASE.

| Test No. | Laboratory Designation | Cut Off, Per cent of Stroke | | | | | Release, Per cent of Stroke | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Right Side | | Left Side | | Average | Right Side | | Left Side | | Average |
| | | Head End | Crank End | Head End | Crank End | | Head End | Crank End | Head End | Crank End | |
| | Code Item☞ | 495 | 496 | 497 | 498 | 499 | 510 | 511 | 512 | 513 | 514 |
| 2009 | 138–16–F | 14.0 | 19.0 | 17.4 | 17.8 | 17.1 | 51.1 | 57.4 | 54.0 | 59.7 | 55.6 |
| 2010 | 193–20–F | 20.1 | 20.6 | 18.0 | 18.0 | 19.2 | 55.4 | 62.3 | 61.0 | 61.9 | 60.2 |
| 2012 | 138–24–F | 24.4 | 22.7 | 22.4 | 23.8 | 23.3 | 63.9 | 66.9 | 63.7 | 65.9 | 65.1 |
| 2013 | 138–32–F | 29.6 | 33.5 | 30.5 | 29.7 | 30.8 | 67.1 | 69.6 | 67.8 | 70.8 | 68.8 |
| 2014 | 193–32–F | 33.4 | 33.1 | 29.0 | 30.1 | 31.4 | 67.0 | 71.2 | 67.3 | 68.0 | 68.4 |
| 2015 | 193–24–F | 22.2 | 24.0 | 21.8 | 22.9 | 22.7 | 61.1 | 64.9 | 59.1 | 65.8 | 62.7 |
| 2016 | 193–16–F | 16.1 | 17.6 | 14.2 | 17.8 | 16.4 | 53.6 | 60.8 | 55.9 | 60.5 | 57.7 |
| 2017 | 83–16–F | 15.0 | 17.1 | 16.3 | 19.2 | 16.9 | 49.9 | 56.4 | 52.4 | 57.5 | 54.1 |
| 2018 | 83–24–F | 20.7 | 22.9 | 22.7 | 23.9 | 22.6 | 56.4 | 63.2 | 59.3 | 60.3 | 59.8 |
| 2019 | 83–32–F | 28.3 | 32.3 | 28.7 | 33.2 | 30.6 | 67.6 | 66.7 | 65.2 | 65.2 | 66.2 |
| 2020 | 83–24–F | 18.3 | 25.5 | 21.4 | 24.4 | 22.4 | 57.1 | 62.6 | 60.4 | 61.8 | 60.5 |
| 2021 | 83–16–F | 12.3 | 15.2 | 14.3 | 17.6 | 14.9 | 48.9 | 54.3 | 50.9 | 54.6 | 52.2 |
| 2022 | 83–32–F | 27.0 | 31.7 | 28.0 | 34.5 | 30.3 | 62.9 | 66.8 | 66.0 | 70.4 | 66.5 |
| 2023 | 138–40–F | 34.0 | 43.1 | 39.3 | 41.2 | 39.4 | 72.1 | 74.5 | 73.2 | 74.9 | 73.7 |
| 2024 | 55–24–F | | | | | | | | | | |
| 2026 | 110–16–F | 15.2 | 24.3 | 18.1 | 18.7 | 19.1 | 52.6 | 55.5 | 56.1 | 59.3 | 55.9 |
| 2027 | 110–24–F | 22.0 | 25.4 | 22.6 | 27.0 | 24.3 | 62.3 | 64.5 | 63.8 | 66.9 | 64.4 |
| 2028 | 55–32–F | 28.6 | 32.1 | 31.4 | 34.8 | 31.7 | 65.6 | 66.7 | 68.9 | 69.9 | 67.8 |
| 2029 | 110–32–F | 28.3 | 30.9 | 30.0 | 33.7 | 30.7 | 69.4 | 70.1 | 68.3 | 72.8 | 70.2 |
| 2030 | 165–24–F | 20.8 | 22.9 | 23.3 | 26.4 | 23.4 | 57.8 | 65.0 | 68.0 | 64.8 | 63.9 |
| 2031 | 83–40–F | 35.9 | 40.4 | 39.1 | 42.4 | 39.5 | 71.6 | 73.9 | 73.9 | 73.9 | 73.3 |
| 2032 | 165–32–F | 28.5 | 31.4 | 27.9 | 33.0 | 30.2 | 65.1 | 71.1 | 70.8 | 72.9 | 70.0 |
| 2033 | 110–48–F | 37.7 | 40.4 | 40.8 | 43.8 | 40.7 | 73.9 | 76.4 | 75.3 | 74.9 | 75.1 |
| 2034 | 193–40–F | 38.3 | 44.9 | 39.1 | 43.4 | 41.4 | 77.0 | 78.2 | 78.3 | 79.3 | 78.2 |
| 2035 | 110–40–F | 36.3 | 39.6 | 41.8 | 41.9 | 39.9 | 73.7 | 75.0 | 76.4 | 75.6 | 75.2 |
| 2037 | 165–40–F | 38.9 | 41.6 | 36.7 | 43.0 | 40.1 | 74.5 | 75.3 | 75.7 | 77.2 | 75.7 |
| 2038 | 55–24–F | | | | | | | | | | |
| 2039 | 110–32–F | 29.9 | 34.9 | 31.8 | 33.4 | 32.5 | 65.9 | 70.7 | 68.8 | 70.3 | 68.9 |
| 2040 | 165–40–F | 41.1 | 41.4 | 41.9 | 41.4 | 41.5 | 75.2 | 76.7 | 76.1 | 76.8 | 76.2 |
| 2041 | 110–40–F | 39.6 | 42.2 | 41.9 | 40.8 | 41.1 | 73.6 | 76.9 | 75.8 | 75.5 | 75.5 |
| 2042 | 110–24–F | 23.6 | 23.5 | 27.9 | 24.3 | 24.8 | 59.5 | 62.8 | 63.5 | 63.8 | 62.4 |
| 2043 | 110–48–F | 47.6 | 49.6 | 48.0 | 48.7 | 48.5 | 79.5 | 79.3 | 83.0 | 79.5 | 80.3 |
| 2044 | 110–56–F | 56.2 | 56.9 | 60.5 | 56.5 | 57.5 | 81.6 | 82.2 | 86.3 | 81.2 | 71.8 |
| 2045 | 110–16–F | 18.4 | 16.8 | 23.5 | 18.2 | 19.2 | 52.0 | 55.5 | 56.5 | 55.6 | 54.9 |
| 2072 | 110–40–F | 41.5 | 41.5 | 41.8 | 41.0 | 41.5 | 75.1 | 74.8 | 75.0 | 75.6 | 75.1 |
| 2073 | 110–32–F | 25.6 | 29.8 | 31.6 | 31.5 | 29.6 | 73.4 | 68.5 | 70.4 | 69.8 | 70.5 |
| 2074 | 165–32–F | 20.3 | 32.7 | 32.5 | 29.7 | 28.8 | 67.7 | 70.8 | 72.3 | 69.5 | 70.1 |
| 2075 | 55–32–F | 29.9 | 33.9 | 33.7 | 31.0 | 32.1 | 66.7 | 68.5 | 70.5 | 69.2 | 68.7 |
| 2076 | 220–32–F | 29.3 | 33.3 | 31.4 | 34.9 | 32.2 | 68.3 | 68.1 | 63.2 | 67.3 | 66.7 |
| 2077 | 110–24–F | 21.5 | 24.4 | 25.5 | 24.4 | 24.0 | 56.1 | 61.7 | 63.0 | 63.1 | 61.0 |
| 2078 | 165–24–F | 22.0 | 24.7 | 26.6 | 22.8 | 24.0 | 58.9 | 64.6 | 64.5 | 65.4 | 63.4 |
| 2079 | 220–24–F | 24.1 | 23.6 | 22.7 | 23.1 | 23.4 | 69.7 | 67.3 | 66.8 | 63.5 | 66.8 |
| 2080 | 110–16–F | 15.3 | 16.6 | 18.9 | 16.8 | 16.9 | 50.6 | 53.0 | 61.0 | 59.3 | 56.0 |
| 2081 | 55–24–F | 23.1 | 24.8 | 26.2 | 22.4 | 24.1 | 57.8 | 60.1 | 62.2 | 61.8 | 60.5 |
| 2082 | 165–40–F | 36.0 | 45.3 | 41.9 | 42.3 | 41.4 | 74.2 | 72.9 | 76.9 | 75.7 | 74.9 |
| 2083 | 165–16–F | 16.2 | 18.2 | 21.8 | 17.4 | 18.4 | 51.4 | 57.5 | 60.8 | 58.7 | 57.1 |
| 2084 | 110–48–F | 47.9 | 49.9 | 48.9 | 47.0 | 48.4 | 79.2 | 79.8 | 80.9 | 78.4 | 79.6 |
| 2085 | 55–40–F | 39.7 | 42.0 | 43.3 | 40.2 | 41.3 | 70.3 | 74.4 | 77.6 | 74.6 | 74.2 |
| 2086 | 55–24–F | 22.0 | 23.8 | 25.7 | 22.0 | 23.4 | 58.5 | 61.3 | 63.2 | 62.3 | 61.3 |
| 2087 | 110–16–F | 15.9 | 15.7 | 17.4 | 17.4 | 16.6 | 49.3 | 54.0 | 56.5 | 54.2 | 53.5 |
| 2088 | 220–16–F | 16.4 | 16.9 | 14.9 | 15.3 | 15.9 | 59.3 | 58.7 | 57.4 | 56.3 | 57.9 |
| 2089 | 220–40–F | 41.2 | 45.4 | 42.0 | 45.4 | 43.5 | 74.5 | 77.8 | 78.4 | 77.4 | 77.0 |
| 2092 | 165–32–F | 30.1 | 31.1 | 30.8 | 29.5 | 30.4 | 70.0 | 70.2 | 70.8 | 70.3 | 70.3 |
| 2093 | 165–48–F | 45.9 | 50.5 | 48.4 | 49.7 | 48.6 | 78.5 | 79.6 | 79.7 | 77.1 | 78.7 |
| 2094 | 110–56–F | 55.7 | 56.3 | 61.1 | 54.7 | 57.0 | 83.1 | 84.5 | 86.9 | 82.5 | 84.3 |
| 2095 | 55–48–F | 47.8 | 50.1 | 51.5 | 47.5 | 49.2 | 80.0 | 79.8 | 81.8 | 79.6 | 80.3 |
| 2096 | 55–40–F | 38.1 | 41.3 | 43.2 | 39.1 | 40.4 | 73.2 | 74.0 | 77.6 | 74.7 | 74.9 |
| 2097 | 55–32–F | 30.6 | 33.2 | 35.0 | 30.5 | 32.3 | 66.4 | 69.4 | 71.0 | 69.0 | 69.0 |
| 2098 | 55–48–F | 47.7 | 50.1 | 51.5 | 47.2 | 49.1 | 79.5 | 80.2 | 82.7 | 80.2 | 80.7 |
| 2090 | 110–24–F | 20.9 | 22.3 | 23.4 | 24.6 | 22.8 | 56.5 | 66.1 | 64.2 | 64.2 | 62.8 |
| 2091 | 165–32–F | 27.5 | 34.0 | 27.6 | 26.0 | 28.8 | 68.5 | 70.5 | 71.8 | 71.3 | 70.5 |

## TABLE 22.

## EVENTS OF STROKE AND PRESSURE FROM INDICATOR CARDS—BEGINNING OF COMPRESSION AND INITIAL PRESSURE.

| Test No. | Laboratory Designation | Beginning of Compression, Per cent of Stroke | | | | | Initial Pressure, lb. per sq. in. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Right Side | | Left Side | | Average | Right Side | | Left Side | | Average |
| | | Head End | Crank End | Head End | Crank End | | Head End | Crank End | Head End | Crank End | |
| | Code Item☞ | 525 | 526 | 527 | 528 | 529 | 540 | 541 | 542 | 543 | 544 |
| 2009 | 138–16–F | 56.0 | 58.3 | 56.7 | 56.2 | 56.8 | 170.9 | 188.6 | 173.0 | 186.2 | 179.7 |
| 2010 | 193–20–F | 57.5 | 66.8 | 71.1 | 59.7 | 63.8 | 150.5 | 159.6 | 144.5 | 162.6 | 154.3 |
| 2012 | 138–24–F | 50.2 | 59.5 | 48.2 | 55.8 | 53.4 | 166.4 | 182.3 | 170.8 | 180.8 | 175.1 |
| 2013 | 138–32–F | 47.4 | 58.7 | 50.4 | 49.1 | 51.4 | 165.0 | 177.9 | 167.7 | 173.1 | 170.9 |
| 2014 | 193–32–F | 77.5 | 76.0 | 80.8 | 59.1 | 73.4 | 156.4 | 154.3 | 166.8 | 173.0 | 162.5 |
| 2015 | 193–24–F | 79.7 | 73.9 | 78.2 | 76.9 | 77.2 | 159.7 | 155.7 | 162.9 | 160.9 | 159.8 |
| 2016 | 193–16–F | 64.0 | 71.2 | 68.5 | 67.2 | 67.7 | 156.1 | 163.6 | 150.1 | 165.1 | 158.7 |
| 2017 | 83–16–F | 50.4 | 58.1 | 49.4 | 56.5 | 53.6 | 177.3 | 187.4 | 177.1 | 181.8 | 180.9 |
| 2018 | 83–24–F | 43.9 | 50.8 | 43.3 | 53.1 | 47.8 | 178.8 | 179.4 | 176.4 | 190.2 | 181.2 |
| 2019 | 83–32–F | 38.2 | 42.4 | 35.8 | 37.6 | 38.5 | 178.3 | 178.6 | 176.5 | 178.7 | 178.0 |
| 2020 | 83–24–F | 39.2 | 43.6 | 39.2 | 42.8 | 41.2 | 175.1 | 177.0 | 177.6 | 180.0 | 177.4 |
| 2021 | 83–16–F | 46.2 | 51.5 | 46.1 | 51.6 | 48.9 | 182.9 | 182.9 | 181.8 | 185.5 | 183.3 |
| 2022 | 83–32–F | 31.2 | 42.6 | 29.0 | 35.5 | 34.6 | 175.5 | 156.6 | 178.7 | 182.1 | 173.2 |
| 2023 | 138–40–F | 33.7 | 35.8 | 30.8 | 34.9 | 33.8 | 168.7 | 178.6 | 169.9 | 178.8 | 174.0 |
| 2024 | 55–24–F | | | | | | | | | | |
| 2026 | 110–16–F | 51.5 | 54.6 | 51.6 | 55.8 | 53.4 | 191.1 | 181.1 | 187.5 | 175.9 | 183.9 |
| 2027 | 110–24–F | 44.9 | 47.6 | 44.6 | 44.8 | 45.5 | 185.4 | 174.5 | 189.2 | 176.2 | 181.3 |
| 2028 | 55–32–F | 32.1 | 35.0 | 35.6 | 36.6 | 34.8 | 186.1 | 184.7 | 192.2 | 191.7 | 188.7 |
| 2029 | 110–32–F | 38.3 | 45.1 | 35.6 | 41.0 | 40.0 | 181.8 | 180.4 | 186.8 | 192.2 | 185.3 |
| 2030 | 165–24–F | 43.6 | 63.4 | 48.3 | 53.6 | 52.2 | 161.3 | 171.9 | 167.4 | 179.7 | 170.1 |
| 2031 | 83–40–F | 28.7 | 32.3 | 30.0 | 31.9 | 30.7 | 183.7 | 177.9 | 187.7 | 186.9 | 184.1 |
| 2032 | 165–32–F | 42.1 | 48.4 | 77.5 | 68.3 | 59.1 | 185.8 | 166.2 | 162.6 | 166.0 | 170.2 |
| 2033 | 110–48–F | 29.7 | 33.7 | 30.5 | 34.1 | 32.0 | 185.6 | 178.4 | 184.2 | 189.3 | 184.4 |
| 2034 | 193–40–F | 62.0 | 70.0 | 71.8 | 71.6 | 68.9 | 168.7 | 172.9 | 179.7 | 170.8 | 173.0 |
| 2035 | 110–40–F | 33.5 | 36.7 | 30.0 | 33.1 | 33.3 | 178.7 | 172.9 | 183.3 | 187.7 | 180.7 |
| 2037 | 165–40–F | 61.6 | 68.5 | 67.4 | 66.5 | 66.0 | 176.7 | 161.4 | 159.3 | 179.8 | 169.3 |
| 2038 | 55–24–F | | | | | | | | | | |
| 2039 | 110–32–F | 41.2 | 46.3 | 40.0 | 39.8 | 41.8 | 175.0 | 194.2 | 181.8 | 189.6 | 185.2 |
| 2040 | 165–40–F | 73.9 | 75.5 | 74.5 | 74.6 | 74.6 | 184.5 | 195.7 | 192.1 | 184.6 | 189.2 |
| 2041 | 110–40–F | 34.5 | 37.3 | 36.2 | 32.8 | 35.2 | 175.5 | 186.1 | 179.6 | 183.1 | 181.1 |
| 2042 | 110–24–F | 46.3 | 51.8 | 47.2 | 46.9 | 48.1 | 180.9 | 176.1 | 188.0 | 173.9 | 179.7 |
| 2043 | 110–48–F | 28.2 | 30.2 | 30.4 | 25.7 | 28.6 | 191.7 | 188.2 | 181.1 | 194.7 | 188.9 |
| 2044 | 110–56–F | 23.8 | 23.3 | 25.7 | 22.4 | 23.8 | 180.1 | 190.8 | 191.4 | 187.4 | 187.4 |
| 2045 | 110–16–F | 54.9 | 56.7 | 52.8 | 53.4 | 54.5 | 185.9 | 181.1 | 193.4 | 195.9 | 189.1 |
| 2072 | 110–40–F | 30.5 | 30.7 | 31.3 | 29.3 | 30.5 | 183.8 | 187.4 | 185.5 | 186.7 | 185.9 |
| 2073 | 110–32–F | 36.8 | 41.4 | 37.7 | 35.5 | 37.9 | 184.8 | 190.3 | 189.7 | 190.2 | 188.8 |
| 2074 | 165–32–F | 43.3 | 43.0 | 41.2 | 44.2 | 42.9 | 178.9 | 173.7 | 178.1 | 173.9 | 176.2 |
| 2075 | 55–32–F | 31.5 | 36.6 | 37.8 | 33.2 | 34.8 | 192.6 | 195.7 | 193.8 | 197.1 | 194.8 |
| 2076 | 220–32–F | 51.4 | 50.5 | 47.4 | 48.9 | 49.6 | 182.7 | 184.5 | 193.1 | 186.5 | 186.7 |
| 2077 | 110–24–F | 48.6 | 47.2 | 48.5 | 47.1 | 47.9 | 185.5 | 193.9 | 190.8 | 191.1 | 190.3 |
| 2078 | 165–24–F | 49.5 | 51.3 | 48.6 | 50.4 | 50.0 | 179.9 | 177.5 | 168.7 | 173.6 | 175.0 |
| 2079 | 220–24–F | 52.2 | 54.7 | 52.3 | 53.5 | 53.2 | 185.1 | 166.4 | 179.6 | 186.1 | 179.3 |
| 2080 | 110–16–F | 54.6 | 54.6 | 53.1 | 52.6 | 53.7 | 190.3 | 178.5 | 176.4 | 179.2 | 181.1 |
| 2081 | 55–24–F | 41.3 | 45.5 | 44.8 | 41.8 | 43.4 | 193.0 | 192.8 | 190.3 | 197.3 | 193.4 |
| 2082 | 165–40–F | 33.2 | 36.4 | 34.0 | 33.0 | 34.2 | 191.9 | 170.6 | 163.6 | 171.6 | 174.4 |
| 2083 | 165–16–F | 51.6 | 59.7 | 48.5 | 54.3 | 53.5 | 169.8 | 182.1 | 173.9 | 176.9 | 175.7 |
| 2084 | 110–48–F | 25.7 | 24.8 | 16.4 | 22.8 | 24.9 | 179.4 | 181.8 | 179.8 | 187.1 | 182.0 |
| 2085 | 55–40–F | 27.3 | 28.1 | 29.6 | 23.7 | 27.2 | 189.4 | 193.6 | 192.9 | 194.2 | 192.5 |
| 2086 | 55–24–F | 39.2 | 41.7 | 39.8 | 37.4 | 39.2 | 192.7 | 192.1 | 189.4 | 193.4 | 191.9 |
| 2087 | 110–16–F | 53.1 | 51.1 | 50.7 | 49.1 | 51.0 | 176.3 | 175.3 | 174.2 | 177.8 | 175.9 |
| 2088 | 220–16–F | 54.9 | 52.5 | 52.1 | 52.4 | 53.0 | 136.4 | 134.0 | 160.9 | 159.9 | 147.8 |
| 2089 | 220–40–F | 44.0 | 43.7 | 42.9 | 41.2 | 43.0 | 163.2 | 162.0 | 160.2 | 170.8 | 164.1 |
| 2092 | 165–32–F | 39.1 | 41.5 | 39.9 | 41.3 | 40.5 | 163.3 | 172.5 | 162.7 | 171.8 | 167.6 |
| 2093 | 165–48–F | 27.6 | 26.2 | 25.5 | 24.8 | 26.0 | 165.4 | 165.4 | 157.2 | 166.7 | 163.7 |
| 2094 | 110–56–F | 19.5 | 17.7 | 20.7 | 15.5 | 18.4 | 172.4 | 171.3 | 170.6 | 178.9 | 173.3 |
| 2095 | 55–48–F | 20.4 | 22.0 | 22.5 | 19.2 | 21.0 | 187.3 | 189.9 | 187.9 | 190.7 | 189.0 |
| 2096 | 55–40–F | 25.1 | 27.1 | 27.8 | 23.6 | 25.9 | 187.8 | 187.9 | 189.1 | 190.2 | 188.8 |
| 2097 | 55–32–F | 30.4 | 33.6 | 34.3 | 29.9 | 32.1 | 192.9 | 191.9 | 192.4 | 192.4 | 192.4 |
| 2098 | 55–48–F | 21.6 | 22.0 | 23.3 | 19.5 | 21.6 | 188.7 | 191.4 | 191.8 | 192.9 | 191.2 |
| 2090 | 110–24–F | 44.4 | 46.1 | 41.4 | 42.6 | 43.6 | 184.7 | 172.4 | 171.2 | 173.4 | 175.4 |
| 2091 | 165–32–F | 39.4 | 43.0 | 36.1 | 39.6 | 39.5 | 164.4 | 174.1 | 165.4 | 175.5 | 169.9 |

## TABLE 23.

## PRESSURE FROM INDICATOR CARDS—CUT-OFF AND RELEASE.

| Test No. | Laboratory Designation | Pressure at Cut-Off, lb. per sq. in. | | | | | Pressure at Release, lb. per sq. in. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Right Side | | Left Side | | Average | Right Side | | Left Side | | Average |
| | | Head End | Crank End | Head End | Crank End | | Head End | Crank End | Head End | Crank End | |
| | Code Item☞ | 566 | 567 | 568 | 569 | 570 | 581 | 582 | 583 | 584 | 585 |
| 2009 | 138–16–F | 146.6 | 129.7 | 130.6 | 132.0 | 134.7 | 44.0 | 45.7 | 46.4 | 42.8 | 44.7 |
| 2010 | 193–20–F | 108.5 | 129.0 | 119.5 | 131.0 | 122.0 | 40.5 | 45.5 | 39.5 | 43.5 | 42.3 |
| 2012 | 138–24–F | 130.7 | 138.0 | 131.3 | 136.8 | 134.2 | 45.6 | 54.0 | 48.4 | 53.4 | 50.4 |
| 2013 | 138–32–F | 123.8 | 134.6 | 122.0 | 139.4 | 130.0 | 52.9 | 64.9 | 55.3 | 63.3 | 59.1 |
| 2014 | 193–32–F | 101.3 | 123.5 | 114.0 | 125.0 | 116.0 | 49.3 | 57.8 | 48.2 | 57.0 | 53.1 |
| 2015 | 193–24–F | 114.8 | 123.6 | 114.0 | 131.8 | 121.1 | 42.8 | 49.4 | 47.2 | 49.1 | 47.1 |
| 2016 | 193–16–F | 113.9 | 133.0 | 114.6 | 132.0 | 123.4 | 38.8 | 41.3 | 34.8 | 42.2 | 39.3 |
| 2017 | 83–16–F | 152.0 | 162.7 | 146.0 | 155.3 | 154.0 | 51.7 | 55.5 | 52.6 | 54.5 | 53.6 |
| 2018 | 83–24–F | 148.0 | 157.7 | 142.0 | 156.7 | 151.1 | 57.9 | 60.5 | 59.1 | 66.4 | 61.0 |
| 2019 | 83–32–F | 150.3 | 158.0 | 145.9 | 155.1 | 152.3 | 58.9 | 75.0 | 66.6 | 79.6 | 70.0 |
| 2020 | 83–24–F | 156.2 | 147.3 | 154.1 | 155.6 | 153.3 | 58.0 | 61.6 | 60.8 | 66.4 | 61.7 |
| 2021 | 83–16–F | 162.0 | 168.9 | 164.5 | 167.0 | 165.6 | 51.1 | 56.3 | 52.6 | 61.7 | 56.7 |
| 2022 | 83–32–F | 149.1 | 129.2 | 151.4 | 154.7 | 146.1 | 65.6 | 66.0 | 70.2 | 77.7 | 69.9 |
| 2023 | 138–40–F | 136.9 | 133.6 | 121.1 | 146.2 | 134.5 | 65.7 | 76.8 | 66.3 | 81.3 | 72.5 |
| 2024 | 55–24–F | | | | | | | | | | |
| 2026 | 110–16–F | 145.6 | 123.6 | 133.2 | 151.6 | 138.4 | 45.1 | 54.7 | 45.1 | 51.6 | 49.1 |
| 2027 | 110–24–F | 139.7 | 146.8 | 143.9 | 144.3 | 143.7 | 49.0 | 58.1 | 51.1 | 57.3 | 53.9 |
| 2028 | 55–32–F | 154.9 | 165.3 | 161.7 | 167.3 | 162.3 | 67.4 | 80.1 | 72.2 | 81.7 | 75.4 |
| 2029 | 110–32–F | 135.3 | 150.7 | 143.7 | 154.5 | 141.1 | 52.0 | 67.9 | 62.7 | 74.0 | 64.2 |
| 2030 | 165–24–F | 122.0 | 139.7 | 129.1 | 136.4 | 131.8 | 48.4 | 56.1 | 43.4 | 61.2 | 52.3 |
| 2031 | 83–40–F | 149.0 | 150.5 | 149.5 | 154.6 | 150.9 | 72.5 | 80.5 | 77.0 | 85.8 | 79.0 |
| 2032 | 165–32–F | 126.2 | 131.8 | 133.0 | 134.7 | 131.4 | 56.4 | 62.5 | 52.6 | 61.8 | 58.3 |
| 2033 | 110–48–F | 139.7 | 146.0 | 141.0 | 145.6 | 143.1 | 70.0 | 75.3 | 75.3 | 82.3 | 75.7 |
| 2034 | 193–40–F | 111.5 | 112.2 | 111.1 | 123.4 | 114.5 | 51.8 | 62.1 | 53.1 | 64.1 | 57.8 |
| 2035 | 110–40–F | 139.5 | 141.4 | 133.9 | 148.2 | 140.8 | 66.1 | 74.7 | 71.0 | 80.3 | 73.0 |
| 2037 | 165–40–F | 117.9 | 124.2 | 125.3 | 132.2 | 124.9 | 58.2 | 69.5 | 60.5 | 72.1 | 65.1 |
| 2038 | 55–24–F | | | | | | | | | | |
| 2039 | 110–32–F | 135.4 | 152.6 | 139.7 | 150.3 | 146.8 | 60.8 | 73.2 | 62.4 | 70.6 | 66.8 |
| 2040 | 165–40–F | 119.7 | 134.9 | 120.3 | 127.4 | 125.6 | 63.7 | 71.2 | 63.6 | 66.8 | 66.3 |
| 2041 | 110–40–F | 136.3 | 150.8 | 141.0 | 149.7 | 144.5 | 71.6 | 80.3 | 75.3 | 78.0 | 76.3 |
| 2042 | 110–24–F | 140.7 | 158.6 | 135.7 | 149.6 | 146.2 | 55.5 | 62.6 | 56.7 | 56.5 | 57.8 |
| 2043 | 110–48–F | 146.5 | 156.1 | 149.8 | 153.0 | 151.4 | 85.1 | 94.5 | 83.8 | 91.9 | 88.8 |
| 2044 | 110–56–F | 144.8 | 154.4 | 139.5 | 152.8 | 147.9 | 96.8 | 103.9 | 92.8 | 105.9 | 99.9 |
| 2045 | 110–16–F | 136.5 | 164.1 | 121.3 | 146.7 | 142.2 | 47.4 | 54.4 | 47.4 | 51.6 | 50.2 |
| 2072 | 110–40–F | 139.2 | 149.1 | 145.2 | 149.4 | 145.7 | 73.1 | 80.9 | 79.4 | 79.7 | 78.3 |
| 2073 | 110–32–F | 155.4 | 157.7 | 149.4 | 148.1 | 152.7 | 51.6 | 70.9 | 68.1 | 67.1 | 64.2 |
| 2074 | 165–32–F | 153.4 | 135.2 | 129.1 | 139.7 | 139.4 | 56.8 | 63.5 | 56.2 | 62.4 | 59.7 |
| 2075 | 55–32–F | 166.3 | 167.0 | 166.1 | 163.8 | 165.8 | 77.1 | 79.9 | 79.8 | 75.4 | 78.1 |
| 2076 | 220–32–F | 116.7 | 117.4 | 113.2 | 116.2 | 115.9 | 51.1 | 58.8 | 55.5 | 57.5 | 55.7 |
| 2077 | 110–24–F | 142.3 | 148.9 | 142.3 | 145.7 | 144.8 | 57.4 | 59.5 | 57.1 | 58.4 | 58.1 |
| 2078 | 165–24–F | 135.2 | 139.0 | 125.8 | 135.9 | 134.0 | 51.7 | 56.0 | 50.5 | 51.1 | 52.3 |
| 2079 | 220–24–F | 112.9 | 121.6 | 124.3 | 119.9 | 119.7 | 36.0 | 47.3 | 44.5 | 43.7 | 42.9 |
| 2080 | 110–16–F | 147.9 | 161.2 | 146.3 | 148.8 | 151.1 | 48.9 | 54.1 | 45.2 | 43.3 | 47.9 |
| 2081 | 55–24–F | 163.2 | 169.9 | 164.6 | 170.6 | 167.1 | 68.8 | 70.5 | 72.0 | 65.1 | 69.1 |
| 2082 | 165–40–F | 134.3 | 124.3 | 123.6 | 124.8 | 126.8 | 64.1 | 76.3 | 64.3 | 68.6 | 68.3 |
| 2083 | 165–16–F | 139.3 | 142.1 | 129.1 | 132.7 | 135.8 | 46.2 | 47.0 | 43.2 | 41.5 | 44.5 |
| 2084 | 110–48–F | 139.1 | 147.6 | 145.8 | 149.1 | 145.4 | 83.8 | 88.6 | 85.6 | 87.1 | 86.3 |
| 2085 | 55–40–F | 162.1 | 169.1 | 164.4 | 165.0 | 165.2 | 91.4 | 90.5 | 90.6 | 85.3 | 89.5 |
| 2086 | 55–24–F | 167.3 | 173.7 | 167.2 | 175.4 | 170.9 | 67.4 | 68.2 | 72.0 | 65.3 | 68.2 |
| 2087 | 110–16–F | 145.9 | 164.8 | 152.6 | 147.8 | 152.8 | 50.9 | 53.4 | 49.6 | 51.6 | 51.4 |
| 2088 | 220–16–F | 99.6 | 108.0 | 126.6 | 130.4 | 116.2 | 25.5 | 32.6 | 34.3 | 35.2 | 31.9 |
| 2089 | 220–40–F | 107.3 | 112.0 | 107.8 | 108.2 | 108.8 | 57.7 | 63.1 | 54.5 | 60.6 | 59.0 |
| 2092 | 165–32–F | 128.3 | 138.9 | 132.7 | 139.8 | 134.9 | 52.4 | 63.6 | 57.7 | 59.3 | 58.3 |
| 2093 | 165–48–F | 128.5 | 131.7 | 127.2 | 129.8 | 129.3 | 73.9 | 82.5 | 75.4 | 81.7 | 78.4 |
| 2094 | 110–56–F | 148.6 | 154.6 | 144.9 | 156.4 | 151.1 | 98.9 | 99.9 | 98.1 | 102.8 | 99.9 |
| 2095 | 55–48–F | 170.2 | 170.6 | 169.3 | 171.3 | 170.4 | 101.0 | 102.7 | 105.2 | 101.2 | 102.5 |
| 2096 | 55–40–F | 169.9 | 171.0 | 168.2 | 172.6 | 170.4 | 90.8 | 93.4 | 92.7 | 89.9 | 91.7 |
| 2097 | 55–32–F | 165.4 | 171.4 | 165.9 | 172.9 | 168.9 | 81.2 | 81.0 | 83.2 | 78.2 | 80.9 |
| 2098 | 55–48–F | 168.9 | 176.5 | 171.9 | 175.5 | 173.2 | 100.5 | 105.2 | 106.1 | 100.6 | 103.1 |
| 2090 | 110–24–F | 145.1 | 155.8 | 151.2 | 143.4 | 148.9 | 55.8 | 53.4 | 54.4 | 55.0 | 54.7 |
| 2091 | 165–32–F | 133.8 | 129.8 | 143.4 | 147.9 | 138.7 | 54.2 | 63.9 | 56.7 | 58.8 | 58.4 |

## TABLE 24.

## PRESSURE FROM INDICATOR CARDS—BEGINNING OF COMPRESSION AND LEAST BACK PRESSURE.

| Test No. | Laboratory Designation | Pressure at Beginning of Compression, lb. per sq. in. |  |  |  |  | Least Back Pressure, lb. per sq. in. |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  | Right Side |  | Left Side |  | Average | Right Side |  | Left Side |  | Average |
|  |  | Head End | Crank End | Head End | Crank End |  | Head End | Crank End | Head End | Crank End |  |
|  | CodeItem☞ | 596 | 597 | 598 | 599 | 600 | 611 | 612 | 613 | 614 | 615 |
| 2009 | 138–16–F | 7.2 | 9.3 | 10.9 | 9.1 | 9.1 | 3.9 | 3.8 | 7.0 | 6.4 | 5.3 |
| 2010 | 193–20–F | 10.4 | 11.5 | 9.0 | 13.3 | 11.1 | 7.3 | 7.6 | 9.0 | 9.0 | 8.2 |
| 2012 | 138–24–F | 11.5 | 12.2 | 14.0 | 10.4 | 12.0 | 7.0 | 7.8 | 7.8 | 8.2 | 7.7 |
| 2013 | 138–32–F | 15.0 | 16.8 | 16.4 | 16.0 | 16.1 | 9.8 | 12.7 | 10.4 | 13.4 | 11.6 |
| 2014 | 193–32–F | 15.8 | 17.5 | 16.8 | 17.3 | 16.9 | 15.4 | 17.9 | 16.6 | 18.3 | 17.1 |
| 2015 | 193–24–F | 11.6 | 13.0 | 13.8 | 13.3 | 12.9 | 10.2 | 13.5 | 11.3 | 13.4 | 12.1 |
| 2016 | 193–16–F | 9.2 | 8.8 | 10.5 | 10.8 | 9.8 | 6.3 | 6.7 | 5.8 | 6.9 | 6.4 |
| 2017 | 83–16–F | 2.8 | 4.6 | 4.1 | 3.1 | 3.7 | 2.7 | 2.1 | 2.4 | 3.2 | 2.6 |
| 2018 | 83–24–F | 4.5 | 6.6 | 5.9 | 4.8 | 5.5 | 4.6 | 3.4 | 3.1 | 4.7 | 4.0 |
| 2019 | 83–32–F | 7.1 | 10.6 | 9.1 | 8.0 | 8.7 | 4.5 | 5.6 | 5.1 | 5.4 | 5.2 |
| 2020 | 83–24–F | 5.3 | 8.0 | 9.3 | 8.4 | 7.8 | 3.6 | 2.4 | 6.6 | 7.8 | 5.1 |
| 2021 | 83–16–F | 4.0 | 5.9 | 8.8 | 5.1 | 6.0 |  | 2.2 | 6.1 | 5.3 | 4.4 |
| 2022 | 83–32–F | 6.7 | 9.4 | 13.3 | 11.2 | 10.2 | 4.1 | 7.2 | 8.2 | 9.4 | 7.2 |
| 2023 | 138–40–F | 22.4 | 27.3 | 28.2 | 28.4 | 26.6 | 17.1 | 18.5 | 20.0 | 22.3 | 19.5 |
| 2024 | 55–24–F |  |  |  |  |  |  |  |  |  |  |
| 2026 | 110–16–F | 5.8 | 8.1 | 9.6 | 5.7 | 7.3 | 2.3 | 2.8 | 4.1 | 3.2 | 3.1 |
| 2027 | 110–24–F | 8.2 | 11.3 | 11.0 | 9.4 | 10.0 | 4.7 | 5.9 | 6.3 | 7.4 | 6.1 |
| 2028 | 55–32–F | 2.2 | 5.5 | 5.9 | 1.5 | 3.8 |  |  |  |  |  |
| 2029 | 110–32–F | 8.2 | 12.7 | 16.5 | 16.0 | 13.4 | 8.1 | 10.4 | 10.0 | 13.6 | 10.5 |
| 2030 | 165–24–F | 13.4 | 13.7 | 16.6 | 20.6 | 16.1 | 10.5 | 12.1 | 9.0 | 14.2 | 11.5 |
| 2031 | 83–40–F | 10.5 | 11.7 | 14.4 | 13.4 | 12.5 | 9.6 | 10.0 | 10.6 | 10.8 | 10.3 |
| 2032 | 165–32–F | 18.8 | 20.0 | 16.2 | 18.7 | 18.4 |  |  |  |  |  |
| 2033 | 110–48–F | 15.4 | 19.9 | 21.4 | 18.8 | 18.9 | 13.8 | 15.3 | 16.9 | 14.6 | 15.2 |
| 2034 | 193–40–F | 21.6 | 23.8 | 24.7 | 26.7 | 24.2 | 22.8 | 24.0 | 24.1 | 25.6 | 24.1 |
| 2035 | 110–40–F | 14.9 | 17.2 | 22.1 | 18.5 | 18.2 | 11.9 | 15.7 | 16.0 | 15.0 | 14.7 |
| 2037 | 165–40–F | 21.1 | 22.8 | 22.1 | 25.1 | 22.8 | 21.8 | 23.4 | 22.5 | 24.7 | 18.1 |
| 2038 | 55–24–F |  |  |  |  |  |  |  |  |  |  |
| 2039 | 110–32–F | 12.6 | 16.8 | 15.8 | 14.4 | 14.9 | 8.5 | 11.5 | 8.4 | 11.1 | 9.9 |
| 2040 | 165–40–F | 24.4 | 24.8 | 23.8 | 23.5 | 24.1 | 22.5 | 24.8 | 22.9 | 24.7 | 23.7 |
| 2041 | 110–40–F | 18.5 | 21.1 | 21.4 | 18.0 | 19.8 | 12.4 | 16.0 | 14.0 | 15.3 | 14.4 |
| 2042 | 110–24–F | 10.4 | 10.7 | 11.2 | 9.6 | 10.5 | 5.0 | 7.0 | 5.4 | 6.4 | 6.0 |
| 2043 | 110–48–F | 23.0 | 26.8 | 26.8 | 25.1 | 25.4 | 18.5 | 22.0 | 20.0 | 21.4 | 20.5 |
| 2044 | 110–56–F | 27.9 | 30.6 | 29.9 | 28.7 | 29.3 | 22.5 | 24.9 | 22.9 | 22.4 | 23.2 |
| 2045 | 110–16–F | 5.9 | 7.2 | 7.6 | 7.1 | 7.0 | 3.0 | 3.8 | 3.1 | 3.3 | 3.3 |
| 2072 | 110–40–F | 12.8 | 15.1 | 14.9 | 13.7 | 14.1 | 8.3 | 10.1 | 8.9 | 10.0 | 9.3 |
| 2073 | 110–32–F | 9.1 | 10.9 | 11.6 | 10.0 | 10.4 | 5.9 | 7.1 | 5.6 | 7.2 | 6.5 |
| 2074 | 165–32–F | 15.5 | 19.5 | 20.5 | 18.5 | 18.5 | 11.4 | 13.9 | 11.3 | 13.5 | 12.5 |
| 2075 | 55–32–F | 4.1 | 3.1 | 2.1 | 2.4 | 2.9 | 2.2 | 1.9 | 1.2 | 1.6 | 1.7 |
| 2076 | 220–32–F | 18.3 | 21.9 | 21.5 | 22.5 | 21.1 | 14.5 | 17.9 | 14.0 | 16.9 | 15.8 |
| 2077 | 110–24–F | 5.6 | 8.5 | 6.6 | 6.5 | 6.8 | 3.0 | 3.4 | 2.8 | 3.5 | 3.2 |
| 2078 | 165–24–F | 10.3 | 14.2 | 12.5 | 12.0 | 12.3 | 5.1 | 8.8 | 7.1 | 8.4 | 7.4 |
| 2079 | 220–24–F | 12.3 | 15.9 | 15.7 | 15.3 | 14.8 | 8.3 | 12.0 | 8.2 | 10.7 | 9.8 |
| 2080 | 110–16–F | 4.3 | 5.6 | 5.6 | 4.5 | 5.0 | 2.0 | 2.1 | 2.0 | 2.0 | 2.0 |
| 2081 | 55–24–F | 2.5 | 2.4 | 1.9 | 2.1 | 2.2 | 2.0 | 0.3 | 0.3 | 0.3 | 0.7 |
| 2082 | 165–40–F | 21.6 | 24.4 | 26.6 | 25.1 | 24.4 | 17.2 | 20.0 | 17.7 | 18.2 | 18.3 |
| 2083 | 165–16–F | 7.7 | 8.9 | 11.5 | 9.1 | 9.3 | 3.3 | 4.9 | 3.3 | 4.2 | 3.9 |
| 2084 | 110–48–F | 15.4 | 17.9 | 18.4 | 17.0 | 17.2 | 12.6 | 13.3 | 10.0 | 13.0 | 12.2 |
| 2085 | 55–40–F | 3.5 | 4.1 | 4.2 | 3.8 | 3.9 | 1.5 | 1.2 | 2.6 | 1.9 | 1.8 |
| 2086 | 55–24–F | 2.7 | 2.3 | 2.1 | 2.8 | 2.5 | 1.6 | 0.4 | 0.0 | 0.5 | 0.6 |
| 2087 | 110–16–F | 4.1 | 6.2 | 6.0 | 4.9 | 5.3 | 2.2 | 2.2 | 2.2 | 2.7 | 2.3 |
| 2088 | 220–16–F | 7.5 | 9.4 | 10.4 | 11.2 | 9.6 | 4.8 | 7.5 | 3.4 | 4.7 | 5.1 |
| 2089 | 220–40–F | 25.2 | 30.3 | 30.2 | 29.7 | 28.9 | 19.8 | 24.5 | 20.8 | 23.0 | 22.0 |
| 2092 | 165–32–F | 15.5 | 18.0 | 18.3 | 17.2 | 17.3 | 9.6 | 14.0 | 10.8 | 12.0 | 11.6 |
| 2093 | 165–48–F | 28.7 | 29.9 | 31.7 | 29.5 | 30.0 | 20.4 | 24.0 | 21.1 | 23.0 | 22.1 |
| 2094 | 110–56–F | 21.9 | 22.1 | 22.9 | 22.3 | 22.3 | 16.0 | 17.1 | 16.3 | 15.8 | 16.3 |
| 2095 | 55–48–F | 5.2 | 4.1 | 4.2 | 4.2 | 4.4 | 2.3 | 1.7 | 1.1 | 2.0 | 1.8 |
| 2096 | 55–40–F | 6.2 |  | 4.2 | 3.8 |  | 2.8 | 1.0 | 1.4 | 1.5 | 1.7 |
| 2097 | 55–32–F | 5.4 | 3.0 | 3.8 | 3.6 | 4.0 | 3.1 | 1.9 | 2.1 | 1.2 | 2.1 |
| 2098 | 55–48–F | 4.6 | 4.8 | 5.1 | 4.0 | 4.6 | 1.7 | 2.0 | 2.3 | 1.9 | 2.0 |
| 2090 | 110–24–F | 8.7 | 11.5 | 11.3 | 10.0 | 10.4 | 5.4 | 6.0 | 5.0 | 5.4 | 5.5 |
| 2091 | 165–32–F | 21.7 | 24.4 | 26.5 | 23.4 | 24.0 | 14.2 | 17.1 | 14.7 | 16.4 | 15.6 |

## TABLE 25.

## BOILER PERFORMANCE—COAL AND EVAPORATION.

| Test No. | Laboratory Designation | Dry Coal Fired, lb. | | Evaporation | | | | | Steam Used at Calorimeter, Safety Valve, Leaks etc., lb. | Dry Steam to Engine per Hour, lb. | Factor of Evaporation |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Per Hour | Per Hour per sq. ft. of Grate Surface | Moist Steam per Hour, lb. | Dry Steam, lb. | | | | | | |
| | | | | | Per Hour | Per Hour per sq. ft. of Heating Surface | Per lb. of Dry Coal | Per lb. of Coal as Fired | | | |
| | Code Item☞ | 626 | 627 | 633 | 634 | 635 | 636 | 637 | 638 | 639 | 641 |
| 2009 | 138–16–F | 2647 | 53.4 | 18 174 | 18 027 | 5.49 | 6.80 | 6.01 | 183 | 18 023 | 1.192 |
| 2010 | 193–20–F | 3834 | 77.4 | 23 887 | 23 668 | 7.21 | 6.17 | 5.44 | 39 | 23 631 | 1.196 |
| 2012 | 138–24–F | 3707 | 74.8 | 23 869 | 23 674 | 7.21 | 6.39 | 5.67 | 89 | 23 632 | 1.194 |
| 2013 | 138–32–F | 4749 | 95.8 | 29 076 | 28 751 | 8.76 | 6.05 | 5.51 | 50 | 28 742 | 1.190 |
| 2014 | 193–32–F | 6199 | 125.1 | 32 648 | 32 272 | 9.83 | 5.21 | 4.72 | 39 | 32 173 | 1.189 |
| 2015 | 193–24–F | 4927 | 99.5 | 27 617 | 27 363 | 8.33 | 5.55 | 4.95 | 50 | 27 330 | 1.193 |
| 2016 | 193–16–F | 3255 | 65.7 | 20 792 | 20 623 | 6.28 | 6.34 | 5.59 | 183 | 20 538 | 1.193 |
| 2017 | 83–16–F | 1957 | 39.5 | 14 778 | 14 683 | 4.47 | 7.50 | 6.74 | 246 | 14 590 | 1.186 |
| 2018 | 83–24–F | 2537 | 51.2 | 17 828 | 17 737 | 5.40 | 6.99 | 6.23 | 157 | 17 643 | 1.195 |
| 2019 | 83–32–F | 3215 | 64.9 | 22 429 | 22 288 | 6.79 | 6.93 | 6.20 | 150 | 22 173 | 1.196 |
| 2020 | 83–24–F | 2472 | 49.9 | 17 650 | 17 560 | 5.35 | 7.10 | 6.48 | 64 | 17 523 | 1.195 |
| 2021 | 83–16–F | 2211 | 44.6 | 14 066 | 14 011 | 4.27 | 6.34 | 5.56 | 502 | 13 758 | 1.204 |
| 2022 | 83–32–F | 3673 | 74.1 | 21 871 | 21 801 | 6.64 | 5.94 | 5.32 | 64 | 21 778 | 1.204 |
| 2023 | 138–40–F | 6687 | 135.0 | 35 199 | 35 025 | 10.67 | 5.24 | 4.54 | 623 | 34 594 | 1.202 |
| 2024 | 55–24–F | 1814 | 36.6 | 15 178 | 15 123 | 4.61 | 8.34 | 7.37 | 265 | 14 974 | 1.203 |
| 2026 | 110–16–F | 2293 | 46.3 | 16 461 | 16 341 | 4.98 | 7.13 | 6.22 | 71 | 16 313 | 1.196 |
| 2027 | 110–24–F | 3256 | 65.7 | 21 022 | 20 892 | 6.36 | 6.42 | 5.60 | 92 | 20 850 | 1.198 |
| 2028 | 55–32–F | 2406 | 48.6 | 16 950 | 16 841 | 5.13 | 7.00 | 6.13 | 169 | 16 775 | 1.188 |
| 2029 | 110–32–F | 4242 | 85.6 | 26 815 | 26 629 | 8.11 | 6.28 | 5.50 | 52 | 26 640 | 1.197 |
| 2030 | 165–24–F | 4013 | 81.0 | 26 326 | 26 126 | 7.96 | 6.51 | 5.80 | 63 | 26 081 | 1.197 |
| 2031 | 83–40–F | 4244 | 85.6 | 27 804 | 27 598 | 8.41 | 6.50 | 5.39 | 303 | 27 419 | 1.197 |
| 2032 | 165–32–F | 5352 | 108.0 | 30 933 | 30 627 | 9.33 | 5.72 | 4.98 | 105 | 30 480 | 1.195 |
| 2033 | 110–48–F | 5126 | 103.5 | 32 341 | 32 030 | 9.76 | 6.25 | 5.38 | 389 | 31 791 | 1.194 |
| 2034 | 193–40–F | 7767 | 156.8 | 38 841 | 38 445 | 11.71 | 4.95 | 4.31 | 414 | 38 330 | 1.194 |
| 2035 | 110–40–F | 5565 | 112.3 | 33 253 | 32 856 | 10.01 | 5.90 | 5.05 | 38 | 32 940 | 1.190 |
| 2037 | 165–40–F | 6554 | 132.3 | 38 440 | 38 056 | 11.59 | 5.81 | 4.99 | 102 | 37 769 | 1.193 |
| | | | | | | | | | | | |
| 2038 | 55–24–F | 2012 | 40.6 | 14 967 | 14 625 | 4.45 | 7.27 | 6.38 | 709 | 14 199 | 1.203 |
| 2039 | 110–32–F | 4517 | 91.2 | 27 927 | 27 762 | 8.46 | 6.15 | 5.42 | 109 | 27 663 | 1.201 |
| 2040 | 165–40–F | 7482 | 151.0 | 38 130 | 37 901 | 11.54 | 5.07 | 4.45 | 70 | 37 787 | 1.201 |
| 2041 | 110–40–F | 5861 | 118.3 | 33 656 | 33 163 | 10.10 | 5.66 | 4.86 | 290 | 32 794 | 1.193 |
| 2042 | 110–24–F | 3356 | 67.7 | 22 539 | 22 431 | 6.83 | 6.68 | 5.77 | 425 | 22 247 | 1.204 |
| 2043 | 110–48–F | 7403 | 149.4 | 38 840 | 38 468 | 11.72 | 5.20 | 4.50 | 314 | 38 213 | 1.196 |
| 2044 | 110–56–F | 8361 | 168.7 | 44 098 | 43 780 | 13.34 | 5.23 | 4.55 | 242 | 43 382 | 1.201 |
| 2045 | 110–16–F | 2427 | 49.0 | 17 228 | 17 151 | 5.22 | 7.07 | 6.13 | 388 | 16 968 | 1.201 |
| | | | | | | | | | | | |
| 2072 | 110–40–F | 5927 | 119.6 | 33 910 | 33 656 | 10.25 | 5.68 | 4.95 | 78 | 33 554 | 1.197 |
| 2073 | 110–32–F | 4359 | 87.9 | 28 029 | 27 866 | 8.49 | 6.39 | 5.54 | 238 | 27 731 | 1.203 |
| 2074 | 165–32–F | 6015 | 121.3 | 34 896 | 34 551 | 10.52 | 5.74 | 5.11 | 216 | 34 354 | 1.195 |
| 2075 | 55–32–F | 2327 | 47.0 | 16 574 | 16 523 | 5.03 | 7.10 | 6.18 | 171 | 16 431 | 1.204 |
| 2076 | 220–32–F | 7831 | 158.0 | 39 261 | 38 820 | 11.82 | 4.96 | 4.20 | 82 | 38 608 | 1.193 |
| 2077 | 110–24–F | 3281 | 66.2 | 21 959 | 21 878 | 6.64 | 6.67 | 5.82 | 141 | 21 770 | 1.203 |
| 2078 | 165–24–F | 4707 | 95.0 | 28 668 | 28 493 | 8.68 | 6.05 | 5.25 | 101 | 28 404 | 1.201 |
| 2079 | 220–24–F | 5783 | 116.7 | 31 849 | 31 597 | 9.62 | 5.46 | 4.99 | 224 | 31 347 | 1.198 |
| 2080 | 110–16–F | 2422 | 48.9 | 17 392 | 17 336 | 5.28 | 7.16 | 6.23 | 214 | 17 244 | 1.203 |
| 2081 | 55–24–F | 1975 | 39.9 | 14 326 | 14 289 | 4.35 | 7.24 | 6.39 | 211 | 14 205 | 1.206 |
| 2082 | 165–40–F | 8994 | 181.5 | 41 078 | 40 738 | 12.41 | 4.53 | 3.99 | 60 | 40 625 | 1.193 |
| 2083 | 165–16–F | 3338 | 67.4 | 22 022 | 21 918 | 6.68 | 6.57 | 5.66 | 425 | 21 656 | 1.201 |
| 2084 | 110–48–F | 7914 | 159.7 | 38 932 | 38 646 | 11.77 | 4.88 | 4.24 | 60 | 38 671 | 1.205 |
| 2085 | 55–40–F | 3058 | 61.7 | 20 730 | 20 674 | 6.30 | 6.76 | 5.91 | 146 | 20 616 | 1.219 |
| 2086 | 55–24–F | 2068 | 41.7 | 14 329 | 14 290 | 4.35 | 6.91 | 6.08 | 213 | 14 219 | 1.208 |
| 2087 | 110–16–F | 2474 | 49.9 | 17 314 | 17 259 | 5.26 | 6.98 | 6.16 | 189 | 17 175 | 1.204 |
| 2088 | 220–16–F | 3353 | 67.7 | 22 459 | 22 328 | 6.80 | 6.66 | 5.77 | 142 | 22 233 | 1.202 |
| 2089 | 220–40–F | 11127 | 224.5 | 45 902 | 45 521 | 13.87 | 4.09 | 3.59 | 42 | 45 498 | 1.198 |
| 2092 | 165–32–F | 5640 | 113.8 | 34 866 | 34 601 | 10.54 | 6.13 | 5.25 | 64 | 34 660 | 1.198 |
| 2093 | 165–48–F | 10216 | 206.2 | 48 416 | 47 994 | 14.62 | 4.70 | 4.04 | 36 | 48 387 | 1.197 |
| 2094 | 110–56–F | 8434 | 170.2 | 45 355 | 44 988 | 13.71 | 5.33 | 4.58 | 80 | 44 709 | 1.198 |
| 2095 | 55–48–F | 3334 | 67.3 | 23 805 | 23 707 | 7.22 | 7.11 | 6.24 | 73 | 23 643 | 1.202 |
| 2096 | 55–40–F | | | 21 222 | 21 057 | 6.41 | | | 285 | 20 837 | 1.196 |
| 2097 | 55–32–F | | | 18 124 | 17 933 | 5.46 | | | 291 | 17 807 | 1.196 |
| 2098 | 55–48–F | | | 24 498 | 24 216 | 7.38 | | | 303 | 23 843 | 1.194 |
| | | | | | | | | | | | |
| 2090 | 110–24–F | 3176 | 64.1 | 21 688 | 21 614 | 6.58 | 6.80 | 6.00 | 73 | 21 542 | 1.203 |
| 2091 | 165–32–F | 5252 | 106.0 | 33 534 | 33 320 | 10.15 | 6.34 | 5.50 | 39 | 33 178 | 1.200 |

## TABLE 26.

## BOILER PERFORMANCE—EQUIVALENT EVAPORATION, HORSE POWER, AND EFFICIENCY.

| Test No. | Laboratory Designation | Dry Steam Loss per Hour Due to Calorimeter, Leaks, Corrections etc., lb. | Dry Coal Loss per Hour Equivalent to Steam Loss, lb. | Equivalent Evaporation From and at 212°F., lb. | | | | | | Boiler Horse Power | Efficiency of Boiler, per cent |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Per Hour | Per Hour per sq. ft. of Total Heating Surface | Per Hour per sq. ft. of Grate Area | Per lb. of Coal as Fired | Per lb. of Dry Coal | Per lb. of Combustible | | |
| | CodeItem☞ | 642 | 643 | 645 | 648 | 656 | 657 | 658 | 659 | 660 | 666 |
| 2009 | 138–16–F | 4 | 1 | 21 669 | 6.60 | 437.3 | 7.23 | 8.19 | 9.36 | 628.1 | 63.30 |
| 2010 | 193–20–F | 37 | 6 | 28 564 | 8.71 | 576.5 | 6.57 | 7.45 | 8.57 | 827.9 | 57.63 |
| 2012 | 138–24–F | 42 | 6 | 28 514 | 8.69 | 575.4 | 6.83 | 7.69 | 8.96 | 826.5 | 60.80 |
| 2013 | 138–32–F | 9 | 1 | 34 612 | 10.54 | 698.5 | 6.63 | 7.29 | 8.48 | 1003.3 | 57.78 |
| 2014 | 193–32–F | 99 | 18 | 38 820 | 11.83 | 783.5 | 5.68 | 6.27 | 7.36 | 1125.2 | 49.92 |
| 2015 | 193–24–F | 33 | 6 | 32 958 | 10.04 | 665.2 | 5.97 | 6.69 | 7.77 | 955.3 | 52.84 |
| 2016 | 193–16–F | 85 | 14 | 24 804 | 7.56 | 500.7 | 6.73 | 7.62 | 9.09 | 719.1 | 61.68 |
| 2017 | 83–16–F | 93 | 12 | 17 513 | 5.33 | 353.4 | 8.05 | 8.96 | 10.35 | 507.6 | 69.88 |
| 2018 | 83–24–F | 94 | 13 | 21 326 | 6.50 | 430.4 | 7.49 | 8.41 | 9.81 | 618.1 | 66.49 |
| 2019 | 83–32–F | 115 | 16 | 26 834 | 8.17 | 541.6 | 7.46 | 8.35 | 9.54 | 777.8 | 64.68 |
| 2020 | 83–24–F | 37 | 5 | 21 092 | 6.42 | 425.7 | 7.79 | 8.53 | 9.77 | 611.4 | 67.33 |
| 2021 | 83–16–F | 253 | 40 | 16 934 | 5.16 | 341.8 | 6.72 | 7.66 | 8.83 | 490.8 | 60.56 |
| 2022 | 83–32–F | 123 | 4 | 26 332 | 8.02 | 531.4 | 6.42 | 7.17 | 8.46 | 763.3 | 58.54 |
| 2023 | 138–40–F | 431 | 82 | 42 329 | 12.89 | 854.3 | 5.49 | 6.33 | 7.35 | 1226.9 | 49.86 |
| 2024 | 55–24–F | 149 | 18 | 18 258 | 5.56 | 368.5 | 8.90 | 10.07 | 11.52 | 529.2 | 76.87 |
| 2026 | 110–16–F | 28 | 4 | 19 700 | 6.00 | 397.6 | 7.50 | 8.59 | 10.01 | 571.0 | 67.75 |
| 2027 | 110–24–F | 42 | 7 | 25 206 | 7.68 | 508.7 | 6.76 | 7.74 | 8.91 | 730.6 | 59.22 |
| 2028 | 55–32–F | 66 | 9 | 20 143 | 6.14 | 406.5 | 7.33 | 8.37 | 9.62 | 583.9 | 64.21 |
| 2029 | 110–32–F | –11 | – 2 | 32 108 | 9.78 | 648.0 | 6.64 | 7.57 | 8.84 | 930.7 | 58.85 |
| 2030 | 165–24–F | 45 | 7 | 31 517 | 9.60 | 636.1 | 7.00 | 7.85 | 8.92 | 913.5 | 59.71 |
| 2031 | 83–40–F | 179 | 28 | 33 281 | 10.14 | 671.7 | 6.49 | 7.84 | 9.22 | 964.7 | 63.43 |
| 2032 | 165–32–F | 147 | 26 | 36 967 | 11.26 | 746.1 | 6.01 | 6.91 | 8.11 | 1071.5 | 54.79 |
| 2033 | 110–48–F | 239 | 38 | 38 624 | 11.77 | 779.5 | 6.49 | 7.54 | 8.82 | 1119.5 | 59.76 |
| 2034 | 193–40–F | 115 | 23 | 46 380 | 14.13 | 936.0 | 5.20 | 5.97 | 7.13 | 1344.4 | 48.95 |
| 2035 | 110–40–F | –84 | –14 | 39 578 | 12.06 | 798.8 | 6.08 | 7.11 | 8.28 | 1147.2 | 55.94 |
| 2037 | 165–40–F | 287 | 49 | 45 859 | 13.97 | 925.5 | 6.01 | 7.00 | 8.14 | 1329.3 | 54.54 |
| 2038 | 55–24–F | 426 | 59 | 17 676 | 5.38 | 356.7 | 7.71 | 8.79 | 10.86 | 512.4 | 74.51 |
| 2039 | 110–32–F | 99 | 16 | 33 529 | 10.21 | 676.7 | 6.55 | 7.42 | 8.90 | 971.9 | 61.38 |
| 2040 | 165–40–F | 114 | 23 | 45 802 | 13.95 | 824.4 | 5.38 | 6.12 | 7.24 | 1327.6 | 48.84 |
| 2041 | 110–40–F | 369 | 65 | 40 168 | 12.24 | 810.7 | 5.89 | 6.85 | 8.01 | 1164.3 | 54.18 |
| 2042 | 110–24–F | 184 | 27 | 27 132 | 8.26 | 547.6 | 6.98 | 8.08 | 9.49 | 787.0 | 63.89 |
| 2043 | 110–48–F | 255 | 49 | 46 472 | 14.16 | 937.9 | 5.43 | 6.28 | 7.30 | 1347.0 | 48.60 |
| 2044 | 110–56–F | 398 | 76 | 52 948 | 16.13 | 1068.6 | 5.50 | 6.33 | 7.48 | 1534.7 | 50.38 |
| 2045 | 110–16–F | 183 | 26 | 20 704 | 6.31 | 417.9 | 7.40 | 8.53 | 10.27 | 600.1 | 69.65 |
| 2072 | 110–40–F | 102 | 18 | 40 590 | 12.36 | 819.2 | 5.97 | 6.85 | 7.95 | 1176.5 | 53.36 |
| 2073 | 110–32–F | 135 | 21 | 33 719 | 10.27 | 680.5 | 6.71 | 7.74 | 8.79 | 977.4 | 58.92 |
| 2074 | 165–32–F | 197 | 34 | 41 701 | 12.70 | 841.6 | 6.16 | 6.93 | 8.01 | 1208.7 | 53.50 |
| 2075 | 55–32–F | 92 | 13 | 19 954 | 6.08 | 402.7 | 7.47 | 8.57 | 9.72 | 578.4 | 65.46 |
| 2076 | 220–32–F | 212 | 43 | 46 838 | 14.27 | 945.3 | 5.07 | 5.98 | 6.86 | 1357.6 | 46.41 |
| 2077 | 110–24–F | 108 | 16 | 26 417 | 8.05 | 533.1 | 7.02 | 8.05 | 9.24 | 765.7 | 61.82 |
| 2078 | 165–24–F | 89 | 15 | 34 431 | 10.49 | 694.9 | 6.34 | 7.31 | 8.39 | 998.0 | 56.66 |
| 2079 | 220–24–F | 250 | 46 | 38 155 | 11.62 | 770.0 | 6.03 | 6.60 | 7.45 | 1105.9 | 50.18 |
| 2080 | 110–16–F | 92 | 13 | 20 923 | 6.37 | 422.3 | 7.52 | 8.64 | 9.72 | 606.5 | 65.28 |
| 2081 | 55–24–F | 84 | 12 | 17 277 | 5.26 | 348.7 | 7.73 | 8.75 | 9.92 | 500.8 | 66.89 |
| 2082 | 165–40–F | 113 | 25 | 49 007 | 14.93 | 989.0 | 4.80 | 5.45 | 6.15 | 1420.5 | 41.87 |
| 2083 | 165–16–F | 262 | 40 | 26 448 | 8.06 | 533.8 | 6.83 | 7.92 | 9.09 | 766.6 | 60.72 |
| 2084 | 110–48–F | –25 | – 5 | 46 913 | 14.29 | 946.7 | 5.15 | 5.93 | 6.94 | 1359.8 | 46.76 |
| 2085 | 55–40–F | 58 | 9 | 25 270 | 7.70 | 509.9 | 7.23 | 8.26 | 9.44 | 732.5 | 63.54 |
| 2086 | 55–24–F | 71 | 10 | 17 308 | 5.27 | 349.3 | 7.36 | 8.37 | 9.65 | 501.7 | 64.49 |
| 2087 | 110–16–F | 84 | 12 | 20 846 | 6.35 | 420.7 | 7.44 | 8.43 | 9.77 | 604.2 | 66.63 |
| 2088 | 220–16–F | 95 | 14 | 26 995 | 8.22 | 544.8 | 6.99 | 8.05 | 9.60 | 782.5 | 64.58 |
| 2089 | 220–40–F | 23 | 6 | 54 989 | 16.75 | 1109.7 | 4.33 | 4.94 | 5.77 | 1593.9 | 38.77 |
| 2092 | 165–32–F | –59 | –10 | 41 770 | 12.72 | 842.9 | 6.34 | 7.41 | 8.43 | 1210.7 | 56.95 |
| 2093 | 165–48–F | –393 | –84 | 57 954 | 17.65 | 1169.5 | 4.88 | 5.67 | 6.52 | 1678.8 | 43.82 |
| 2094 | 110–56–F | 279 | 52 | 54 336 | 16.55 | 1096.5 | 5.53 | 6.44 | 7.41 | 1575.0 | 50.33 |
| 2095 | 55–48–F | 64 | 9 | 28 614 | 8.72 | 577.4 | 7.53 | 8.58 | 10.03 | 829.4 | 67.61 |
| 2096 | 55–40–F | 220 | | 25 382 | 7.73 | 512.2 | | | | 735.7 | |
| 2097 | 55–32–F | 126 | | 21 676 | 6.60 | 437.4 | | | | 628.3 | |
| 2098 | 55–48–F | 373 | | 29 251 | 8.91 | 590.3 | | | | 847.9 | |
| 2090 | 110–24–F | 72 | 11 | 26 091 | 7.95 | 526.5 | 7.24 | 8.22 | 9.41 | 756.3 | 63.33 |
| 2091 | 165–32–F | 142 | 23 | 40 240 | 12.26 | 812.0 | 6.64 | 7.66 | 8.74 | 1166.4 | 58.76 |

## TABLE 27.

## ENGINE PERFORMANCE—MEAN EFFECTIVE PRESSURE AND NUMBER OF EXPANSIONS.

| Test No. | Laboratory Designation | Mean Effective Pressure, lb. per sq. in. | | | | | Number of Expansions | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Right Side | | Left Side | | Average | Right Side | | Left Side | |
| | | Head End | Crank End | Head End | Crank End | | Head End | Crank End | Head End | Crank End |
| | Code Item☞ | 674 | 675 | 676 | 677 | 678 | 697 | 698 | 699 | 700 |
| 2009 | 138–16–F | 32.8 | 32.9 | 32.9 | 37.7 | 34.1 | 2.51 | 2.34 | 2.35 | 2.50 |
| 2010 | 193–20–F | 28.3 | 36.9 | 29.0 | 36.4 | 32.6 | 2.18 | 2.38 | 2.55 | 2.56 |
| 2012 | 138–24–F | 45.5 | 53.1 | 48.0 | 53.7 | 50.1 | 2.15 | 2.36 | 2.29 | 2.24 |
| 2013 | 138–32–F | 56.8 | 65.5 | 57.4 | 65.9 | 61.4 | 1.95 | 1.84 | 1.93 | 2.03 |
| 2014 | 193–32–F | 44.7 | 52.2 | 41.6 | 49.7 | 47.1 | 1.78 | 1.89 | 1.99 | 1.94 |
| 2015 | 193–24–F | 33.8 | 40.3 | 32.8 | 40.3 | 36.8 | 2.21 | 2.21 | 2.18 | 2.30 |
| 2016 | 193–16–F | 23.0 | 28.4 | 21.1 | 29.3 | 25.5 | 2.44 | 2.58 | 2.74 | 2.53 |
| 2017 | 83–16–F | 42.9 | 46.8 | 45.8 | 50.8 | 46.6 | 2.40 | 2.47 | 2.39 | 2.31 |
| 2018 | 83–24–F | 59.5 | 65.4 | 63.7 | 69.7 | 64.6 | 2.17 | 2.24 | 2.13 | 2.07 |
| 2019 | 83–32–F | 76.2 | 85.1 | 79.9 | 89.6 | 82.7 | 2.03 | 1.82 | 1.95 | 1.74 |
| 2020 | 83–24–F | 58.2 | 63.5 | 62.4 | 68.6 | 63.2 | 2.38 | 2.05 | 2.25 | 2.08 |
| 2021 | 83–16–F | 42.2 | 43.8 | 47.3 | 52.7 | 46.5 | 2.65 | 2.57 | 2.53 | 2.34 |
| 2022 | 83–32–F | 77.2 | 86.7 | 80.6 | 89.5 | 83.5 | 1.97 | 1.85 | 2.01 | 1.80 |
| 2023 | 138–40–F | 70.2 | 76.4 | 69.8 | 81.3 | 74.4 | 1.87 | 1.59 | 1.69 | 1.66 |
| 2024 | 55–24–F | 69.8 | 74.8 | 72.5 | 78.8 | 74.0 | | | | |
| 2026 | 110–16–F | 36.5 | 40.6 | 38.1 | 48.7 | 41.0 | 2.49 | 1.92 | 2.37 | 2.41 |
| 2027 | 110–24–F | 55.3 | 59.3 | 57.3 | 65.2 | 59.3 | 2.26 | 2.11 | 2.28 | 2.08 |
| 2028 | 55–32–F | 88.2 | 93.2 | 94.2 | 103.9 | 94.9 | 1.96 | 1.83 | 1.91 | 1.78 |
| 2029 | 110–32–F | 71.8 | 76.6 | 73.8 | 84.7 | 76.7 | 2.08 | 1.97 | 1.96 | 1.89 |
| 2030 | 165–24–F | 42.4 | 47.5 | 45.5 | 49.4 | 46.2 | 2.21 | 2.29 | 2.35 | 2.05 |
| 2031 | 83–40–F | 91.9 | 95.8 | 95.3 | 104.6 | 96.9 | 1.78 | 1.67 | 1.71 | 1.60 |
| 2032 | 165–32–F | 53.8 | 56.8 | 53.6 | 62.5 | 56.7 | 1.95 | 1.97 | 2.14 | 1.93 |
| 2033 | 110–48–F | 88.9 | 91.6 | 88.1 | 93.1 | 90.4 | 1.75 | 1.72 | 1.68 | 1.58 |
| 2034 | 193–40–F | 54.7 | 56.4 | 52.2 | 60.6 | 56.0 | 1.80 | 1.61 | 1.80 | 1.67 |
| 2035 | 110–40–F | 83.3 | 85.0 | 86.8 | 93.2 | 87.1 | 1.81 | 1.72 | 1.67 | 1.65 |
| 2037 | 165–40–F | 63.1 | 65.4 | 62.9 | 71.2 | 65.7 | 1.73 | 1.66 | 1.84 | 1.64 |
| 2038 | 55–24–F | 66.6 | 73.6 | 73.1 | 80.3 | 73.4 | | | | |
| 2039 | 110–32–F | 69.3 | 77.0 | 71.9 | 79.4 | 74.4 | 1.87 | 1.78 | 1.87 | 1.83 |
| 2040 | 165–40–F | 61.8 | 66.2 | 62.1 | 68.1 | 64.5 | 1.65 | 1.67 | 1.65 | 1.68 |
| 2041 | 110–40–F | 83.3 | 90.4 | 85.8 | 91.1 | 87.7 | 1.67 | 1.65 | 1.65 | 1.67 |
| 2042 | 110–24–F | 54.3 | 60.7 | 60.8 | 63.8 | 59.9 | 2.03 | 2.14 | 1.92 | 2.12 |
| 2043 | 110–48–F | 97.4 | 101.3 | 98.3 | 102.5 | 99.9 | 1.54 | 1.49 | 1.60 | 1.52 |
| 2044 | 110–56–F | 103.0 | 107.0 | 104.8 | 108.7 | 105.9 | 1.38 | 1.37 | 1.36 | 1.37 |
| 2045 | 110–16–F | 37.2 | 42.0 | 40.3 | 45.0 | 41.1 | 2.13 | 2.39 | 1.97 | 2.28 |
| 2072 | 110–40–F | 94.1 | 99.3 | 96.9 | 100.2 | 97.6 | 1.64 | 1.63 | 1.63 | 1.67 |
| 2073 | 110–32–F | 77.4 | 81.7 | 80.4 | 83.6 | 80.8 | 2.29 | 1.95 | 1.92 | 1.90 |
| 2074 | 165–32–F | 61.7 | 67.8 | 61.8 | 67.5 | 64.7 | 2.50 | 1.87 | 1.92 | 1.98 |
| 2075 | 55–32–F | 93.8 | 99.9 | 101.7 | 100.3 | 98.9 | 1.89 | 1.77 | 1.83 | 1.91 |
| 2076 | 220–32–F | 53.0 | 53.2 | 47.1 | 52.6 | 51.5 | 1.96 | 1.79 | 1.76 | 1.71 |
| 2077 | 110–24–F | 56.0 | 63.9 | 61.9 | 67.6 | 62.4 | 2.05 | 2.05 | 2.04 | 2.09 |
| 2078 | 165–24–F | 48.2 | 55.6 | 50.0 | 54.0 | 52.0 | 2.10 | 2.12 | 2.02 | 2.26 |
| 2079 | 220–24–F | 40.9 | 43.2 | 40.2 | 41.8 | 41.5 | 2.28 | 2.26 | 2.32 | 2.18 |
| 2080 | 110–16–F | 38.8 | 45.5 | 43.5 | 47.7 | 43.9 | 2.32 | 2.32 | 2.43 | 2.53 |
| 2081 | 55–24–F | 72.1 | 78.1 | 80.8 | 77.6 | 77.1 | 2.01 | 1.99 | 1.98 | 2.18 |
| 2082 | 165–40–F | 71.7 | 78.3 | 71.9 | 75.9 | 74.5 | 1.81 | 1.49 | 1.67 | 1.63 |
| 2083 | 165–16–F | 33.2 | 39.3 | 39.0 | 37.3 | 37.2 | 2.28 | 2.35 | 2.20 | 2.45 |
| 2084 | 110–48–F | 103.3 | 108.1 | 103.5 | 107.8 | 105.7 | 1.53 | 1.49 | 1.54 | 1.54 |
| 2085 | 55–40–F | 112.7 | 120.0 | 120.3 | 118.1 | 117.8 | 1.60 | 1.61 | 1.64 | 1.67 |
| 2086 | 55–24–F | 71.9 | 77.2 | 81.1 | 78.0 | 77.1 | 2.09 | 2.08 | 2.03 | 2.22 |
| 2087 | 110–16–F | 39.2 | 44.5 | 43.7 | 47.6 | 43.7 | 2.22 | 2.43 | 2.40 | 2.30 |
| 2088 | 220–16–F | 28.0 | 27.1 | 29.7 | 27.5 | 28.1 | 2.54 | 2.50 | 2.67 | 2.56 |
| 2089 | 220–40–F | 57.8 | 60.8 | 56.2 | 60.1 | 58.7 | 1.63 | 1.57 | 1.69 | 1.57 |
| 2092 | 165–32–F | 61.9 | 68.0 | 63.9 | 67.3 | 65.3 | 1.96 | 1.93 | 1.97 | 2.01 |
| 2093 | 165–48–F | 81.1 | 86.9 | 82.0 | 86.6 | 84.2 | 1.57 | 1.47 | 1.53 | 1.45 |
| 2094 | 110–56–F | 115.8 | 120.8 | 118.5 | 121.3 | 119.1 | 1.41 | 1.42 | 1.36 | 1.42 |
| 2095 | 55–48–F | 131.8 | 136.7 | 136.7 | 136.2 | 135.4 | 1.54 | 1.49 | 1.49 | 1.55 |
| 2096 | 55–40–F | 115.1 | 121.2 | 123.2 | 121.3 | 120.2 | 1.72 | 1.63 | 1.64 | 1.71 |
| 2097 | 55–32–F | 97.0 | 103.7 | 103.2 | 102.2 | 101.5 | 1.85 | 1.82 | 1.79 | 1.93 |
| 2098 | 55–48–F | 133.0 | 138.1 | 141.3 | 137.7 | 137.5 | 1.54 | 1.49 | 1.50 | 1.57 |
| 2090 | 110–24–F | 52.7 | 59.1 | 59.2 | 61.4 | 58.1 | 2.10 | 2.32 | 2.20 | 2.11 |
| 2091 | 165–32–F | 54.2 | 60.2 | 58.1 | 60.9 | 58.3 | 2.05 | 1.81 | 2.16 | 2.22 |

## TABLE 28.

## ENGINE PERFORMANCE—INDICATED HORSE POWER.

| Test No. | Laboratory Designation | Indicated Horse Power | | | | | |
|---|---|---|---|---|---|---|---|
| | | Right Side | | Left Side | | Total | Maximum |
| | | Head End | Crank End | Head End | Crank End | | |
| | Code Item☞ | 707 | 708 | 709 | 710 | 711 | 721 |
| 2009 | 138–16–F | 131.9 | 128.8 | 135.1 | 149.7 | 545.5 | 687.5 |
| 2010 | 193–20–F | 161.2 | 203.4 | 167.9 | 204.5 | 737.0 | 902.5 |
| 2012 | 138–24–F | 183.9 | 207.4 | 197.3 | 214.2 | 802.8 | 1023.7 |
| 2013 | 138–32–F | 229.6 | 256.5 | 237.0 | 263.2 | 986.3 | 1224.6 |
| 2014 | 193–32–F | 258.4 | 292.0 | 245.0 | 283.6 | 1079.0 | 1374.9 |
| 2015 | 193–24–F | 195.8 | 225.9 | 193.4 | 230.5 | 845.6 | 1051.3 |
| 2016 | 193–16–F | 133.1 | 158.9 | 124.6 | 167.2 | 583.8 | 774.8 |
| 2017 | 83–16–F | 99.3 | 104.8 | 107.9 | 116.1 | 428.1 | 624.5 |
| 2018 | 83–24–F | 137.6 | 146.6 | 150.3 | 159.3 | 593.8 | 742.0 |
| 2019 | 83–32–F | 177.6 | 192.1 | 189.7 | 206.3 | 765.7 | 948.8 |
| 2020 | 83–24–F | 135.4 | 143.2 | 148.1 | 157.7 | 584.4 | 596.3 |
| 2021 | 83–16–F | 97.8 | 98.3 | 111.7 | 120.8 | 428.6 | 441.7 |
| 2022 | 83–32–F | 180.0 | 195.9 | 191.8 | 206.2 | 773.9 | 797.9 |
| 2023 | 138–40–F | 282.4 | 297.5 | 286.0 | 322.8 | 1188.7 | 1217.6 |
| 2024 | 55–24–F | 102.3 | 106.2 | 108.3 | 114.2 | 431.0 | 450.9 |
| 2026 | 110–16–F | 115.5 | 124.5 | 122.7 | 152.3 | 515.0 | 527.6 |
| 2027 | 110–24–F | 175.9 | 182.7 | 185.7 | 204.8 | 749.1 | |
| 2028 | 55–32–F | 128.4 | 131.3 | 139.7 | 149.3 | 548.7 | 557.6 |
| 2029 | 110–32–F | 228.1 | 235.8 | 238.9 | 265.8 | 968.6 | 974.7 |
| 2030 | 165–24–F | 208.0 | 225.2 | 227.1 | 239.3 | 899.6 | 924.4 |
| 2031 | 83–40–F | 226.9 | 229.5 | 241.1 | 256.4 | 953.9 | 992.4 |
| 2032 | 165–32–F | 261.4 | 267.3 | 265.7 | 300.2 | 1094.6 | 1125.8 |
| 2033 | 110–48–F | 282.6 | 282.0 | 285.4 | 292.3 | 1142.3 | 1155.3 |
| 2034 | 193–40–F | 314.0 | 313.6 | 305.3 | 343.8 | 1276.7 | 1299.0 |
| 2035 | 110–40–F | 269.4 | 266.1 | 285.9 | 297.7 | 1119.1 | 1148.6 |
| 2037 | 165–40–F | 309.1 | 310.1 | 313.9 | 344.6 | 1277.7 | 1307.5 |
| | | | | | | | |
| 2038 | 55–24–F | 97.8 | 104.7 | 109.4 | 116.5 | 428.4 | 442.6 |
| 2039 | 110–32–F | 220.5 | 237.1 | 233.0 | 249.4 | 940.0 | 967.5 |
| 2040 | 165–40–F | 301.4 | 312.8 | 308.7 | 328.1 | 1251.0 | 1294.3 |
| 2041 | 110–40–F | 265.4 | 279.3 | 278.7 | 287.0 | 1110.4 | 1133.7 |
| 2042 | 110–24–F | 172.8 | 186.5 | 197.0 | 197.4 | 753.9 | 799.9 |
| 2043 | 110–48–F | 309.0 | 311.3 | 318.1 | 321.2 | 1259.6 | 1284.3 |
| 2044 | 110–56–F | 326.7 | 328.7 | 338.7 | 340.6 | 1334.7 | 1391.1 |
| 2045 | 110–16–F | 118.2 | 129.2 | 130.5 | 141.2 | 519.1 | 534.7 |
| | | | | | | | |
| 2072 | 110–40–F | 299.3 | 305.9 | 313.8 | 314.8 | 1233.8 | 1253.2 |
| 2073 | 110–32–F | 245.8 | 251.3 | 260.1 | 262.1 | 1019.3 | 1047.8 |
| 2074 | 165–32–F | 303.7 | 322.9 | 309.7 | 328.0 | 1264.3 | 1282.6 |
| 2075 | 55–32–F | 137.9 | 142.3 | 152.7 | 145.7 | 578.6 | 597.2 |
| 2076 | 220–32–F | 353.5 | 344.0 | 320.0 | 346.8 | 1364.3 | 1390.0 |
| 2077 | 110–24–F | 179.9 | 198.8 | 202.7 | 214.3 | 795.7 | 813.9 |
| 2078 | 165–24–F | 236.4 | 263.8 | 249.5 | 261.5 | 1011.2 | 1023.2 |
| 2079 | 220–24–F | 275.4 | 281.1 | 275.3 | 278.1 | 1109.9 | 1152.3 |
| 2080 | 110–16–F | 124.3 | 141.3 | 142.0 | 150.9 | 558.5 | 576.0 |
| 2081 | 55–24–F | 105.9 | 111.2 | 120.9 | 112.5 | 450.5 | 467.6 |
| 2082 | 165–40–F | 353.1 | 373.7 | 361.1 | 369.4 | 1457.3 | 1484.1 |
| 2083 | 165–16–F | 163.9 | 188.0 | 196.4 | 182.1 | 730.4 | 756.1 |
| 2084 | 110–48–F | 331.6 | 336.0 | 338.5 | 341.4 | 1347.5 | 1366.2 |
| 2085 | 55–40–F | 167.2 | 172.4 | 181.7 | 173.0 | 694.3 | 705.6 |
| 2086 | 55–24–F | 106.5 | 111.4 | 123.2 | 114.9 | 456.0 | 466.4 |
| 2087 | 110–16–F | 126.5 | 138.9 | 143.5 | 151.4 | 560.3 | 574.9 |
| 2088 | 220–16–F | 189.5 | 177.7 | 204.9 | 184.0 | 756.1 | 791.4 |
| 2089 | 220–40–F | 386.6 | 393.6 | 382.6 | 397.1 | 1559.9 | 1588.6 |
| 2092 | 165–32–F | 302.2 | 321.4 | 319.6 | 324.1 | 1267.3 | 1427.0 |
| 2093 | 165–48–F | 396.0 | 411.5 | 408.3 | 417.7 | 1633.5 | 1654.2 |
| 2094 | 110–56–F | 372.2 | 375.9 | 388.3 | 385.0 | 1521.4 | 1570.1 |
| 2095 | 55–48–F | 196.2 | 197.1 | 211.4 | 200.2 | 804.9 | 822.7 |
| 2096 | 55–40–F | 172.0 | 175.3 | 187.4 | 178.9 | 713.6 | 733.1 |
| 2097 | 55–32–F | 146.7 | 152.0 | 159.0 | 152.8 | 610.6 | 621.0 |
| 2098 | 55–48–F | 200.0 | 201.1 | 216.6 | 204.5 | 822.2 | 837.5 |
| | | | | | | | |
| 2090 | 110–24–F | 169.7 | 184.2 | 194.2 | 195.0 | 743.1 | 757.3 |
| 2091 | 165–32–F | 266.6 | 286.6 | 291.0 | 295.8 | 1140.0 | 1170.8 |

## TABLE 29.

## ENGINE PERFORMANCE—COAL, STEAM, AND B.T.U. PER INDICATED HORSE POWER HOUR.

| Test No. | Laboratory Designation | Consumed per Indicated Horse Power per Hour | | | |
|---|---|---|---|---|---|
| | | Dry Coal, lb. | B.t.u. in Coal | Dry Steam, lb. | B.t.u. in Steam Above 32°F. |
| | Code Item☞ | 734 | 735 | 736 | 737 |
| 2009 | 138–16–F | 4.85 | 60 882 | 33.06 | |
| 2010 | 193–20–F | 5.19 | 64 527 | 32.07 | |
| 2012 | 138–24–F | 4.61 | 56 611 | 29.44 | |
| 2013 | 138–32–F | 4.81 | 58 884 | 29.14 | |
| 2014 | 193–32–F | 5.72 | 69 692 | 29.82 | |
| 2015 | 193–24–F | 5.82 | 71 627 | 32.32 | |
| 2016 | 193–16–F | 5.55 | 66 556 | 35.18 | |
| 2017 | 83–16–F | 4.54 | 56 396 | 34.08 | |
| 2018 | 83–24–F | 4.25 | 52 126 | 29.71 | |
| 2019 | 83–32–F | 4.18 | 52 346 | 28.96 | |
| 2020 | 83–24–F | 4.22 | 51 914 | 29.99 | 35 943 |
| 2021 | 83–16–F | 5.07 | 62 229 | 32.10 | 38 478 |
| 2022 | 83–32–F | 4.75 | 56 406 | 28.14 | 33 723 |
| 2023 | 138–40–F | 5.56 | 68 449 | 29.10 | 34 876 |
| 2024 | 55–24–F | 4.17 | 53 009 | 34.74 | 41 650 |
| 2026 | 110–16–F | 4.44 | 54 652 | 31.67 | 37 969 |
| 2027 | 110–24–F | 4.34 | 55 066 | 27.84 | 33 377 |
| 2028 | 55–32–F | 4.37 | 55 294 | 30.57 | 36 653 |
| 2029 | 110–32–F | 4.38 | 55 188 | 27.51 | 32 984 |
| 2030 | 165–24–F | 4.45 | 56 769 | 28.99 | 34 756 |
| 2031 | 83–40–F | 4.42 | 52 991 | 28.75 | 34 468 |
| 2032 | 165–32–F | 4.87 | 59 619 | 27.84 | 33 365 |
| 2033 | 110–48–F | 4.45 | 54 481 | 27.83 | 33 363 |
| 2034 | 193–40–F | 6.07 | 71 838 | 30.02 | 35 982 |
| 2035 | 110–40–F | 4.99 | 61 522 | 29.43 | 35 278 |
| 2037 | 165–40–F | 5.09 | 63 325 | 29.56 | 35 439 |
| 2038 | 55–24–F | 4.56 | 52 176 | 33.14 | 39 745 |
| 2039 | 110–32–F | 4.79 | 56 206 | 29.43 | 35 287 |
| 2040 | 165–40–F | 5.96 | 72 497 | 30.21 | 36 206 |
| 2041 | 110–40–F | 5.22 | 64 081 | 29.54 | 35 407 |
| 2042 | 110–24–F | 4.42 | 54 247 | 29.52 | 35 391 |
| 2043 | 110–48–F | 5.84 | 73 134 | 30.34 | 36 372 |
| 2044 | 110–56–F | 6.21 | 75 712 | 32.50 | 38 948 |
| 2045 | 110–16–F | 4.63 | 55 028 | 32.69 | 39 195 |
| 2072 | 110–40–F | 4.79 | 59 683 | 27.19 | 32 601 |
| 2073 | 110–32–F | 4.26 | 54 319 | 27.20 | 32 586 |
| 2074 | 165–32–F | 4.73 | 59 480 | 27.17 | 32 574 |
| 2075 | 55–32–F | 4.00 | 50 872 | 28.40 | 34 052 |
| 2076 | 220–32–F | 5.71 | 71 483 | 28.30 | 33 929 |
| 2077 | 110–24–F | 4.10 | 51 795 | 27.36 | 32 788 |
| 2078 | 165–24–F | 4.63 | 57 954 | 28.09 | 33 677 |
| 2079 | 220–24–F | 5.17 | 66 005 | 28.25 | 33 869 |
| 2080 | 110–16–F | 4.31 | 55 375 | 30.87 | 37 016 |
| 2081 | 55–24–F | 4.36 | 55 372 | 31.53 | 37 808 |
| 2082 | 165–40–F | 6.15 | 77 650 | 27.88 | 33 423 |
| 2083 | 165–16–F | 4.52 | 57 223 | 29.65 | 35 553 |
| 2084 | 110–48–F | 5.88 | 72 353 | 28.69 | 34 391 |
| 2085 | 55–40–F | 4.39 | 55 411 | 29.69 | 35 598 |
| 2086 | 55–24–F | 4.51 | 56 763 | 31.18 | 37 388 |
| 2087 | 110–16–F | 4.39 | 53 383 | 30.65 | 36 752 |
| 2088 | 220–16–F | 4.42 | 53 460 | 29.40 | 35 251 |
| 2089 | 220–40–F | 7.10 | 87 728 | 29.18 | 34 978 |
| 2092 | 165–32–F | 4.46 | 56 285 | 27.34 | 32 783 |
| 2093 | 165–48–F | 6.31 | 79 197 | 29.62 | 35 500 |
| 2094 | 110–56–F | 5.58 | 69 337 | 29.39 | 35 233 |
| 2095 | 55–48–F | 4.15 | 51 107 | 29.37 | 35 215 |
| 2096 | 55–40–F | | | | 35 005 |
| 2097 | 55–32–F | | | | 34 978 |
| 2098 | 55–48–F | | | | 34 783 |
| 2090 | 110–24–F | 4.26 | 53 650 | 28.99 | 34 762 |
| 2091 | 165–32–F | 4.58 | 58 080 | 29.10 | 34 894 |

## TABLE 30.

## GENERAL PERFORMANCE—DRAWBAR HORSE POWER AND MILLIONS OF FOOT POUNDS AT DRAWBAR.

| Test No. | Laboratory Designation | Drawbar Horse Power | Consumed per D.H.P. Hour | | | Millions of Foot Pounds at Drawbar per Hour | Per Million Ft. lb. at Drawbar | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | Dry Coal, lb. | Dry Steam, lb. | B.t.u. | | Dry Coal, lb. | Dry Steam, lb. | B.t.u. |
| | Code Item☞ | 743 | 744 | 745 | 746 | 750 | 752 | 753 | 754 |
| 2009 | 138–16–F | | | | | | | | |
| 2010 | 193–20–F | | | | | | | | |
| 2012 | 138–24–F | 684.9 | 5.40 | 34.50 | 66 312 | 1357 | 2.73 | 17.4 | 33 524 |
| 2013 | 138–32–F | 863.7 | 5.50 | 33.28 | 67 331 | 1711 | 2.77 | 16.8 | 33 910 |
| 2014 | 193–32–F | 853.1 | 7.25 | 37.71 | 88 334 | 1689 | 3.66 | 19.0 | 44 593 |
| 2015 | 193–24–F | 626.2 | 7.86 | 43.64 | 96 733 | 1240 | 3.97 | 22.0 | 48 859 |
| 2016 | 193–16–F | 418.2 | 7.75 | 49.11 | 92 938 | 828 | 3.92 | 24.8 | 47 009 |
| 2017 | 83–16–F | 357.1 | 5.45 | 40.85 | 67 700 | 707 | 2.75 | 20.6 | 34 161 |
| 2018 | 83–24–F | 511.7 | 4.93 | 34.48 | 60 466 | 1014 | 2.49 | 17.4 | 30 540 |
| 2019 | 83–32–F | 683.1 | 4.68 | 32.46 | 58 608 | 1353 | 2.36 | 16.4 | 29 554 |
| 2020 | 83–24–F | 508.9 | 4.85 | 34.46 | 59 665 | 1009 | 2.44 | 17.4 | 30 017 |
| 2021 | 83–16–F | 346.3 | 6.27 | 39.72 | 76 958 | 686 | 3.16 | 20.0 | 38 786 |
| 2022 | 83–32–F | 674.6 | 5.44 | 32.29 | 64 600 | 1336 | 2.75 | 16.3 | 32 656 |
| 2023 | 138–40–F | 1070.5 | 6.16 | 32.31 | 75 836 | 2120 | 3.11 | 16.3 | 38 287 |
| 2024 | 55–24–F | 355.6 | 5.05 | 42.34 | 64 196 | 706 | 2.54 | 21.2 | 32 288 |
| 2026 | 110–16–F | 415.1 | 5.51 | 39.31 | 67 823 | 822 | 2.78 | 19.8 | 34 219 |
| 2027 | 110–24–F | 633.6 | 5.13 | 32.92 | 65 089 | 1255 | 2.59 | 16.6 | 32 862 |
| 2028 | 55–32–F | 488.1 | 4.91 | 34.37 | 62 126 | 967 | 2.48 | 17.3 | 31 379 |
| 2029 | 110–32–F | 820.8 | 5.17 | 32.50 | 64 553 | 1626 | 2.61 | 16.4 | 32 588 |
| 2030 | 165–24–F | 725.8 | 5.52 | 35.95 | 70 419 | 1438 | 2.79 | 18.1 | 35 592 |
| 2031 | 83–40–F | 869.7 | 4.85 | 31.52 | 58 147 | 1723 | 2.45 | 15.9 | 29 373 |
| 2032 | 165–32–F | 922.8 | 5.77 | 33.22 | 70 636 | 1828 | 2.91 | 16.7 | 35 624 |
| 2033 | 110–48–F | 1007.9 | 5.05 | 31.54 | 61 827 | 1996 | 2.55 | 15.9 | 31 220 |
| 2034 | 193–40–F | 961.7 | 8.05 | 39.86 | 95 272 | 1905 | 4.07 | 20.1 | 48 168 |
| 2035 | 110–40–F | 942.9 | 5.92 | 34.99 | 72 988 | 1868 | 2.99 | 17.6 | 36 864 |
| 2037 | 165–40–F | 1045.3 | 6.22 | 36.09 | 77 383 | 2070 | 3.14 | 18.2 | 39 065 |
| 2038 | 55–24–F | 368.4 | 5.30 | 38.54 | 60 643 | 730 | 2.68 | 19.4 | 30 665 |
| 2039 | 110–32–F | 824.2 | 5.46 | 33.56 | 64 068 | 1631 | 2.76 | 17.0 | 32 386 |
| 2040 | 165–40–F | 1133.6 | 6.58 | 33.33 | 80 039 | 2244 | 3.32 | 16.8 | 40 384 |
| 2041 | 110–40–F | 1007.7 | 5.75 | 32.54 | 70 587 | 1995 | 2.91 | 16.4 | 35 723 |
| 2042 | 110–24–F | 674.8 | 4.93 | 32.97 | 60 506 | 1336 | 2.49 | 16.7 | 30 560 |
| 2043 | 110–48–F | 1158.5 | 6.35 | 32.98 | 79 521 | 2293 | 3.21 | 16.7 | 40 199 |
| 2044 | 110–56–F | 1257.7 | 6.59 | 34.49 | 80 345 | 2490 | 3.33 | 17.4 | 40 599 |
| 2045 | 110–16–F | 436.6 | 5.50 | 38.86 | 65 368 | 865 | 2.78 | 19.6 | 33 040 |
| 2072 | 110–40–F | 1107.8 | 5.33 | 30.28 | 66 412 | 2193 | 2.69 | 15.3 | 33 517 |
| 2073 | 110–32–F | 898.3 | 4.83 | 30.87 | 61 587 | 1779 | 2.44 | 15.6 | 31 112 |
| 2074 | 165–32–F | 1107.3 | 5.40 | 31.03 | 67 905 | 2193 | 2.73 | 15.7 | 34 330 |
| 2075 | 55–32–F | 501.4 | 4.62 | 32.76 | 58 757 | 993 | 2.33 | 16.5 | 29 633 |
| 2076 | 220–32–F | 1157.1 | 6.73 | 33.36 | 84 253 | 2291 | 3.40 | 16.9 | 42 565 |
| 2077 | 110–24–F | 670.2 | 4.87 | 32.48 | 61 523 | 1328 | 2.46 | 16.4 | 31 077 |
| 2078 | 165–24–F | 833.8 | 5.62 | 34.06 | 70 346 | 1651 | 2.84 | 17.1 | 35 548 |
| 2079 | 220–24–F | 928.5 | 6.18 | 33.84 | 78 900 | 1839 | 3.12 | 17.0 | 39 833 |
| 2080 | 110–16–F | | | | | | | | |
| 2081 | 55–24–F | | | | | | | | |
| 2082 | 165–40–F | 1214.6 | 7.38 | 33.45 | 93 180 | 2405 | 3.73 | 16.9 | 47 095 |
| 2083 | 165–16–F | 583.6 | 5.65 | 37.10 | 71 529 | 1156 | 2.85 | 18.7 | 36 081 |
| 2084 | 110–48–F | 1197.2 | 6.62 | 32.30 | 81 459 | 2371 | 3.34 | 16.3 | 41 099 |
| 2085 | 55–40–F | 614.5 | 4.96 | 33.55 | 62 605 | 1217 | 2.51 | 16.9 | 31 681 |
| 2086 | 55–24–F | 386.0 | 5.33 | 36.83 | 67 083 | 764 | 2.69 | 18.6 | 33 856 |
| 2087 | 110–16–F | 437.6 | 5.63 | 39.24 | 69 091 | 867 | 2.84 | 19.8 | 34 852 |
| 2088 | 220–16–F | 631.3 | 5.29 | 35.21 | 63 983 | 1250 | 2.67 | 17.8 | 32 294 |
| 2089 | 220–40–F | 1321.6 | 8.38 | 34.42 | 103 543 | 2617 | 4.25 | 17.4 | 52 513 |
| 2092 | 165–32–F | 1117.6 | 5.06 | 31.01 | 63 857 | 2213 | 2.55 | 15.7 | 32 181 |
| 2093 | 165–48–F | 1431.6 | 7.19 | 33.80 | 90 242 | 2835 | 3.63 | 17.1 | 45 560 |
| 2094 | 110–56–F | 1354.1 | 6.27 | 33.02 | 77 911 | 2681 | 3.17 | 16.7 | 39 390 |
| 2095 | 55–48–F | 718.0 | 4.66 | 32.92 | 57 388 | 1422 | 2.35 | 16.6 | 28 940 |
| 2096 | 55–40–F | 622.8 | | 33.46 | | 1234 | | 16.9 | |
| 2097 | 55–32–F | 525.2 | | 33.91 | | 1040 | | 17.1 | |
| 2098 | 55–48–F | 732.2 | | 32.56 | | 1450 | | 16.4 | |
| 2090 | 110–24–F | 614.9 | 5.15 | 35.03 | 64 859 | 1218 | 2.60 | 17.7 | 32 744 |
| 2091 | 165–32–F | 985.6 | 5.31 | 33.66 | 67 044 | 1952 | 2.68 | 17.0 | 33 838 |

## Table 31.

## General Performance—Indicated Horse Power, Drawbar Horse Power, and Tractive Force.

| Test No. | Laboratory Designation | Indicated Horse Power | | Drawbar Horse Power | | Tractive Force Based on M.E.P., lb. |
|---|---|---|---|---|---|---|
| | | Per sq. ft. of Heating Surface | Per sq. ft. of Grate Surface | Per sq. ft. of Heating Surface | Per sq. ft. of Grate Surface | |
| | Code Item☞ | 755 | 756 | 757 | 758 | 764 |
| 2009 | 138–16–F | 0.17 | 11.01 | | | 8 096 |
| 2010 | 193–20–F | 0.22 | 14.89 | | | 7 745 |
| 2012 | 138–24–F | 0.24 | 16.23 | 0.21 | 13.84 | 11 876 |
| 2013 | 138–32–F | 0.30 | 19.91 | 0.26 | 17.43 | 14 581 |
| 2014 | 193–32–F | 0.33 | 21.84 | 0.26 | 17.27 | 11 153 |
| 2015 | 193–24–F | 0.26 | 17.09 | 0.19 | 12.65 | 8 736 |
| 2016 | 193–16–F | 0.18 | 11.83 | 0.13 | 8.48 | 6 031 |
| 2017 | 83–16–F | 0.13 | 8.69 | 0.11 | 7.25 | 11 050 |
| 2018 | 83–24–F | 0.18 | 12.04 | 0.16 | 10.39 | 15 325 |
| 2019 | 83–32–F | 0.23 | 15.53 | 0.21 | 13.86 | 19 621 |
| 2020 | 83–24–F | 0.18 | 11.81 | 0.16 | 10.29 | 14 994 |
| 2021 | 83–16–F | 0.13 | 8.81 | 0.11 | 7.12 | 11 050 |
| 2022 | 83–32–F | 0.24 | 15.64 | 0.21 | 13.62 | 19 827 |
| 2023 | 138–40–F | 0.37 | 24.29 | 0.33 | 21.87 | 17 659 |
| 2024 | 55–24–F | 0.13 | 8.79 | 0.11 | 7.25 | 17 555 |
| 2026 | 110–16–F | 0.16 | 10.41 | 0.13 | 8.39 | 9 728 |
| 2027 | 110–24–F | 0.23 | 15.15 | 0.19 | 12.82 | 14 065 |
| 2028 | 55–32–F | 0.17 | 11.11 | 0.15 | 9.89 | 22 533 |
| 2029 | 110–32–F | 0.29 | 19.54 | 0.25 | 16.56 | 18 216 |
| 2030 | 165–24–F | 0.27 | 18.19 | 0.22 | 14.67 | 10 967 |
| 2031 | 83–40–F | 0.29 | 19.38 | 0.27 | 17.66 | 22 967 |
| 2032 | 165–32–F | 0.34 | 22.20 | 0.28 | 18.71 | 13 445 |
| 2033 | 110–48–F | 0.35 | 23.22 | 0.31 | 20.49 | 21 459 |
| 2034 | 193–40–F | 0.39 | 25.85 | 0.29 | 19.47 | 13 280 |
| 2035 | 110–40–F | 0.34 | 22.53 | 0.29 | 18.98 | 20 674 |
| 2037 | 165–40–F | 0.39 | 25.99 | 0.32 | 21.26 | 15 593 |
| 2038 | 55–24–F | 0.13 | 8.91 | 0.12 | 7.66 | 17 431 |
| 2039 | 110–32–F | 0.29 | 19.04 | 0.25 | 16.69 | 17 659 |
| 2040 | 165–40–F | 0.38 | 25.33 | 0.35 | 22.95 | 15 304 |
| 2041 | 110–40–F | 0.34 | 22.65 | 0.31 | 20.57 | 20 819 |
| 2042 | 110–24–F | 0.23 | 15.33 | 0.21 | 13.73 | 14 148 |
| 2043 | 110–48–F | 0.39 | 25.59 | 0.36 | 23.54 | 23 710 |
| 2044 | 110–56–F | 0.41 | 27.22 | 0.39 | 25.63 | 25 115 |
| 2045 | 110–16–F | 0.16 | 10.59 | 0.13 | 8.91 | 9 769 |
| 2072 | 110–40–F | 0.38 | 24.97 | 0.34 | 22.43 | 23 256 |
| 2073 | 110–32–F | 0.31 | 20.67 | 0.27 | 18.22 | 19 249 |
| 2074 | 165–32–F | 0.39 | 25.67 | 0.34 | 22.48 | 15 387 |
| 2075 | 55–32–F | 0.18 | 11.75 | 0.15 | 10.18 | 23 627 |
| 2076 | 220–32–F | 0.42 | 27.68 | 0.35 | 23.48 | 12 247 |
| 2077 | 110–24–F | 0.24 | 16.14 | 0.21 | 13.60 | 14 850 |
| 2078 | 165–24–F | 0.31 | 20.47 | 0.25 | 16.88 | 12 351 |
| 2079 | 220–24–F | 0.34 | 22.58 | 0.29 | 18.89 | 9 893 |
| 2080 | 110–16–F | 0.17 | 11.33 | | | 10 451 |
| 2081 | 55–24–F | 0.14 | 9.14 | | | 18 382 |
| 2082 | 165–40–F | 0.45 | 29.49 | 0.37 | 24.58 | 17 741 |
| 2083 | 165–16–F | 0.23 | 14.92 | 0.18 | 11.92 | 8 860 |
| 2084 | 110–48–F | 0.41 | 27.19 | 0.36 | 24.16 | 25 218 |
| 2085 | 55–40–F | 0.21 | 14.05 | 0.19 | 12.43 | 28 047 |
| 2086 | 55–24–F | 0.14 | 9.25 | 0.12 | 7.83 | 18 361 |
| 2087 | 110–16–F | 0.17 | 11.37 | 0.13 | 8.87 | 10 409 |
| 2088 | 220–16–F | 0.23 | 15.33 | 0.19 | 12.79 | 6 671 |
| 2089 | 220–40–F | 0.48 | 31.50 | 0.40 | 26.68 | 13 962 |
| 2092 | 165–32–F | 0.39 | 25.53 | 0.34 | 22.51 | 15 531 |
| 2093 | 165–48–F | 0.49 | 32.69 | 0.43 | 28.66 | 20 158 |
| 2094 | 110–56–F | 0.47 | 30.89 | 0.42 | 27.50 | 28 336 |
| 2095 | 55–48–F | 0.25 | 16.28 | 0.22 | 14.53 | 32 426 |
| 2096 | 55–40–F | 0.22 | 14.55 | 0.19 | 12.70 | 28 605 |
| 2097 | 55–32–F | 0.19 | 12.41 | 0.16 | 10.68 | 24 205 |
| 2098 | 55–48–F | 0.25 | 16.85 | 0.23 | 15.01 | 32 818 |
| 2090 | 110–24–F | 0.23 | 15.05 | 0.19 | 12.45 | 13 879 |
| 2091 | 165–32–F | 0.35 | 23.10 | 0.30 | 19.98 | 13 900 |

## TABLE 32.

## GENERAL PERFORMANCE—MACHINE FRICTION, EFFICIENCIES, AND RATIOS.

| Test No. | Laboratory Designation | Machine Friction of Locomotive in Terms of | | | | Machine Efficiency of Locomotive, per cent | Efficiency of Locomotive per cent | Ratios | |
|---|---|---|---|---|---|---|---|---|---|
| | | Horse Power | Mean Effective Pressure, lb. per sq. in. | Drawbar Pull, lb. | Per cent of Indicated Horse Power | | | Total Weight of Locomotive to Maximum I.H.P. | Total Heating Surface to Maximum I.H.P. |
| | Code Item ☞ | 770 | 771 | 772 | 773 | 778 | 779 | 785 | 786 |
| 2009 | 138–16–F | | | | | | | 324.4 | 4.8 |
| 2010 | 193–20–F | | | | | | | 247.1 | 3.6 |
| 2012 | 138–24–F | 117.9 | 7.35 | 1746 | 14.7 | 85.3 | 3.84 | 217.8 | 3.2 |
| 2013 | 138–32–F | 122.6 | 7.63 | 1812 | 12.4 | 87.6 | 3.78 | 182.1 | 2.7 |
| 2014 | 193–32–F | 225.9 | 9.86 | 2337 | 20.9 | 79.1 | 2.88 | 162.2 | 2.4 |
| 2015 | 193–24–F | 219.4 | 9.56 | 2267 | 26.0 | 74.1 | 2.63 | 212.1 | 3.1 |
| 2016 | 193–16–F | 165.6 | 7.22 | 1711 | 28.4 | 71.6 | 2.74 | 287.8 | 4.2 |
| 2017 | 83–16–F | 71.0 | 7.72 | 1837 | 16.6 | 83.4 | 3.76 | 357.1 | 5.3 |
| 2018 | 83–24–F | 82.1 | 8.93 | 2120 | 13.8 | 86.2 | 4.21 | 300.5 | 4.4 |
| 2019 | 83–32–F | 82.6 | 8.92 | 2118 | 10.8 | 89.2 | 4.35 | 235.0 | 3.5 |
| 2020 | 83–24–F | 75.5 | 8.16 | 1938 | 12.9 | 87.1 | 4.27 | 374.0 | 5.5 |
| 2021 | 83–16–F | 82.3 | 8.93 | 2123 | 19.2 | 80.8 | 3.31 | 504.9 | 7.4 |
| 2022 | 83–32–F | 99.3 | 10.71 | 2545 | 12.8 | 87.2 | 3.94 | 279.5 | 4.1 |
| 2023 | 138–40–F | 118.2 | 7.40 | 1757 | 9.9 | 90.1 | 3.36 | 183.2 | 2.7 |
| 2024 | 55–24–F | 75.4 | 12.94 | 3073 | 17.5 | 82.5 | 3.97 | 494.6 | 7.3 |
| 2026 | 110–16–F | 99.9 | 7.95 | 1887 | 19.4 | 80.6 | 3.75 | 422.7 | 6.2 |
| 2027 | 110–24–F | 115.5 | 9.14 | 2169 | 15.4 | 84.6 | 3.91 | | |
| 2028 | 55–32–F | 60.6 | 10.47 | 2489 | 11.0 | 89.0 | 4.10 | 399.9 | 5.9 |
| 2029 | 110–32–F | 147.8 | 11.71 | 2781 | 15.3 | 84.7 | 3.94 | 235.3 | 3.4 |
| 2030 | 165–24–F | 173.8 | 8.93 | 2118 | 19.3 | 80.7 | 3.62 | 241.2 | 3.6 |
| 2031 | 83–40–F | 84.2 | 8.56 | 2023 | 8.8 | 91.2 | 4.38 | 224.7 | 3.3 |
| 2032 | 165–32–F | 171.8 | 8.90 | 2112 | 15.7 | 84.3 | 3.61 | 198.1 | 2.9 |
| 2033 | 110–48–F | 134.4 | 10.64 | 2527 | 11.8 | 88.2 | 4.12 | 193.0 | 2.8 |
| 2034 | 193–40–F | 315.0 | 13.81 | 3279 | 24.7 | 75.3 | 2.67 | 171.7 | 2.5 |
| 2035 | 110–40–F | 176.2 | 13.70 | 3256 | 15.7 | 84.3 | 3.49 | 194.2 | 2.9 |
| 2037 | 165–40–F | 232.4 | 11.94 | 2836 | 18.2 | 81.8 | 3.29 | 170.6 | 2.5 |
| 2038 | 55–24–F | 60.0 | 10.28 | 2440 | 14.0 | 86.0 | 4.20 | 503.8 | 7.4 |
| 2039 | 110–32–F | 115.8 | 9.17 | 2175 | 12.3 | 87.7 | 3.97 | 230.5 | 3.4 |
| 2040 | 165–40–F | 117.4 | 6.05 | 1437 | 9.4 | 90.6 | 3.18 | 172.3 | 2.5 |
| 2041 | 110–40–F | 102.7 | 8.11 | 1923 | 9.3 | 90.8 | 3.61 | 196.8 | 2.9 |
| 2042 | 110–24–F | 78.9 | 6.27 | 1483 | 10.5 | 89.5 | 4.21 | 278.9 | 4.1 |
| 2043 | 110–48–F | 101.1 | 8.02 | 1903 | 8.0 | 92.0 | 3.20 | 173.7 | 2.6 |
| 2044 | 110–56–F | 77.0 | 6.10 | 1449 | 5.8 | 94.2 | 3.17 | 160.3 | 2.4 |
| 2045 | 110–16–F | 82.5 | 6.53 | 1551 | 15.9 | 84.1 | 3.90 | 417.1 | 6.1 |
| 2072 | 110–40–F | 126.0 | 9.97 | 2375 | 10.2 | 89.8 | 3.83 | 177.9 | 2.6 |
| 2073 | 110–32–F | 121.0 | 9.59 | 2284 | 11.9 | 88.1 | 4.13 | 212.8 | 3.1 |
| 2074 | 165–32–F | 157.0 | 8.04 | 1913 | 12.4 | 87.6 | 3.75 | 173.9 | 2.6 |
| 2075 | 55–32–F | 77.2 | 13.20 | 3154 | 13.3 | 86.7 | 4.33 | 373.4 | 5.5 |
| 2076 | 220–32–F | 207.2 | 7.81 | 1861 | 15.2 | 84.8 | 3.02 | 160.4 | 2.4 |
| 2077 | 110–24–F | 125.5 | 9.83 | 2342 | 15.8 | 84.2 | 4.14 | 274.0 | 4.0 |
| 2078 | 165–24–F | 177.4 | 9.10 | 2169 | 17.5 | 82.5 | 3.62 | 217.9 | 3.2 |
| 2079 | 220–24–F | 181.4 | 6.78 | 1615 | 16.3 | 83.7 | 3.23 | 193.5 | 2.9 |
| 2080 | 110–16–F | | | | | | | 387.2 | 5.7 |
| 2081 | 55–24–F | | | | | | | 476.9 | 7.0 |
| 2082 | 165–40–F | 242.7 | 12.40 | 2954 | 16.7 | 83.4 | 2.73 | 150.3 | 2.2 |
| 2083 | 165–16–F | 146.8 | 7.46 | 1780 | 20.1 | 79.9 | 3.56 | 294.9 | 4.3 |
| 2084 | 110–48–F | 150.3 | 11.79 | 2813 | 11.2 | 88.9 | 3.13 | 163.2 | 2.4 |
| 2085 | 55–40–F | 79.8 | 13.54 | 3224 | 11.5 | 88.5 | 4.07 | 316.0 | 4.7 |
| 2086 | 55–24–F | 70.0 | 11.83 | 2817 | 15.4 | 84.7 | 3.80 | 478.1 | 7.0 |
| 2087 | 110–16–F | 122.7 | 9.57 | 2280 | 21.9 | 78.1 | 3.69 | 387.9 | 5.7 |
| 2088 | 220–16–F | 124.8 | 4.64 | 1101 | 16.5 | 83.5 | 3.98 | 281.8 | 4.2 |
| 2089 | 220–40–F | 238.3 | 8.97 | 2134 | 15.3 | 84.7 | 2.46 | 140.4 | 2.1 |
| 2092 | 165–32–F | 149.7 | 7.71 | 1834 | 11.8 | 88.2 | 3.99 | 156.3 | 2.3 |
| 2093 | 165–48–F | 201.9 | 10.41 | 2491 | 12.4 | 87.6 | 2.82 | 134.8 | 2.0 |
| 2094 | 110–56–F | 167.3 | 13.10 | 3117 | 11.0 | 89.0 | 3.27 | 142.0 | 2.1 |
| 2095 | 55–48–F | 86.9 | 14.62 | 3501 | 10.8 | 89.2 | 4.44 | 271.1 | 4.0 |
| 2096 | 55–40–F | 90.8 | 15.28 | 3641 | 12.7 | 87.3 | | 304.2 | 4.5 |
| 2097 | 55–32–F | 85.4 | 14.20 | 3385 | 14.0 | 86.0 | | 359.1 | 5.3 |
| 2098 | 55–48–F | 90.0 | 15.06 | 3592 | 11.0 | 89.1 | | 266.3 | 3.9 |
| 2090 | 110–24–F | 128.2 | 10.02 | 2394 | 17.3 | 82.8 | 3.93 | 294.5 | 4.3 |
| 2091 | 165–32–F | 154.4 | 7.89 | 1884 | 13.5 | 86.5 | 3.80 | 190.5 | 2.8 |

## Table 33.

## Analysis of Ash, Front-end Cinders, and Stack Cinders.

| Test No. | Laboratory Designation | Analysis of Ash | | | Analysis of Front-end Cinders | | | Analysis of Stack Cinders | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Carbon, per cent | Earthy Matter, per cent | Moisture, per cent | Carbon, per cent | Earthy Matter, per cent | Moisture, per cent | Carbon, per cent | Earthy Matter, per cent | Moisture, per cent |
| | Code Item☞ | 831 | 832 | 833 | 841 | 842 | 843 | 846 | 847 | 848 |
| 2009 | 138–16–F | 34.90 | 51.97 | 10.92 | 57.47 | 35.56 | 4.50 | 44.35 | 51.75 | 0.84 |
| 2010 | 193–20–F | 30.26 | 60.22 | 7.33 | 42.75 | 54.85 | 0.58 | 61.97 | 34.90 | 0.54 |
| 2012 | 138–24–F | 29.34 | 63.97 | 4.21 | 42.52 | 54.33 | 0.73 | 52.71 | 43.88 | 0.84 |
| 2013 | 138–32–F | 31.53 | 62.82 | 2.86 | 37.46 | 59.41 | 0.66 | 60.26 | 37.00 | 0.54 |
| 2014 | 193–32–F | 25.41 | 66.84 | 4.66 | 38.60 | 57.42 | 1.50 | 66.76 | 28.48 | 2.23 |
| 2015 | 193–24–F | 33.03 | 58.52 | 4.48 | 18.19 | 79.68 | 0.62 | 63.90 | 32.16 | 1.26 |
| 2016 | 193–16–F | 30.75 | 59.56 | 6.43 | 39.95 | 55.40 | 0.74 | 56.47 | 40.76 | 0.81 |
| 2017 | 83–16–F | 30.12 | 63.55 | 3.78 | 29.99 | 65.99 | 1.34 | 40.17 | 50.54 | 7.02 |
| 2018 | 83–24–F | 27.49 | 67.01 | 3.28 | 44.08 | 52.97 | 0.88 | 50.62 | 44.44 | 2.64 |
| 2019 | 83–32–F | 24.59 | 71.63 | 1.58 | 45.95 | 48.93 | 2.71 | 55.30 | 40.83 | 1.27 |
| 2020 | 83–24–F | 44.49 | 50.94 | 4.57 | 37.26 | 44.31 | 18.43 | 46.34 | 52.40 | 1.26 |
| 2021 | 83–16–F | 35.90 | 62.95 | 1.15 | 45.85 | 53.00 | 1.15 | 51.63 | 46.90 | 1.47 |
| 2022 | 83–32–F | 26.24 | 72.63 | 1.13 | 27.25 | 72.05 | 0.70 | 64.48 | 33.95 | 1.57 |
| 2023 | 138–40–F | 33.00 | 64.76 | 2.24 | 51.32 | 47.83 | 0.85 | 71.14 | 28.56 | 0.30 |
| 2024 | 55–24–F | 29.04 | 68.25 | 2.70 | 39.46 | 38.60 | 21.94 | 43.59 | 55.04 | 1.37 |
| 2026 | 110–16–F | 41.33 | 51.24 | 7.43 | 48.85 | 50.37 | 0.78 | 56.14 | 42.77 | 1.09 |
| 2027 | 110–24–F | 33.08 | 63.71 | 3.21 | 48.80 | 50.38 | 0.82 | 56.19 | 43.24 | 0.57 |
| 2028 | 55–32–F | 27.48 | 70.63 | 1.89 | 49.59 | 49.88 | 0.53 | 53.17 | 44.05 | 2.78 |
| 2029 | 110–32–F | 37.00 | 60.61 | 2.39 | 33.77 | 65.56 | 0.67 | 60.79 | 38.27 | 0.94 |
| 2030 | 165–24–F | 38.29 | 56.74 | 4.97 | 48.88 | 51.11 | 0.01 | 67.31 | 32.34 | 0.35 |
| 2031 | 83–40–F | 29.05 | 69.49 | 1.46 | 20.82 | 78.66 | 0.52 | 66.13 | 32.96 | 0.91 |
| 2032 | 165–32–F | 41.04 | 55.71 | 3.25 | 52.59 | 46.76 | 0.65 | 33.80 | 65.28 | 0.92 |
| 2033 | 110–48–F | 31.93 | 66.04 | 2.03 | 19.52 | 79.89 | 0.59 | 67.86 | 31.48 | 0.66 |
| 2034 | 193–40–F | 36.31 | 61.59 | 2.10 | 43.05 | 56.35 | 0.60 | 71.39 | 28.04 | 0.57 |
| 2035 | 110–40–F | 37.81 | 60.00 | 2.19 | 40.73 | 58.72 | 0.55 | 66.60 | 32.69 | 0.71 |
| 2037 | 165–40–F | 40.50 | 56.91 | 2.59 | 45.64 | 53.40 | 0.96 | 69.72 | 29.61 | 0.67 |
| 2038 | 55–24–F | 42.04 | 57.58 | 0.38 | 46.39 | 52.05 | 1.56 | 38.95 | 60.16 | 0.89 |
| 2039 | 110–32–F | 29.59 | 69.28 | 1.13 | 42.74 | 55.44 | 1.82 | 58.50 | 40.89 | 0.61 |
| 2040 | 165–40–F | 38.57 | 59.98 | 1.45 | 34.78 | 64.71 | 0.51 | 70.55 | 28.96 | 0.49 |
| 2041 | 110–40–F | 34.91 | 63.69 | 1.40 | 46.43 | 52.35 | 1.22 | 66.22 | 33.30 | 0.48 |
| 2042 | 110–24–F | 36.54 | 62.11 | 1.35 | 47.78 | 51.63 | 0.59 | 57.25 | 42.08 | 0.67 |
| 2043 | 110–48–F | 32.99 | 64.35 | 2.66 | 10.55 | 89.37 | 0.08 | 69.26 | 30.14 | 0.60 |
| 2044 | 110–56–F | 29.99 | 67.45 | 2.56 | 26.19 | 73.50 | 0.31 | 73.92 | 25.64 | 0.44 |
| 2045 | 110–16–F | 33.14 | 65.06 | 1.80 | 52.44 | 47.41 | 0.15 | 46.58 | 53.03 | 0.39 |
| 2072 | 110–40–F | 26.72 | 70.98 | 2.30 | 32.50 | 67.18 | 0.32 | 68.08 | 31.37 | 0.55 |
| 2073 | 110–32–F | 35.55 | 60.82 | 3.63 | 34.42 | 65.08 | 0.50 | 64.61 | 34.96 | 0.43 |
| 2074 | 165–32–F | 29.52 | 68.03 | 2.45 | 40.97 | 58.68 | 0.35 | 67.18 | 32.41 | 0.41 |
| 2075 | 55–32–F | 30.33 | 69.36 | 0.31 | 43.76 | 56.18 | 0.06 | 46.69 | 52.84 | 0.47 |
| 2076 | 220–32–F | 33.20 | 64.08 | 2.72 | 44.16 | 55.50 | 0.34 | 75.82 | 23.79 | 0.39 |
| 2077 | 110–24–F | 51.92 | 40.16 | 7.92 | 56.25 | 43.34 | 0.41 | 56.19 | 43.00 | 0.81 |
| 2078 | 165–24–F | 28.41 | 69.20 | 2.39 | 42.75 | 56.88 | 0.37 | 64.45 | 34.98 | 0.57 |
| 2079 | 220–24–F | 38.08 | 60.98 | 0.94 | 42.02 | 57.69 | 0.29 | 67.47 | 32.03 | 0.50 |
| 2080 | 110–16–F | 30.34 | 67.79 | 1.87 | 44.20 | 53.12 | 2.68 | 37.30 | 61.96 | 0.74 |
| 2081 | 55–24–F | 32.01 | 66.82 | 1.17 | 41.82 | 57.98 | 0.20 | 41.10 | 58.37 | 0.53 |
| 2082 | 165–40–F | 32.66 | 64.62 | 2.72 | 45.85 | 53.65 | 0.50 | 73.05 | 26.41 | 0.54 |
| 2083 | 165–16–F | 34.94 | 61.87 | 3.19 | 53.99 | 45.66 | 0.35 | 62.11 | 37.19 | 0.70 |
| 2084 | 110–48–F | 31.38 | 66.85 | 1.97 | 24.30 | 53.52 | 22.18 | 73.52 | 25.92 | 0.56 |
| 2085 | 55–40–F | 33.16 | 66.19 | 0.65 | 51.37 | 48.42 | 0.21 | 64.35 | 35.23 | 0.42 |
| 2086 | 55–24–F | 29.70 | 69.64 | 0.66 | 22.63 | 54.43 | 22.94 | 38.94 | 53.81 | 7.25 |
| 2087 | 110–16–F | 29.94 | 69.47 | 0.59 | 39.45 | 60.35 | 0.20 | 45.33 | 53.70 | 0.97 |
| 2088 | 220–16–F | 28.16 | 71.15 | 0.69 | 19.32 | 74.04 | 6.64 | 57.39 | 41.93 | 0.68 |
| 2089 | 220–40–F | 25.16 | 72.85 | 1.99 | 41.25 | 58.48 | 0.27 | 75.83 | 23.84 | 0.33 |
| 2092 | 165–32–F | 34.23 | 63.49 | 2.28 | 36.73 | 62.95 | 0.32 | 69.76 | 29.65 | 0.59 |
| 2093 | 165–48–F | 44.12 | 54.79 | 1.09 | 41.35 | 58.42 | 0.23 | 71.65 | 28.01 | 0.34 |
| 2094 | 110–56–F | 37.66 | 61.46 | 0.88 | 42.96 | 55.06 | 1.98 | 72.55 | 26.94 | 0.51 |
| 2095 | 55–48–F | 31.83 | 67.47 | 0.70 | 62.66 | 37.00 | 0.34 | 57.88 | 41.66 | 0.46 |
| 2096 | 55–40–F | | | | | | | | | |
| 2097 | 55–32–F | | | | | | | | | |
| 2098 | 55–48–F | | | | | | | | | |
| 2090 | 110–24–F | 34.88 | 64.52 | 0.60 | 51.66 | 48.03 | 0.31 | 57.23 | 41.93 | 0.84 |
| 2091 | 165–32–F | 36.60 | 61.98 | 1.42 | 55.00 | 44.32 | 0.68 | 67.47 | 31.79 | 0.74 |

## TABLE 34.

## HEAT BALANCE—BRITISH THERMAL UNITS.

| Test No. | Laboratory Designation | B.t.u. Absorbed by Boiler per lb. of Coal as Fired | B.t.u. Loss Per Pound of Coal as Fired | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Due to Moisture in Coal | Due to Moisture in Air | Due to Hydrogen in Coal | Due to Escaping Gases | Due to Incomplete Combustion | Due to Combustible in Front-end Cinders | Due to Combustible in Stack Cinders | Due to Combustible in Ash | Due to Radiation, and Unaccounted-for |
| | Code Item ☞ | 851 | 852 | 853 | 854 | 855 | 856 | 857 | 858 | 860 | 869 |
| 2009 | 138–16–F | 7007 | 143 | 46 | 491 | 1691 | 0 | 27 | 368 | 116 | 1194 |
| 2010 | 193–20–F | 6376 | 163 | 57 | 542 | 3688 | 0 | 11 | 882 | 61 | -821 |
| 2012 | 138–24–F | 6618 | 150 | 77 | 522 | 3525 | 30 | 7 | 681 | 187 | -896 |
| 2013 | 138–32–F | 6435 | 122 | 78 | 542 | 3506 | 47 | 7 | 1036 | 184 | -821 |
| 2014 | 193–32–F | 5512 | 126 | 75 | 532 | 2698 | 22 | 7 | 1272 | 81 | 713 |
| 2015 | 193–24–F | 5638 | 144 | 64 | 518 | 2171 | 0 | 4 | 1243 | 355 | 826 |
| 2016 | 193–16–F | 6531 | 153 | 68 | 490 | 2323 | 15 | 8 | 662 | 250 | 87 |
| 2017 | 83–16–F | 7812 | 130 | 64 | 512 | 2213 | 106 | 9 | 175 | 209 | - 50 |
| 2018 | 83–24–F | 7269 | 141 | 49 | 510 | 2220 | 106 | 24 | 377 | 168 | 268 |
| 2019 | 83–32–F | 7239 | 140 | 62 | 532 | 2211 | 124 | 11 | 618 | 162 | 93 |
| 2020 | 83–24–F | 7560 | | | | | 0 | 16 | 362 | 182 | |
| 2021 | 83–16–F | 6521 | 154 | 22 | 409 | 1786 | 0 | 28 | 370 | 476 | 1003 |
| 2022 | 83–32–F | 6231 | 133 | 34 | 412 | 1811 | 0 | 4 | 1039 | 267 | 711 |
| 2023 | 138–40–F | 5317 | 174 | 19 | 419 | 1336 | 0 | 4 | 1856 | 264 | 1297 |
| 2024 | 55–24–F | 8637 | 146 | 18 | 419 | 1942 | 0 | 28 | 181 | 210 | -345 |
| 2026 | 110–16–F | 7278 | 162 | 26 | 409 | 1928 | 0 | 31 | 408 | 415 | 85 |
| 2027 | 110–24–F | 6550 | 162 | 29 | 418 | 1908 | 0 | 15 | 661 | 267 | 1069 |
| 2028 | 55–32–F | 7113 | 157 | 30 | 414 | 1812 | 0 | 16 | 457 | 270 | 808 |
| 2029 | 110–32–F | 6414 | 157 | 33 | 410 | 1512 | 0 | 10 | 1068 | 376 | 968 |
| 2030 | 165–24–F | 6793 | 138 | 40 | 429 | 1566 | 0 | 10 | 1127 | 280 | 993 |
| 2031 | 83–40–F | 6299 | 220 | 33 | 388 | 1360 | 96 | 5 | 1188 | 240 | 91 |
| 2032 | 165–32–F | 5822 | 171 | | 411 | 1431 | 0 | 19 | 794 | 326 | |
| 2033 | 110–48–F | 6289 | 180 | 35 | 405 | 1400 | 0 | 3 | 1519 | 241 | 467 |
| 2034 | 193–40–F | 5045 | 168 | 36 | 405 | 1347 | 0 | 7 | 1736 | 386 | 1180 |
| 2035 | 110–40–F | 5900 | 186 | 39 | 403 | 1707 | 0 | 4 | 1191 | 379 | 738 |
| 2037 | 165–40–F | 5813 | 185 | 39 | 413 | 1400 | 0 | 9 | 1599 | 347 | 889 |
| 2038 | 55–24–F | 7482 | 154 | 18 | 383 | 1622 | 0 | 29 | 173 | 411 | -231 |
| 2039 | 110–32–F | 6356 | 152 | 22 | 402 | 1478 | 0 | 13 | 900 | 269 | 762 |
| 2040 | 165–40–F | 5180 | 160 | 18 | 417 | 1271 | 38 | 6 | 1941 | 261 | 1396 |
| 2041 | 110–40–F | 5716 | 185 | 20 | 416 | 1455 | 45 | 5 | 1477 | 258 | 974 |
| 2042 | 110–24–F | 6774 | 175 | 22 | 405 | 1557 | 0 | 5 | 719 | 283 | 662 |
| 2043 | 110–48–F | 5231 | 179 | 16 | 426 | 1299 | 154 | 2 | 1838 | 225 | 1471 |
| 2044 | 110–56–F | 5306 | 177 | 17 | 427 | 1408 | 156 | 4 | 1961 | 187 | 551 |
| 2045 | 110–16–F | 7180 | 170 | 21 | 391 | 1474 | 0 | 3 | 515 | 321 | 236 |
| 2072 | 110–40–F | 5793 | 169 | | 505 | | | 0 | 1664 | 157 | |
| 2073 | 110–32–F | 6511 | 175 | 19 | 516 | 1638 | 56 | 9 | 1042 | 261 | 824 |
| 2074 | 165–32–F | 5978 | 148 | 21 | 529 | 1568 | 229 | 2 | 1372 | 266 | 1061 |
| 2075 | 55–32–F | 7249 | 166 | 28 | 509 | 1981 | 69 | 13 | 427 | 154 | 478 |
| 2076 | 220–32–F | 4920 | 205 | 18 | 516 | 1352 | 74 | 16 | 2159 | 283 | 1059 |
| 2077 | 110–24–F | 6812 | 165 | 22 | 507 | 1765 | 29 | 16 | 540 | 221 | 941 |
| 2078 | 165–24–F | 6152 | 172 | 22 | 508 | 1738 | 0 | 16 | 1132 | 185 | 932 |
| 2079 | 220–24–F | 5851 | 113 | 22 | 549 | 1676 | 0 | 13 | 1361 | 502 | 1573 |
| 2080 | 110–16–F | 7297 | 166 | 28 | 512 | 1982 | 142 | 16 | 193 | 316 | 527 |
| 2081 | 55–24–F | 7501 | 147 | 29 | 511 | 2121 | 0 | 19 | 169 | 359 | 358 |
| 2082 | 165–40–F | 4658 | 160 | 20 | 545 | 1468 | 0 | 14 | 2034 | 324 | 1902 |
| 2083 | 165–16–F | 6629 | 177 | 23 | 504 | 1672 | 0 | 26 | 613 | 448 | 824 |
| 2084 | 110–48–F | 4998 | 175 | 26 | 510 | 1566 | 0 | 9 | 1746 | 304 | 1354 |
| 2085 | 55–40–F | 7016 | 160 | 26 | 510 | 1725 | 0 | 18 | 590 | 369 | 628 |
| 2086 | 55–24–F | 7142 | 151 | 26 | 495 | 1750 | 68 | 8 | 269 | 363 | 803 |
| 2087 | 110–16–F | 7220 | 147 | 28 | 496 | 1725 | 0 | 14 | 284 | 379 | 543 |
| 2088 | 220–16–F | 6773 | 169 | 25 | 476 | 1517 | 0 | 7 | 599 | 335 | 586 |
| 2089 | 220–40–F | 4202 | 167 | 14 | 528 | 1207 | 126 | 10 | 2551 | 214 | 1819 |
| 2092 | 165–32–F | 6152 | 192 | 18 | 522 | 1475 | 22 | 11 | 1292 | 326 | 792 |
| 2093 | 165–48–F | 4736 | 188 | 15 | 530 | 1187 | 62 | 10 | 2414 | 540 | 1125 |
| 2094 | 110–56–F | 5666 | 190 | 21 | 517 | 1282 | 35 | 20 | 2069 | 315 | 847 |
| 2095 | 55–48–F | 7307 | 158 | 23 | 505 | 1656 | 0 | 24 | 560 | 315 | 260 |
| 2096 | 55–40–F | | | | | | | | | | |
| 2097 | 55–32–F | | | | | | | | | | |
| 2098 | 55–48–F | | | | | | | | | | |
| 2090 | 110–24–F | 7026 | 152 | 25 | 508 | 1524 | 26 | 29 | 730 | 447 | 627 |
| 2091 | 165–32–F | 6443 | 174 | 23 | 514 | 1470 | 0 | 31 | 1399 | 461 | 449 |

## TABLE 35.

## HEAT BALANCE—PERCENTAGE.

| Test No. | Laboratory Designation | Per cent of Heat of Coal as Fired | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Absorbed by Boiler | To Moisture in Coal | To Moisture in Air | To Hydrogen in Coal | To Escaping Gases | To Incomplete Combustion | To Combustible in Front-end Cinders | To Combustible in Stack Cinders | To Combustible in Ash | To Radiation, and Unaccounted-for |
| | Code Item ☞ | 881 | 882 | 883 | 884 | 885 | 886 | 887 | 888 | 890 | 899 |
| 2009 | 138–16–F | 63.2 | 1.3 | 0.4 | 4.4 | 15.3 | 0.0 | 0.2 | 3.3 | 1.1 | 10.8 |
| 2010 | 193–20–F | 58.2 | 1.5 | 0.5 | 5.0 | 33.7 | 0.0 | 0.1 | 8.1 | 0.6 | − 7.5 |
| 2012 | 138–24–F | 60.7 | 1.4 | 0.7 | 4.8 | 32.3 | 0.3 | 0.1 | 6.3 | 1.7 | − 8.2 |
| 2013 | 138–32–F | 57.8 | 1.1 | 0.7 | 4.9 | 31.5 | 0.4 | 0.1 | 9.3 | 1.7 | − 7.4 |
| 2014 | 193–32–F | 49.9 | 1.1 | 0.7 | 4.8 | 24.4 | 0.2 | 0.1 | 11.5 | 0.7 | 6.5 |
| 2015 | 193–24–F | 51.4 | 1.3 | 0.6 | 4.7 | 19.8 | 0.0 | 0.0 | 11.3 | 3.2 | 7.5 |
| 2016 | 193–16–F | 61.7 | 1.4 | 0.6 | 4.6 | 21.9 | 0.1 | 0.1 | 6.3 | 2.4 | 0.8 |
| 2017 | 83–16–F | 69.9 | 1.2 | 0.6 | 4.6 | 19.8 | 1.0 | 0.1 | 1.6 | 1.9 | − 0.4 |
| 2018 | 83–24–F | 66.5 | 1.3 | 0.5 | 4.7 | 20.3 | 1.0 | 0.2 | 3.4 | 1.5 | 2.5 |
| 2019 | 83–32–F | 64.7 | 1.3 | 0.6 | 4.8 | 19.8 | 1.1 | 0.1 | 5.5 | 1.5 | 0.8 |
| 2020 | 83–24–F | 67.3 | | | | | 0.0 | 0.1 | 3.2 | 1.6 | |
| 2021 | 83–16–F | 60.6 | 1.4 | 0.2 | 3.8 | 16.6 | 0.0 | 0.3 | 3.4 | 4.4 | 9.3 |
| 2022 | 83–32–F | 58.6 | 1.3 | 0.3 | 3.9 | 17.0 | 0.0 | 0.0 | 9.8 | 2.5 | 6.7 |
| 2023 | 138–40–F | 49.8 | 1.6 | 0.2 | 3.9 | 12.5 | 0.0 | 0.0 | 17.4 | 2.5 | 12.1 |
| 2024 | 55–24–F | 76.9 | 1.3 | 0.2 | 3.7 | 17.3 | 0.0 | 0.3 | 1.6 | 1.9 | − 3.1 |
| 2026 | 110–16–F | 67.8 | 1.5 | 0.2 | 3.8 | 18.0 | 0.0 | 0.3 | 3.8 | 3.9 | 0.8 |
| 2027 | 110–24–F | 59.1 | 1.5 | 0.3 | 3.8 | 17.2 | 0.0 | 0.1 | 6.0 | 2.4 | 9.7 |
| 2028 | 55–32–F | 64.2 | 1.4 | 0.3 | 3.7 | 16.4 | 0.0 | 0.1 | 4.1 | 2.4 | 7.3 |
| 2029 | 110–32–F | 58.6 | 1.4 | 0.3 | 3.8 | 13.8 | 0.0 | 0.1 | 9.8 | 3.4 | 8.8 |
| 2030 | 165–24–F | 59.7 | 1.2 | 0.4 | 3.8 | 13.8 | 0.0 | 0.1 | 9.9 | 2.5 | 8.7 |
| 2031 | 83–40–F | 63.4 | 2.2 | 0.3 | 3.9 | 13.7 | 1.0 | 0.1 | 12.0 | 2.4 | 0.9 |
| 2032 | 165–32–F | 54.7 | 1.6 | | 3.9 | | 0.0 | 0.2 | 7.5 | 3.1 | |
| 2033 | 110–48–F | 59.7 | 1.7 | 0.3 | 3.8 | 13.3 | 0.0 | 0.0 | 14.0 | 2.3 | 4.4 |
| 2034 | 193–40–F | 48.9 | 1.6 | 0.4 | 3.9 | 13.1 | 0.0 | 0.1 | 16.8 | 3.7 | 11.5 |
| 2035 | 110–40–F | 55.9 | 1.8 | 0.4 | 3.8 | 16.2 | 0.0 | 0.0 | 11.3 | 3.6 | 7.0 |
| 2037 | 165–40–F | 54.4 | 1.7 | 0.4 | 3.9 | 13.1 | 0.0 | 0.1 | 15.0 | 3.2 | 8.3 |
| 2038 | 55–24–F | 74.5 | 1.5 | 0.2 | 3.8 | 16.2 | 0.0 | 0.3 | 1.7 | 4.1 | − 2.3 |
| 2039 | 110–32–F | 61.4 | 1.5 | 0.2 | 3.9 | 14.3 | 0.0 | 0.1 | 8.7 | 2.6 | 7.4 |
| 2040 | 165–40–F | 48.5 | 1.5 | 0.2 | 3.9 | 11.9 | 0.4 | 0.1 | 18.2 | 2.5 | 13.1 |
| 2041 | 110–40–F | 54.2 | 1.8 | 0.2 | 3.9 | 13.8 | 0.4 | 0.0 | 14.0 | 2.4 | 9.2 |
| 2042 | 110–24–F | 63.9 | 1.7 | 0.2 | 3.8 | 14.7 | 0.0 | 0.1 | 6.8 | 2.7 | 6.3 |
| 2043 | 110–48–F | 48.3 | 1.7 | 0.2 | 3.9 | 12.0 | 1.4 | 0.0 | 17.0 | 2.1 | 13.6 |
| 2044 | 110–56–F | 50.1 | 1.7 | 0.2 | 4.0 | 13.3 | 1.5 | 0.0 | 18.5 | 1.8 | 5.2 |
| 2045 | 110–16–F | 69.6 | 1.7 | 0.2 | 3.8 | 14.3 | 0.0 | 0.0 | 5.0 | 3.1 | 2.3 |
| 2072 | 110–40–F | | | | | | | | | | |
| 2073 | 110–32–F | 58.9 | 1.6 | 0.2 | 4.7 | 14.8 | 0.5 | 0.1 | 9.4 | 2.4 | 7.4 |
| 2074 | 165–32–F | 53.5 | 1.3 | 0.2 | 4.7 | 14.0 | 2.1 | 0.0 | 12.3 | 2.4 | 9.5 |
| 2075 | 55–32–F | 65.5 | 1.5 | 0.3 | 4.6 | 17.9 | 0.7 | 0.1 | 3.9 | 1.4 | 4.3 |
| 2076 | 220–32–F | 46.4 | 1.9 | 0.2 | 4.9 | 12.8 | 0.7 | 0.1 | 20.4 | 2.7 | 10.0 |
| 2077 | 110–24–F | 61.8 | 1.5 | 0.2 | 4.6 | 16.0 | 0.3 | 0.1 | 4.9 | 2.0 | 8.5 |
| 2078 | 165–24–F | 56.7 | 1.6 | 0.2 | 4.7 | 16.0 | 0.0 | 0.1 | 10.4 | 1.7 | 8.6 |
| 2079 | 220–24–F | 50.2 | 1.0 | 0.2 | 4.7 | 15.0 | 0.0 | 0.1 | 11.7 | 4.3 | 13.5 |
| 2080 | 110–16–F | 65.3 | 1.5 | 0.3 | 4.6 | 17.7 | 1.3 | 0.1 | 1.7 | 2.8 | 4.7 |
| 2081 | 55–24–F | 66.9 | 1.3 | 0.3 | 4.6 | 18.9 | 0.0 | 0.2 | 1.5 | 3.2 | 3.2 |
| 2082 | 165–40–F | 41.9 | 1.4 | 0.2 | 4.9 | 13.2 | 0.0 | 0.1 | 18.3 | 2.9 | 17.1 |
| 2083 | 165–16–F | 60.7 | 1.6 | 0.2 | 4.6 | 15.3 | 0.0 | 0.2 | 5.6 | 4.1 | 7.6 |
| 2084 | 110–48–F | 46.8 | 1.6 | 0.3 | 4.8 | 14.6 | 0.0 | 0.1 | 16.3 | 2.8 | 12.7 |
| 2085 | 55–40–F | 63.5 | 1.5 | 0.2 | 4.6 | 15.6 | 0.0 | 0.2 | 5.3 | 3.3 | 5.7 |
| 2086 | 55–24–F | 64.5 | 1.4 | 0.2 | 4.5 | 15.8 | 0.6 | 0.1 | 2.4 | 3.3 | 7.2 |
| 2087 | 110–16–F | 66.6 | 1.4 | 0.3 | 4.6 | 15.9 | 0.0 | 0.1 | 2.6 | 3.5 | 5.0 |
| 2088 | 220–16–F | 64.6 | 1.6 | 0.2 | 4.5 | 14.5 | 0.0 | 0.1 | 5.7 | 3.2 | 5.6 |
| 2089 | 220–40–F | 38.8 | 1.5 | 0.1 | 4.9 | 11.1 | 1.2 | 0.1 | 23.5 | 2.0 | 16.8 |
| 2092 | 165–32–F | 57.0 | 1.8 | 0.2 | 4.8 | 13.7 | 0.2 | 0.1 | 12.0 | 3.0 | 7.3 |
| 2093 | 165–48–F | 43.8 | 1.7 | 0.1 | 4.9 | 11.0 | 0.6 | 0.1 | 22.3 | 5.0 | 10.4 |
| 2094 | 110–56–F | 50.3 | 1.8 | 0.2 | 4.9 | 12.0 | 0.3 | 0.2 | 19.4 | 3.0 | 7.9 |
| 2095 | 55–48–F | 67.6 | 1.5 | 0.2 | 4.7 | 15.3 | 0.0 | 0.2 | 5.2 | 2.9 | 2.4 |
| 2096 | 55–40–F | | | | | | | | | | |
| 2097 | 55–32–F | | | | | | | | | | |
| 2098 | 55–48–F | | | | | | | | | | |
| 2090 | 110–24–F | 63.3 | 1.4 | 0.2 | 4.6 | 13.7 | 0.2 | 0.3 | 6.6 | 4.0 | 5.6 |
| 2091 | 165–32–F | 58.8 | 1.6 | 0.2 | 4.7 | 13.4 | 0.0 | 0.3 | 12.8 | 4.2 | 4.1 |

## TABLE 36.

## INFORMATION CONCERNING THE INDICATOR DIAGRAMS SHOWN IN FIG. 56, 57, and 58.

| Diagram No. | Right or Left Side | Head or Crank End | Test No. | Laboratory Designation | Nominal Speed, M. P. H. | Speed, R. P. M. |
|---|---|---|---|---|---|---|
| 1 | R | H | 2086 | 55–24–F | 10 | 51.3 |
| 2 | R | C | " | " | " | " |
| 3 | L | H | " | " | " | " |
| 4 | L | C | " | " | " | " |
| 5 | R | H | 2077 | 110–24–F | 20 | 110.7 |
| 6 | R | C | " | " | " | " |
| 7 | L | H | " | " | " | " |
| 8 | L | C | " | " | " | " |
| 9 | R | H | 2083 | 165–16–F | 30 | 170.3 |
| 10 | R | C | " | " | " | " |
| 11 | L | H | " | " | " | " |
| 12 | L | C | " | " | " | " |
| 13 | R | H | 2088 | 220–16–F | 40 | 234.2 |
| 14 | R | C | " | " | " | " |
| 15 | L | H | " | " | " | " |
| 16 | L | C | " | " | " | " |
| 17 | R | H | 2095 | 55–48–F | 10 | 51.3 |
| 18 | R | C | " | " | " | " |
| 19 | L | H | " | " | " | " |
| 20 | L | C | " | " | " | " |
| 21 | R | H | 2084 | 110–48–F | 20 | 110.4 |
| 22 | R | C | " | " | " | " |
| 23 | L | H | " | " | " | " |
| 24 | L | C | " | " | " | " |
| 25 | R | H | 2093 | 165–48–F | 30 | 167.4 |
| 26 | R | C | " | " | " | " |
| 27 | L | H | " | " | " | " |
| 28 | L | C | " | " | " | " |
| 29 | R | H | 2089 | 220–40–F | 40 | 230.7 |
| 30 | R | C | " | " | " | " |
| 31 | L | H | " | " | " | " |
| 32 | L | C | " | " | " | " |
| 33 | R | H | 2028 | 55–32–F | 10 | 50.3 |
| 34 | R | C | " | " | " | " |
| 35 | L | H | " | " | " | " |
| 36 | L | C | " | " | " | " |
| 37 | R | H | 2029 | 110–32–F | 20 | 109.8 |
| 38 | R | C | " | " | " | " |
| 39 | L | H | " | " | " | " |
| 40 | L | C | " | " | " | " |
| 41 | R | H | 2030 | 165–24–F | 30 | 169.4 |
| 42 | R | C | " | " | " | " |
| 43 | L | H | " | " | " | " |
| 44 | L | C | " | " | " | " |
| 45 | R | H | 2034 | 193–40–F | 35 | 198.5 |
| 46 | R | C | " | " | " | " |
| 47 | L | H | " | " | " | " |
| 48 | L | C | " | " | " | " |

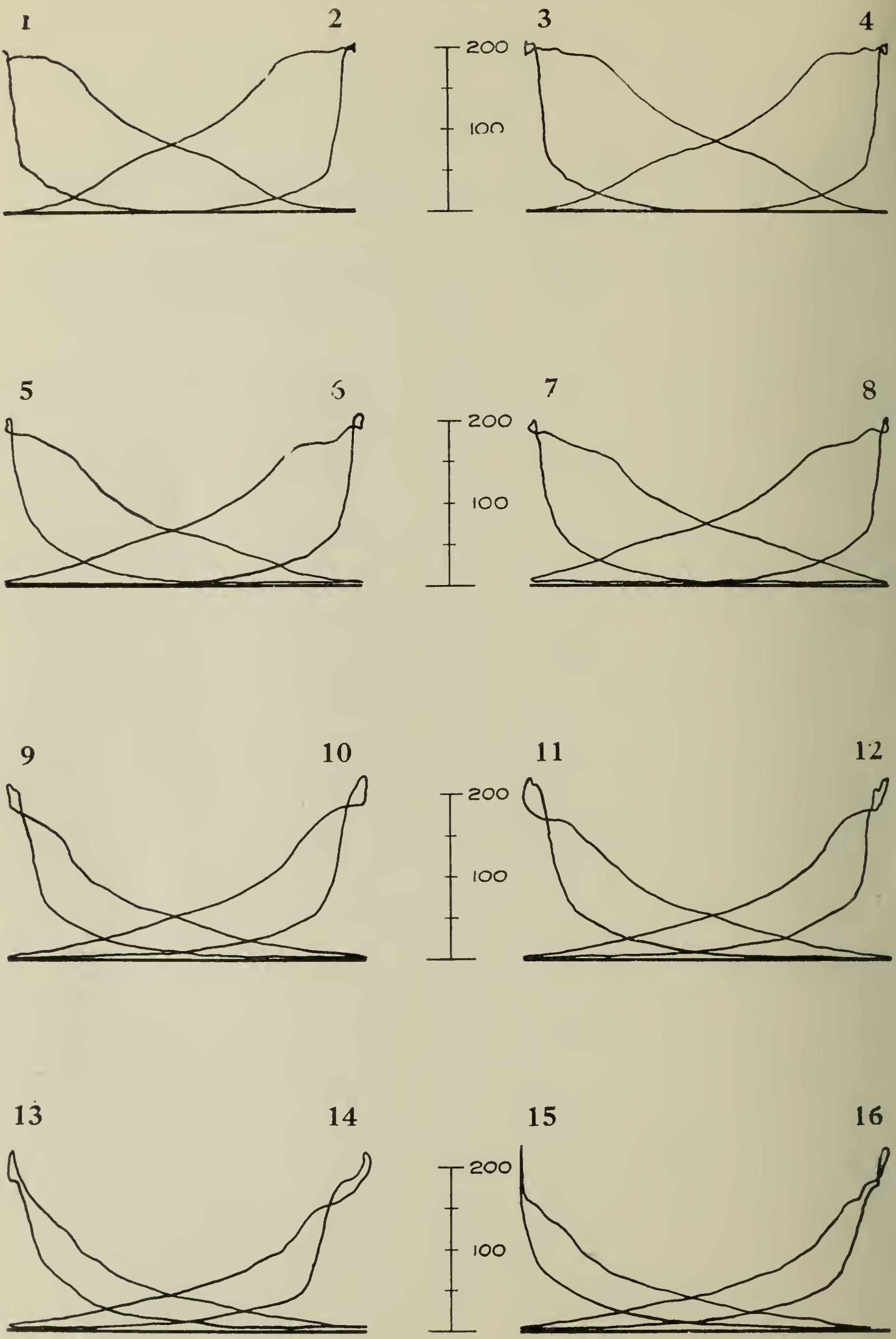


Fig. 56. Representative Indicator Diagrams for Series 2.
For Data, See Table 36.

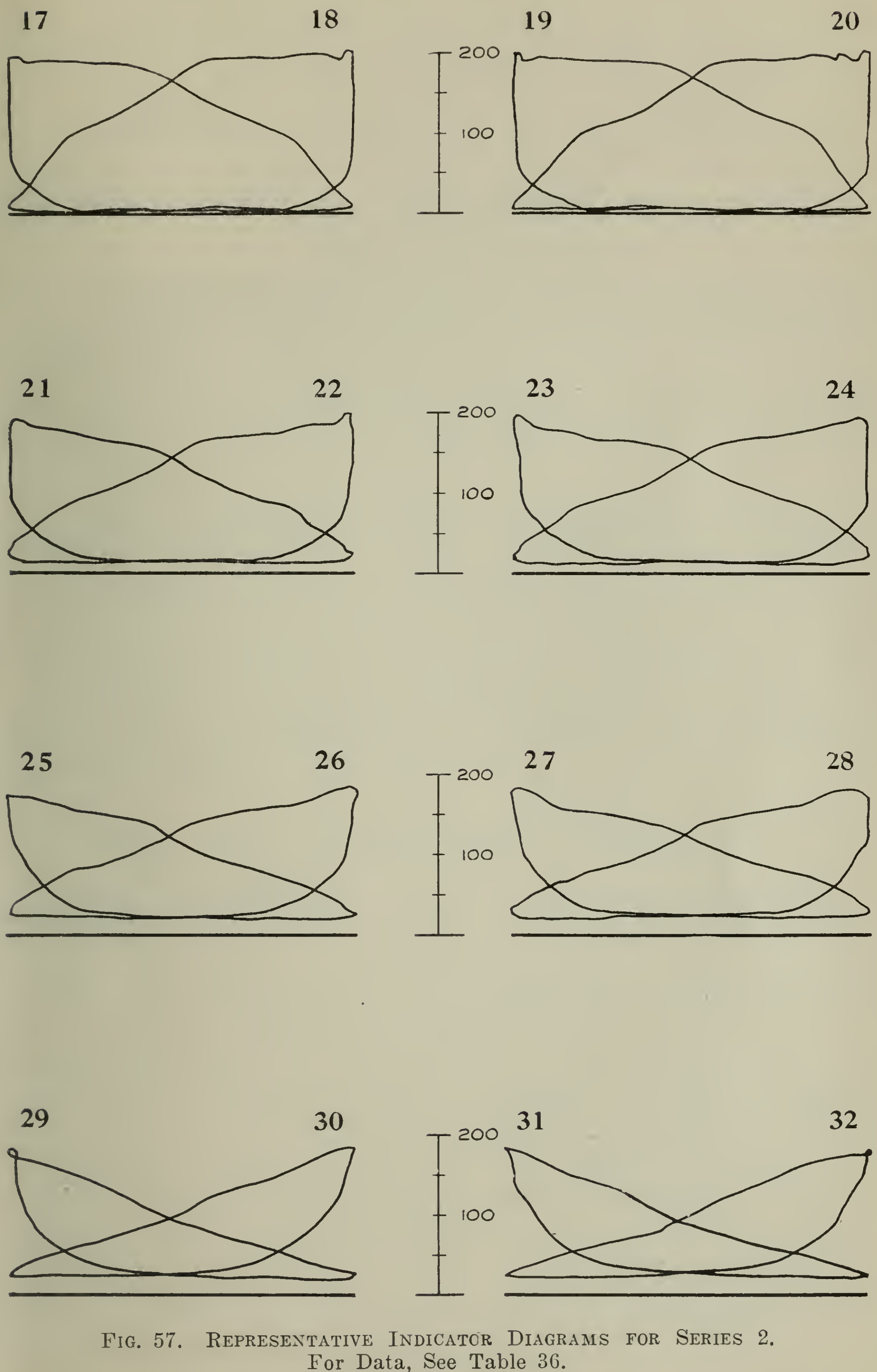


Fig. 57. Representative Indicator Diagrams for Series 2.
For Data, See Table 36.

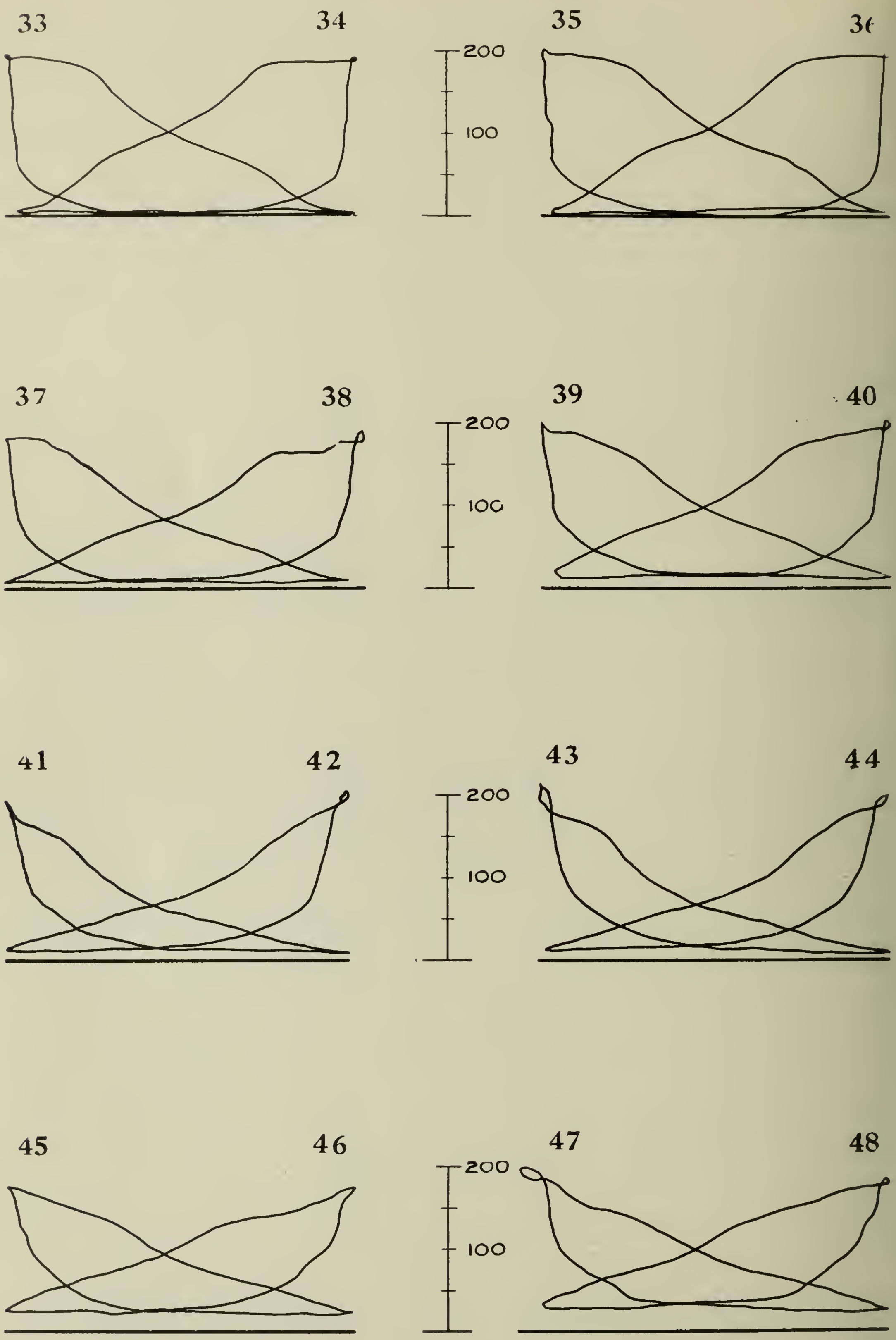


Fig. 58. Representative Indicator Diagrams for Series 1.
For Data, See Table 36.

## APPENDIX 5.

### METHODS OF CALCULATION.

Appendix 5 presents in detail the methods of calculation used in determining all results except for those items whose determination is self-evident.

The Marks and Davis steam tables for saturated and superheated steam have been used in all calculations pertaining to the properties of steam.

The events of the stroke and the corresponding pressures were determined for each indicator diagram by inspection and measurement. Horse power calculations were made, in like manner, for each indicator diagram. The values tabulated in Appendix 4 are averages of the determinations made from the individual diagrams.

Methods of estimating the ultimate analyses for the individual coal samples and of estimating the calorific values for the samples of ash and cinders have been presented in Appendix 4 in the consideration of Test Methods.

Item 318. Constant for dynamometer horse power.

$$\frac{\text{Item 19}}{33\,000}$$

Item 319. Constant for indicated horse power. Right, head end.

$$.000001983 \times (\text{Item } 68)^2 \times \text{Item } 77$$

Item 320. Constant for indicated horse power. Right, crank end.

$$.000001983 \times [(\text{Item } 68)^2 - (\text{Item } 135)^2] \times \text{Item } 77$$

Item 321. Constant for indicated horse power. Left, head end.

$$.000001983 \times (\text{Item } 69)^2 \times \text{Item } 78$$

Item 322. Constant for indicated horse power. Left, crank end.

$.000001983 \times [(\text{Item } 69)^2 - (\text{Item } 136)^2 \times \text{Item } 78$

Item 332. Constant for piston displacement. Right, head end.

$229.17 \times \text{Item } 319$

Item 333. Constant for piston displacement. Right, crank end.

$229.17 \times \text{Item } 320$

Item 334. Constant for piston displacement. Left, head end.

$229.17 \times \text{Item } 321$

Item 335. Constant for piston displacement. Left, crank end.

$229.17 \times \text{Item } 322$

Item 352. Average revolutions per minute.

$$\frac{\text{Item } 351}{60 \times \text{Item } 345}$$

Item 353. Equivalent speed, miles per hour.

$$\frac{\text{Item } 352 \times \text{Item } 19}{88}$$

Item 354. Equivalent piston speed in feet per minute.

$$\text{Item } 352 \times \left[\frac{\text{Item } 77 + \text{Item } 78}{12}\right]$$

Item 388. Barometric pressure in laboratory. The observed value has been corrected for the expansion of the mercury and brass by means of the formula:

$H = h_1[1.0026 - 0.000091\ t_1]$.

This method is in accordance with that of the United States Weather Bureau as described in Bulletin No. 472, page 29.

Item 407. Quality of steam, average.

Quality of steam in the dome has been determined by means of a throttling calorimeter and the formula:

$$x_o = \frac{H_o + 0.47 \times [t_s - t_o] - q_o}{r_o}$$

$x_o$ = quality of steam
$t_s$ = observed temperature in calorimeter
$t_o$ = temperature of saturated steam at pressure in calorimeter
$q_o$ = heat of liquid due to boiler pressure
$H_o$ = total heat of dry steam at calorimeter pressure
$r_o$ = latent heat of dry steam due to boiler pressure

Item 412. Factor of correction for quality of steam.

$$\frac{q + xr - h}{q + r - h}$$

$q$ = heat of liquid due to average boiler pressure
$h$ = heat of liquid due to average feed water temperature
$x$ = quality of steam, average
$r$ = latent heat of dry steam due to average boiler pressure.

Item 419. Total pounds of dry coal fired.

$$\text{Item } 418 \times \left[\frac{100 - \text{Item } 440}{100}\right]$$

Item 420. Total pounds of combustible by analysis.

$$\text{Item } 418 \times \left[\frac{100 - (\text{Item } 440 + \text{Item } 441)}{100}\right]$$

Item 421. Total pounds of ash by analysis.

$$\text{Item } 418 \times \left[\frac{\text{Item } 441}{100}\right]$$

Item 424. Total pounds of front end and stack cinders.

Item 422 + Item 423.

Item 435. Pounds of moisture per pound of dry air has been obtained from item 368, item 369, and the psychrometric chart and formula described by W. H. Carrier in the November, 1911, Journal of the American Society of Mechanical Engineers.

Item 458. Calorific value of dry coal in B.t.u. per pound.

$$\left[\frac{\text{Item 443}}{100 - \text{Item 440}}\right] \times 100$$

Item 459. Calorific value of combustible in B.t.u. per pound.

$$\left[\frac{\text{Item 443}}{100 - [\text{Item 440} + \text{Item 441}]}\right] \times 100$$

Item 478. Correction for change of water level and pressure in the boiler from start to close of test has been calculated by the formula:

$$\frac{W_i\ [q + xr - q_i] - W_f\ [q + xr - q_f]}{q + xr - h}$$

$W_i$ = initial weight of water in the boiler, pounds
$W_f$ = final weight of water in the boiler, pounds
$q$ = heat of liquid due to average boiler pressure
$x$ = quality of steam, average
$r$ = latent heat of dry steam due to average boiler pressure
$q_i$ = heat of liquid at start of test
$q_f$ = heat of liquid at close of test
$h$ = heat of liquid due to average feed water temperature

Item 480. Total hot water losses, corrected, pounds.

$$\text{Item 479} \times \left[\frac{xr}{q + xr - h}\right]$$

Item 481. Water delivered to boiler and presumably evaporated, pounds.

Item 476 — Item 480 + Item 478

Item 626. Dry coal fired per hour, pounds.

$$\frac{\text{Item 419}}{\text{Item 345}}$$

Item 627. Dry coal fired per hour per square foot of grate surface, pounds.

$$\frac{\text{Item 626}}{\text{Item 252}}$$

Item 633. Moist steam evaporated per hour, pounds.

$$\frac{\text{Item 481}}{\text{Item 345}}$$

Item 634. Dry steam evaporated per hour, pounds.

$$\text{Item 633} \times \text{Item 412}$$

Item 635. Dry steam evaporated per hour per square foot of heating surface, pounds.

$$\frac{\text{Item 634}}{\text{Item 275}}$$

Item 636. Dry steam evaporated per pound of dry coal, pounds.

$$\frac{\text{Item 634}}{\text{Item 626}}$$

Item 637. Dry steam evaporated per pound of coal as fired, pounds.

$$\text{Item 634} \div \left[\frac{\text{Item 418}}{\text{Item 345}}\right]$$

Item 639. Dry steam to engine per hour, pounds.

$$[\text{Item 476} + \text{Item 477} - \text{Item 479} - \text{Item 638}] \times \left[\frac{\text{Item 412}}{\text{Item 345}}\right]$$

Item 641. Factor of evaporation.

$$\frac{q + xr - h}{970.4}$$

Item 642. Dry steam loss per hour due to calorimeter, leaks, corrections, etc., pounds.

$$\text{Item } 634 - \text{Item } 639$$

Item 642. Dry coal loss per hour equivalent to steam loss, pounds.

$$\frac{\text{Item } 642}{\text{Item } 636}$$

Item. 645. Equivalent evaporation per hour from and at 212°F., pounds.

$$\text{Item } 633 \times \text{Item } 641.$$

Item 648. Equivalent evaporation per hour per square foot of total heating surface, pounds.

$$\frac{\text{Item } 645}{\text{Item } 275}$$

Item 656. Equivalent evaporation per hour per square foot of grate surface, pounds.

$$\frac{\text{Item } 645}{\text{Item } 252}$$

Item 657. Equivalent evaporation per hour per pound of coal as fired, pounds.

$$\text{Item } 645 \div \left[\frac{\text{Item } 418}{\text{Item } 345}\right]$$

Item 658. Equivalent evaporation per hour per pound of dry coal, pounds.

$$\frac{\text{Item } 645}{\text{Item } 626}$$

Item 659. Equivalent evaporation per hour per pound of combustible, pounds.

$$\text{Item } 645 \div \left[\frac{\text{Item } 420}{\text{Item } 345}\right]$$

Item 660. Boiler horse power.

$$\frac{\text{Item } 645}{34.5}$$

Item 666. Efficiency of the boiler, per cent.

$$\frac{\text{Item } 657 \times 970.4 \times 100}{\text{Item } 443.}$$

Item 697. Number of expansions, right, head end.

$$\frac{\text{Item } 510 + \text{Item } 86}{\text{Item } 495 + \text{Item } 86}$$

Item 698. Number of expansions, right, crank end.

$$\frac{\text{Item } 511 + \text{Item } 87}{\text{Item } 496 + \text{Item } 87}$$

Item 699. Number of expansions, left, head end.

$$\frac{\text{Item } 512 + \text{Item } 88}{\text{Item } 497 + \text{Item } 88}$$

Item 700. Number of expansions, left, crank end.

$$\frac{\text{Item } 513 + \text{Item } 89}{\text{Item } 498 + \text{Item } 89}$$

Item 734. Dry coal used by engine per indicated horse power per hour, pounds.

$$\left[\frac{\text{Item } 639}{\text{Item } 636}\right] \div \text{Item } 711$$

Item 735. B.t.u. in dry coal per indicated horse power per hour.

$$\text{Item } 734 \times \text{Item } 458$$

Item 736. Dry steam per indicated horse power per hour, pounds.

$$\frac{\text{Item } 639}{\text{Item } 711}$$

Item 737. B.t.u. in steam above 32°F. per indicated horse power per hour.

$$\text{Item } 736 \times [q + r]$$

Item 743. Drawbar horse power.

$$\text{Item } 318 \times \text{Item } 352 \times \text{Item } 487$$

Item 744. Dry coal per drawbar horse power per hour, pounds.

$$\left[\frac{\text{Item } 639}{\text{Item } 636}\right] \div \text{Item } 743$$

Item 745. Dry steam per drawbar horse power per hour, pounds.

$$\frac{\text{Item } 639}{\text{Item } 743}$$

Item 746. B.t.u. per drawbar horse power per hour.

$$\text{Item } 744 \times \text{Item } 458$$

Item 750. Millions of foot pounds at drawbar per hour.

$$\frac{\text{Item } 487 \times \text{Item } 19 \times \text{Item } 351}{\text{Item } 345 \times 1\,000\,000}$$

Item 752. Dry coal per million foot pounds at drawbar, pounds.

$$\left[\frac{\text{Item } 639}{\text{Item } 636}\right] \div \text{Item } 750$$

Item 753. Dry steam per million foot pounds at drawbar, pounds.

$$\frac{\text{Item } 639}{\text{Item } 750}$$

Item 754. B.t.u. per million foot pounds at drawbar.

$$\text{Item } 752 \times \text{Item } 458$$

Item 755. Indicated horse power per square foot of heating surface.

$$\left[\frac{\text{Item } 711}{\text{Item } 275}\right] \times \left[\frac{\text{Item } 634}{\text{Item } 639}\right]$$

Item 756. Indicated horse power per square foot of grate surface.

$$\left[\frac{\text{Item } 711}{\text{Item } 252}\right] \times \left[\frac{\text{Item } 634}{\text{Item } 639}\right]$$

Item 757. Drawbar horse power per square foot of heating surface.

$$\left[\frac{\text{Item } 743}{\text{Item } 275}\right] \times \left[\frac{\text{Item } 634}{\text{Item } 639}\right]$$

Item 758. Drawbar horse power per square foot of grate surface.

$$\left[\frac{\text{Item } 743}{\text{Item } 252}\right] \times \left[\frac{\text{Item } 634}{\text{Item } 639}\right]$$

Item 764. Tractive force based on mean effective pressure, pounds.

$$\left[\frac{33\,000}{\text{Item } 19}\right] \times \left[\frac{\text{Item } 711}{\text{Item } 352}\right]$$

Item 770. Machine friction of the locomotive in terms of horse power.

$$\text{Item } 711 - \text{Item } 743$$

Item 771. Machine friction of the locomotive in terms of mean effective pressure, pounds.

$$\frac{\text{Item } 770}{\text{Item } 352 \times [\text{Item } 319 + \text{Item } 320 + \text{Item } 321 + \text{Item } 322]}$$

Item 772. Machine friction of the locomotive in terms of drawbar pull, pounds.

$$\left[\frac{33\,000}{\text{Item } 19}\right] \times \left[\frac{\text{Item } 770}{\text{Item } 352}\right]$$

Item 773. Machine friction of the locomotive in per cent of indicated horse power.

$$\left[\frac{\text{Item } 770}{\text{Item } 711}\right] \times 100$$

Item 778. Machine efficiency of the locomotive, per cent.

$$\left[\frac{\text{Item } 743}{\text{Item } 711}\right] \times 100$$

Item 779. Efficiency of the locomotive, per cent.

$$\frac{254\,655.8}{\text{Item } 746}$$

$$\text{Constant } 254\,655.8 = \left[\frac{33\,000 \times 60}{777.52}\right] \times 100$$

Item 785. Ratio of total weight of the locomotive to the maximum indicated horse power.

$$\frac{\text{Item } 63}{\text{Item } 721}$$

Item 786. Ratio of total heating surface to maximum indicated horse power.

$$\frac{\text{Item } 275}{\text{Item } 721}$$

Item 851. B.t.u. absorbed by the boiler per pound of coal as fired.

$$\text{Item } 657 \times 970.4$$

*Items 852, 853, 854, 855, and 856 see next page.*

Item 857. B.t.u. loss due to combustible in front-end cinders.

$$\frac{\text{Item } 422 \times \text{Item } 461}{\text{Item } 418}$$

Item 858. B.t.u. loss due to combustible in stack cinders.

$$\frac{\text{Item } 423 \times \text{Item } 462}{\text{Item } 418}$$

Item 860. B.t.u. loss due to combustible in ash.

$$\frac{\text{Item } 428 \times \text{Item } 463}{\text{Item } 418}$$

Item 869. B.t.u. loss due to radiation and unaccounted-for.

$$\text{Item } 443 - [\text{Item } 851 + \text{Item } 852 + \text{Item } 853 + \text{Item } 854 + \text{Item } 855 + \text{Item } 856 + \text{Item } 857 + \text{Item } 858 + \text{Item } 860]$$

Item 852. B.t.u. loss per pound of coal as fired due to moisture in the coal.

$$\frac{\text{Item } 440}{100} \times \left[ (211 - \text{Item } 368) + 970.4 + 0.47 \times (\text{Item } 367 - 211) \right]$$

Item 853. B.t.u. loss per pound of coal as fired due to moisture in the air.

$$\left[ \frac{[\text{Item } 418 \times \text{Item } 449] - [\text{Item } 428 \times \text{Item } 831] - [\text{Item } 422 \times \text{Item } 841] - [\text{Item } 423 \times \text{Item } 846]}{\text{Item } 418 \times 100} \right]$$

$$\times \left[ \frac{3.032 \times \text{Item } 469}{\text{Item } 468 + \text{Item } 467} \right] \times \left[ 0.47 \times (\text{Item } 367 - \text{Item } 368) \right] \times \text{Item } 435$$

Item 854. B.t.u. loss per pound of coal as fired due to hydrogen in the coal.

$$9 \times \left[ \frac{\text{Item } 450}{100} \right] \times \left[ (211 - \text{Item } 368) + 970.4 + 0.47 \times (\text{Item } 367 - 211) \right]$$

Item 855. B.t.u. loss per pound of coal as fired due to escaping gases.

$$\left[ \frac{[\text{Item } 418 \times \text{Item } 449] - [\text{Item } 428 \times \text{Item } 831] - [\text{Item } 422 \times \text{Item } 841] - [\text{Item } 423 \times \text{Item } 846]}{\text{Item } 418 \times 100} \right] \times$$

$$\left[ \frac{[4 \times \text{Item } 468] + \text{Item } 466 + 700}{3 \times [\text{Item } 468 + \text{Item } 467]} \right] \times \left[ 0.24 \times [\text{Item } 367 - \text{Item } 368] \right]$$

Item 856. B.t.u. loss per pound of coal as fired due to incomplete combustion.

$$\left[ \frac{[\text{Item } 418 \times \text{Item } 449] - [\text{Item } 428 \times \text{Item } 831] - [\text{Item } 422 \times \text{Item } 841] - [\text{Item } 423 \times \text{Item } 846]}{\text{Item } 418 \times 100} \right] \times$$

$$\left[ \frac{\text{Item } 467}{\text{Item } 468 + \text{Item } 467} \right] \times 10\,150$$

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# MAGNETIC AND OTHER PROPERTIES OF IRON-SILICON ALLOYS, MELTED IN VACUO

BY

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## I. INTRODUCTION.*

1. *Preliminary.*—Probably no series of iron alloys has received more attention in recent years than has the iron-silicon series. This is largely due to the fact that some of these alloys have proved their superiority over pure iron and other iron alloys for certain very important purposes in the electrical industry. The iron-silicon alloys have been thoroughly investigated by chemists, metallurgists, and physicists, so that their properties are generally very well known. The reason that this series of alloys has again been the object of an investigation is the discovery by the writer about two years ago that the magnetic properties of pure iron that is melted in vacuo are far superior to any other grade of iron known at that time.† It was evident that this superiority was due to the purity of the iron produced by the vacuum process, the iron thus produced containing only 0.01 per cent carbon. A new standard of iron purity being thus produced, it became desirable to repeat previous investigations on the properties of iron alloys with this new standard as a basis, and the present bulletin deals primarily with iron-silicon alloys containing from 0.001 to 8.5 per cent silicon.‡ Besides the magnetic properties the bulletin contains data concerning the electrical and mechanical properties and gives chemical analyses. Photomicrographs are also shown for the various alloys.

The writer wishes to express his appreciation of the sympathy shown and assistance rendered by members of various departments in the University of Illinois. In particular he wishes to mention Mr. W. A. Gatward, Fellow in the Engineering Experiment Station, for the conscientious work done during the year, largely in connection with the magnetic testing, but also in many other ways. The chemical analysis has been done by Mr. J. M. Lindgren and Mr. F. H. Whittum of the Chemistry department who deserve great credit for the thoroughness with which they have accomplished their part of the investigation.

## II. HISTORICAL REVIEW.

2. *Mechanical Properties.*—The effect of silicon upon the mechanical properties of iron was noticed as early as the beginning of

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*A paper containing much of the information given in this bulletin was presented before the American Institute of Electrical Engineers, St. Louis, October 20, 1915. (Trans. Am. Inst. of Elect. Engrs. Vol. 34, No. 10, Oct., 1915).

†Magnetic and other Properties of Electrolytic Iron Melted in Vacuo. Bulletin No. 72, Engineering Experiment Station. Univ. of Illinois, 1914. Trans. Am. Inst. of Elect. Engrs. Vol. 33, Part I, p. 451, 1914. (A. I. E. E. Proceedings, February, 1915, p. 237).

‡Bulletin No. 77 published by the Engineering Experiment Station deals with iron-boron alloys, melted in vacuo, the maximum boron content being 0.50 per cent.

the last century, when Mushet found that quartz sand applied to molten iron made the iron harder and more brittle. These results were later confirmed by other investigators,* and by 1880 it was generally accepted that silicon increases the strength of iron and at the same time it tends to make it hard and brittle if added in quantities above a few per cent. It was not until towards the end of the century, however, that systematic investigations of the iron-silicon alloys were made. In 1887 Tilden, Roberts-Austen, and Turner† carried on extensive investigations on the mechanical properties of low silicon alloys, containing from 0.10 to 0.50 per cent silicon. These investigators found that silicon increases the hardness, raises the elastic limit and ultimate strength, and decreases the elongation. The investigations were continued by Hadfield in 1889,‡ who carried the silicon content up to 8.83 per cent. With a silicon content of 7.23 per cent and 8.83 per cent, however, the ingots could not be forged, so that the highest silicon alloy tested contained 4.90 per cent silicon. The results obtained by Hadfield as well as by some of the later investigators are shown graphically in Figs. 1a, b, and 2a, b. Pages 7, 8 and 9. As the carbon content in the alloys used in the above investigations was comparatively high, varying from 0.14 to 0.26 per cent, Baker,¶ in his research on iron-silicon alloys in 1903, reduced the carbon content to 0.04 per cent. This was accomplished by first melting the iron alone, and then, after removing the slag, adding the ferrosilicon. The charge was kept molten for 15 minutes longer, and was then cast into ingots. This procedure produced an iron not only low in carbon, but low in manganese as well. Guillet§ in 1904 made an extensive investigation of the microstructure of the iron-silicon alloys with silicon contents up to 30 per cent. He also studied the mechanical properties of the alloys, but his results, although in general confirming those obtained by his predecessors, are not strictly comparable to them, on account of the high and varying manganese and carbon content. Furthermore, only four of his alloys were forgeable, namely, those containing 5.12 per cent silicon or less. However, Guillet confirmed the results of Hadfield with regard to the effect of carbon upon the limit of forgeability of the iron-silicon alloys. Thus he found that, with a carbon content of 0.94 per cent, the limit lies below 5 per cent silicon. Bisset** states that silicon adds strength to iron comparable to the effect of carbon, but in other respects leaves the iron unchanged. Paglianti,* in 1912 investigated the properties of iron silicon alloys, using as a basis electro-converted iron with a carbon content of 0.10 per cent and with manganese varying from 0.22 to 0.60 per cent. The silicon was added in the form of 50 per cent ferrosilicon, and the specimens were obtained from ingots weighing 80 to 100 kg. The results obtained by these various investigators, as shown in Fig. 1a, b differ considerably in numerical values, probably due partly to the difference in the purity of the iron used as a basis, and partly to the method of melting and the subsequent mechanical-and heat-treatment.

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*See: P. Paglianti, Metallurgie, Vol. 9, p. 217, 1912, where references are given to early investigators.

†Report of British Assoc. for the Advancement of Science, 1888.

‡Journ. Iron and Steel Institute, Vol. 36, 1889, II, p. 222.

¶Journ. Iron and Steel Institute, Vol. 64, 1903, II, p. 312.

§Rev. de Metallurgie, Memoirs, Vol. 1, p. 46, 1904.

**Iron Age, Aug. 25, 1910.

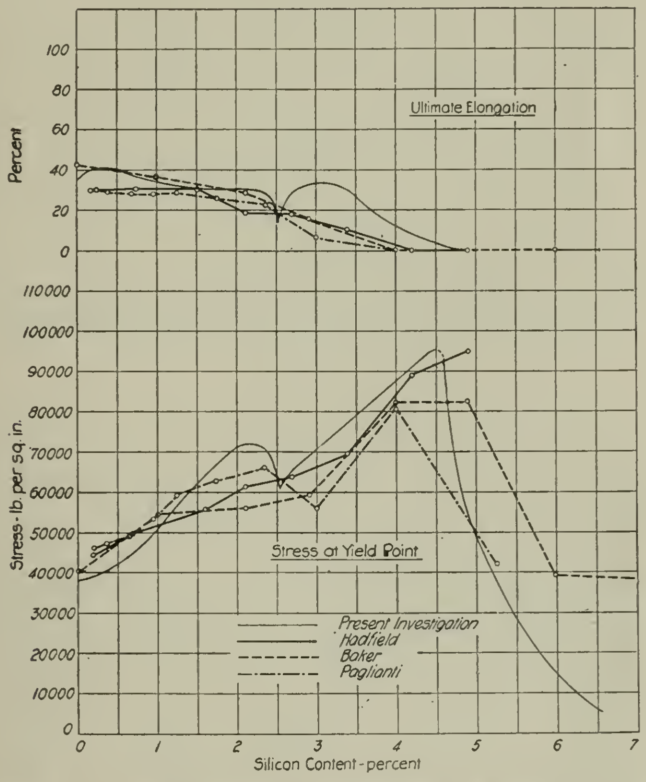

FIG. 1a. MECHANICAL PROPERTIES OF IRON-SILICON ALLOYS, ACCORDING TO VARIOUS INVESTIGATORS. AS FORGED.

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*Metallurgie, Vol. 9, p. 217, 1912.
Rev. de Metal. Extracts, Vol. 11, p. 4, Jan., 1914.

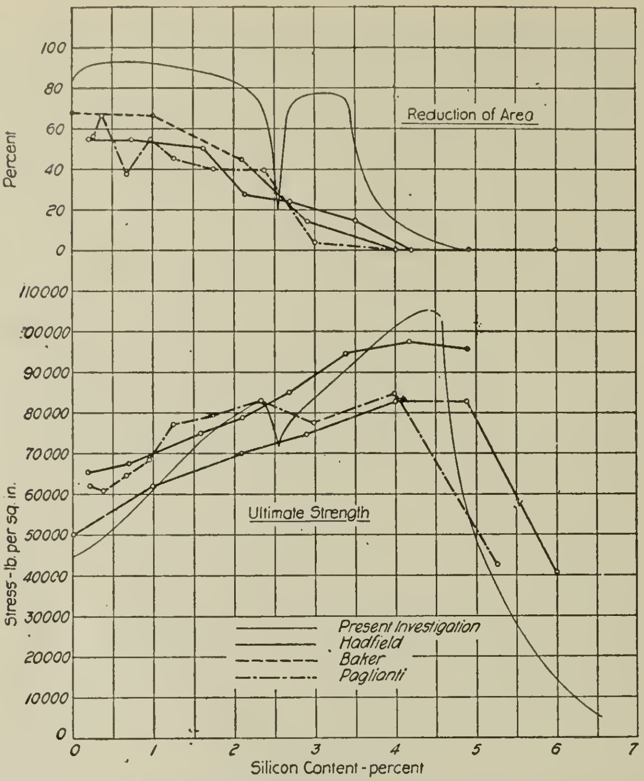

Fig. 1b. Mechanical Properties of Iron-Silicon Alloys, According to Various Investigators. As Forged.

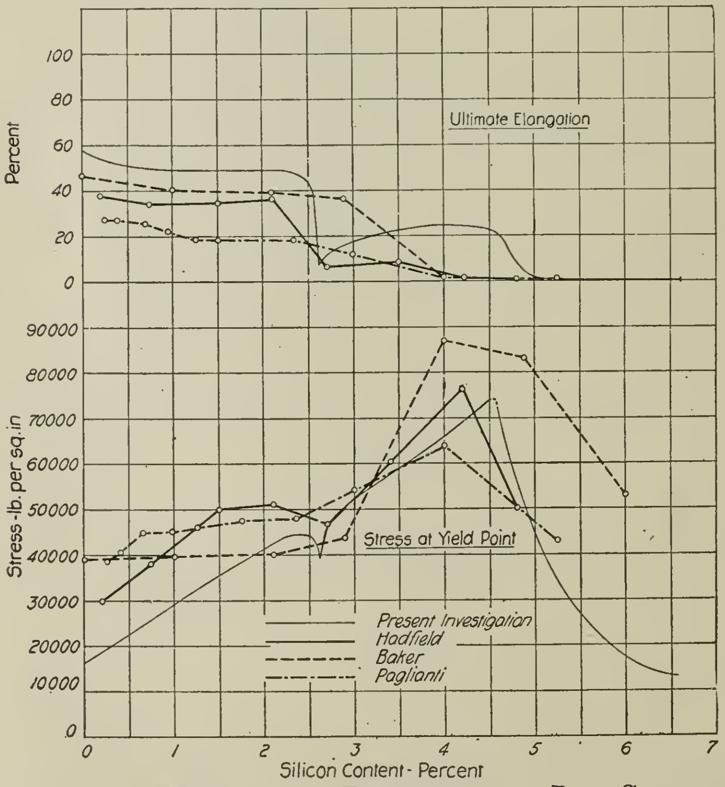

Fig. 2a. Mechanical Properties of Iron-Silicon Alloys, According to Various Investigators. Annealed.

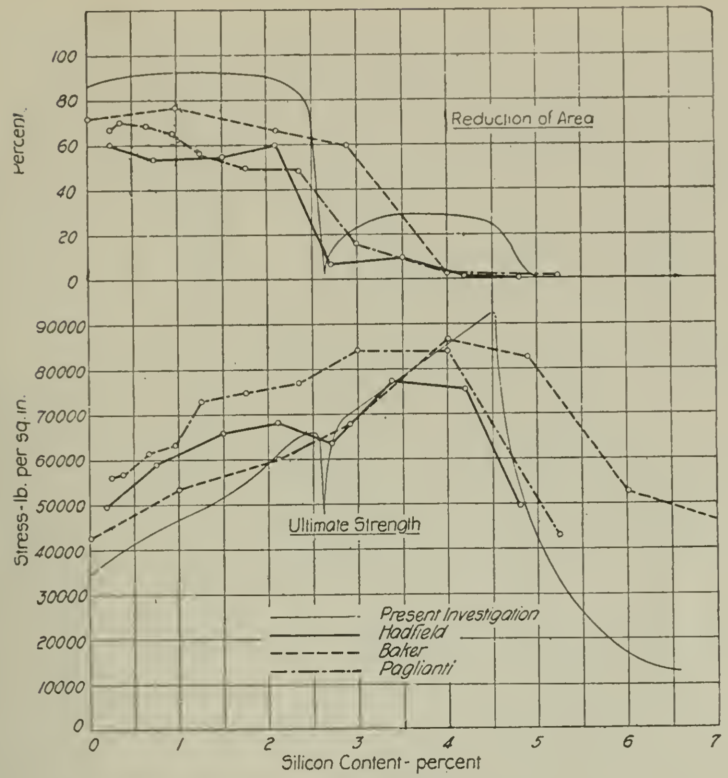

FIG. 2b. MECHANICAL PROPERTIES OF IRON-SILICON ALLOYS, ACCORDING TO VARIOUS INVESTIGATORS. ANNEALED.

## TABLE 1.

## MAGNETIC AND ELECTRICAL PROPERTIES OF HADFIELD'S IRON-SILICON ALLOYS.

| Mark | Silicon % | Max. Induction for H = 45. | Permeability for H = 8 | Retentivity in Terms of B | Coercive Force in Terms of H | Energy Dissipated per Complete Cycle. ergs per cc. | Spec. Elec. Resistance. microhms |
|---|---|---|---|---|---|---|---|
| S. C. I. | .07 | ..... | .... | .... | ... | ..... | 10.2 |
| B | .14 | 16800 | 1625 | 9770 | 1.66 | 10760 | 10.9 |
| 898 E | 2.50 | 16420 | 1680 | 4080 | .90 | 7900 | 42.1 |
| 898 H | 5.50 | 15980 | 1630 | 3430 | .85 | 6500 | 65.2 |

The results due to Baker, for example, show the tensile test curves to lie considerably to the right of those due to the other investigators. This fact, in conjunction with the appearance of his photomicrographs, suggests that part of the silicon in his alloys may be in the form of silica, caused by the method of melting. In view of these differences it is not strange that the results of the above investigators should disagree to some extent.

3. *Magnetic and Electrical Properties.*—In connection with his investigation of the mechanical properties of iron-silicon alloys in

1889, Hadfield* touched upon their magnetic and electrical properties. The results arrived at were not favorable, as it was found "that the material (containing 4.43 per cent silicon and .18 per cent carbon) had less susceptibility and more retentiveness than good soft iron, and that it had enormously less retentiveness than hard steel suitable for magnet making." However, Hadfield continued his investigations on the magnetic properties of iron-silicon alloys and in 1900, assisted by Barrett and Brown, he was able to give a report that was quite different from the one above referred to.† Among the large number of alloys investigated, the report includes two iron-silicon alloys, containing 2.5 and 5.5 per cent silicon respectively. The results show (see Table 1) that both alloys have a higher maximum permeability and a decidedly lower hysteresis loss than pure iron, and that the electrical resistance increases at the rate of 10 to 12 microhms per c. c. for each per cent of silicon added.‡ This remarkable discovery caused an almost immediate adoption of the silicon-steel for use in electrical machinery, particularly in transformers. It also gave a new impetus to the investigation of iron and steel alloys for electrical purposes, and a large number of investigators have been searching for new treasures in this field. Except for permanent magnets, however, silicon still holds the first place as the alloying element, giving best results for electromagnetic machinery.¶ Among those who have investigated the magnetic and electrical properties of iron-silicon since Hadfield's discovery, only a few will be mentioned in the following review.

Gumlich and Schmidt§ investigated a large number of commercial steels and among these certain iron-silicon alloys, in general confirming the results of Hadfield. Baker,** as already mentioned, used a very pure grade of iron, the carbon content being only 0.04 per cent. His results are summarized in Table 2. From this table it is seen that the permeability increases and the coercive force and hys-

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*Journ. Iron & Steel Inst., Vol. 36, 1889, II, p. 237.

†Barrett, Brown & Hadfield; Scient. Trans. Royal Dublin Soc. VII Ser. 2, PaPrt 4, Jan., 1900.
Barrett, Brown & Hadfield: Jour. Inst. of Elect. Engrs. Vol. 31, p. 674, 1901-02.
Reviewed by E. Gumlich: Stahl u Eisen 1902, No. 6, p. 230.

‡The electrical conductivity of iron alloys is described in detail by Barrett: Proc. Royal Soc. Vol. 69, p. 480, 1902.

¶The iron-cobalt alloy $Fe_2Co$, first investigated by Weiss, has recently been investigated by the writer, and may prove to be a valuable material on account of its high permeability in medium and high fields. See Gen. Elect. Review. Vol. 18, p. 881, Sept., 1915.

§Elektrotech. Zeitsch. Vol. 22, p. 691, 1901.

**Journ. Iron and Steel Inst. Vol. 64, 1903, II, p. 312.
Journ. Inst. of Elect. Engrs. Vol. 34, p. 498, 1904-05.

TABLE 2.

MAGNETIC PROPERTIES OF IRON-SILICON ALLOYS, (FROM BAKER).

| Silicon % | Max. Induction H = 20 | Permeability for H = 4 | Retentivity | Coercive Force | Energy Dissipated per Complete Cycle |
|---|---|---|---|---|---|
| 0.02 | 16000 | 2325 | 8375 | 1.8 | 10550 ergs per cc |
| 1.02 | 16200 | 2562 | 8000 | 1.7 | 8798 ” ” ” |
| 2.90 | 15500 | 2750 | 7325 | 1.5 | 8081 ” ” ” |
| 4.89 | 14750 | 2665 | 7200 | 1.2 | 6110 ” ” ” |
| 7.47 | 14000 | 2937 | 9000 | 1.0 | 5613 ” ” ” |

teresis loss decrease with increasing silicon content up to the limit of forgeability.

Dillner and Engstrom,* in contradiction to other investigators, found silicon to reduce the permeability and increase the hysteresis loss of sheet iron. By using silicon and aluminum together in certain proportions the best results were obtained.

Guertler† investigated the electrical conductivity of numerous alloys of iron with all the more common elements. From the curves given the resistance of the 3.4 per cent iron-silicon alloy is shown to be 50 microhms per cu. cm. confirming the results obtained by Barrett.

Burgess and Aston‡ have investigated a large number of iron alloys, using doubly refined electrolytic iron as a basis. This iron, containing only 0.03 per cent impurities, was melted in an electric resistance furnace together with the alloying element. This method of melting, however, proved to involve some danger, inasmuch as the iron would decompose the carbon monoxide gases in the furnace and combine with small quantities of carbon and oxygen.¶ Burgess and Aston found that silicon improves the iron magnetically, the 4.65 per cent alloy having a decidedly lower hysteresis loss than their pure iron standard.

Gumlich and Goerens§ in 1912 carried out researches on iron-silicon alloys using samples made for them for the purpose by the firm of Krupp at Essen. The results are summarized in Table 3. It is seen from this table that for rods the coercive force and probably also the hysteresis loss is a minimum in the region of 4 per cent silicon, as shown by previous investigators, while for sheets the minimum occurs at 0.50 per cent silicon. This difference between sheets

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*Journ. Iron and Steel Inst. Vol. 67, 1905, I, p. 474.

†Zeitschr. Anorg. Chemie. Vol. 51, p. 397, 1906.

‡Met. and Chem. Eng'n. March, 1910.

¶This was shown by the writer to be the case in his own preliminary experiments, described in Bulletin No. 72 of the Eng. Exp. Sta. Univ. of Ill. 1914.

§Trans. Faraday Soc. Vol. 8, p. 98, 1912.
Ferrum 10 (12) p. 33, 1912; Chem. Zentr. Bl. [5] 17 (13) p. 380.

## TABLE 3.

## MAGNETIC PROPERTIES OF IRON SILICON ALLOYS (FROM GUMLICH AND GOERENS.) SAMPLES ANNEALED AT 800° C.

| Silicon % | Coercive Force | | Maximum Permeability | | Saturation Value, $4\pi I_s$ |
|---|---|---|---|---|---|
| | Sheets | Rods | Sheets | Rods | |
| .06 | 1.80 | 1.25 | 3000 | 3000 | 21200 |
| .15 | .70 | 1.10 | .... | .... | 21300 |
| .30 | .65 | 1.15 | .... | .... | 21200 |
| .40 | .54 | 1.20 | 11600 | 2800 | 21150 |
| .60 | .80 | 1.30 | .... | 2400 | 21050 |
| .90 | .75 | 1.50 | .... | .... | 20950 |
| 1.00 | .80 | 1.30 | 8500 | 3500 | 20900 |
| 1.65 | .70 | 1.20 | .... | .... | 20600 |
| 1.95 | .75 | 1.25 | .... | 2900 | 20400 |
| 2.35 | .85 | 1.00 | 7700 | 2700 | 20300 |
| 3.70 | .85 | .65 | .... | 4900 | 19750 |
| 4.00 | ... | ... | 9400 | 4100 | ..... |
| 4.50 | 1.05 | .65 | 5500 | 4400 | 19100 |
| 5.25 | ... | .75 | .... | 4300 | 18300 |
| 8.60 | ... | .95 | .... | 3000 | 16100 |

These values are only approximate as they are obtained from curves.
The flux densities are not given.

and rods could not be accounted for at the time of the report, but was being investigated. For sheets the maximum permeability curve has two maxima, one, of 11600, occuring at 0.40 per cent silicon, and the other, of 9400, at 4 per cent silicon. For rods, only one maximum occurs, namely 4900, at 3.7 per cent silicon. In view of the saturation values, and of the photomicrographs and chemical analysis, the authors are of the opinion, previously suggested by Hadfield and Hopkinson,* that silicon has not directly any improving influence on the iron, but that the improvement is due to the neutralizing effect of silicon upon the carbon contained in the iron. Thus, silicon appears to convert all the carbon from the dissolved form into either cementite or graphite; and even to prevent the formation of hardening carbon by quenching. The correctness of this opinion is apparently borne out by the values for the coercive force before and after annealing for a number of silicon alloys containing relatively large amounts of carbon.

The conclusions arrived at by Gumlich and Goerens have received further confirmation by the results obtained by Paglianti.† In Table 4 the magnetic properties of a few of his alloys have been given. The carbon content is approximately 0.10 per cent. It is seen from this table that quenching increases the coercive force 4 to 7 times for low silicon iron, and only twice for high silicon iron. Similarly the hysteresis loss is increased 2½ to 4½ times for low, and only 1¼

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*Journ. Inst. Elect. Engrs. Vol. 46, p. 235, 1911.

†Metallurgie, Vol. 9, p 217, 1912; Rev. de Metall. Extracts, Vol. 11, p. 4, Jan., 1914.

## TABLE 4.

### MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-SILICON ALLOYS (FROM PAGLIANTI)

| Alloy No. | Silicon % | Heat treatment | Maximum Permeability $\mu^{max}$ | Coercive Force for $B_{max}$ = 16000 gilberts per cm. | Retentivity for $B_{max}$ = 16000. gausses | Hysteresis Loss for $B_{max}$ = 16000. ergs per cc. per cycle | Spec. Elect. Resistance microhms |
|---|---|---|---|---|---|---|---|
| 1 | .24 | A<br>As forged | 980 | 4.75 | 10075 | 17865 | 16.98 |
| 3 | .67 | | 985 | 4.25 | 9100 | 15700 | 22.20 |
| 7 | 2.35 | | 1200 | 3.10 | 7375 | 13890 | 42.50 |
| 10 | 5.26 | | 1900 | 1.75 | 7900 | 9650 | 66.00 |
| 1 | .24 | B<br>Annealed at 1100° C, cooled in 36 hours | 1475 | 2.37 | 7200 | 10700 | 15.62 |
| 3 | .67 | | 1600 | 2.42 | 7650 | 11550 | 21.00 |
| 7 | 2.35 | | 3200 | .78 | 4625 | 3900 | 39.10 |
| 10 | 5.26 | | 2760 | .85 | 4000 | 4900 | 64.50 |
| 1 | .24 | K<br>Annealed at 1100° Slowly cooled to 900°. Quenched in water from 900° | 500 | 8.37 | 7975 | 28600 | 16.66 |
| 3 | .67 | | 320 | 15.87 | 8975 | 46100 | 22.20 |
| 7 | 2.35 | | 1400 | 1.50 | 3500 | 7350 | 41.50 |
| 10 | 5.26 | | 925 | 1.75 | 2400 | 6500 | 66.00 |

times for high silicon iron. Again, the retentivity is increased for low silicon but considerably decreased for high silicon. All these results apparently point in the direction suggested by Gumlich. Taking into account both the hysteresis loss and eddycurrent loss as well as the mechanical properties of the alloys, Paglianti concludes that the 2 per cent alloy has the most favorable characteristics for electromagnetic machinery. This conclusion is based on the assumption that the eddycurrent loss varies inversely as the resistance of the material, an assumption which according to recent investigations by Ball and Ruder* is not strictly correct. No law could be found by them for expressing the dependence of eddycurrent loss on resistivity, except that the loss in general decreases with increasing resistance, but at a slower rate. However, this discovery would not change Paglianti's conclusion, as it would favor the low rather than the high silicon content, and a silicon content much below 2 per cent would be out of the question in any case.

4. *Metallography.*—The effect of silicon upon some of the physical properties of iron having been dealt with in the two previous sections, it remains to inquire into the manner in which silicon has been found to influence the structure of the iron, and the relation between the

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*Gen. Elect. Review, Vol. 17, p. 487, May, 1914.
Electrician Vol. 74, p. 55, 1914.

structural and physical properties, although this has already been touched upon.

The metallography of the iron-silicon series was investigated by Osmond* in 1890, followed by Arnold† and Baker.‡ By their experiments the polymorphic transformation points, $Ar_2$ and $Ar_3$ were established with some degree of certainty for alloys containing up to about 8 per cent silicon. Later, Guertler and Tammann¶ investigated the complete series, but only that part which deals with crystallization from the liquid to the solid state is as yet available, (Fig. 3.)

However, from the data thus obtained Gontermann§ has worked out an equilibrium diagram for the iron-silicon alloys up to the first eutectic point, 21.4 per cent silicon. The diagram is shown in Fig. 4, where the curves that have not been established by experiments are shown broken. According to this diagram, silicon in quantities of about 15 per cent or less remains dissolved in the iron throughout all the allotropic modifications of the latter. If the solution contains more than 15 per cent silicon, two different crystals are formed upon cooling, namely the mixed saturated crystal of iron and silicon, containing about 15 per cent silicon, and a crystal of the composition FeSi.**

The results of the above investigations are confirmed by the photomicrographs published by Baker,‡ Guillet,†† Guertler and Tammann,¶ Gumlich and Goerens,‡‡ and Paglianti.¶¶ These shows that with a silicon content of 15 per cent or less there is only one kind of crystal present in the alloys, with the exception—in the low silicon alloys—of the cementite crystals caused by the carbon present in the iron to a more or less extent. With a higher silicon content, above 3 or 4 per cent, carbon is no longer precipitated as cementite, but as graphite, and appears in the forms of black spots on the surface of the polished specimens.

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*Journ. Iron & Steel Inst. Vol. 37, 1890, I, p. 62.

†Journ. Iron & Steel Inst. Vol. 45, 1894, I, p. 107.

‡Journ. Iron & Steel Inst. Vol. 64, 1903, II, p. 312.

¶Zeitschr. Anorg. Chem. Vol. 47, p. 163, 1905.

§Zeitschr. Anorg. Chem. Vol. 59, p. 384, 1908.
Journ. Iron & Steel Inst. Vol. 83, 1911, I, p. 431.

**According to Guertler and Tammann the 20 per cent alloy forms the compound $Fe_2Si$, and this in turn forms a eutectic with FeSi when the alloy contains 21.4 per cent silicon.

††Rev. de Metallurgie, Memoirs, Vol. 1, p. 46, 1904.

‡‡Trans. Faraday Soc. Vol. 8, p. 98, 1912.

¶¶Metallurgie, Vol. 9, p. 217, 1912.
Rev. de Metallurgie, Extracts, Vol. 11, p. 4, 1914.

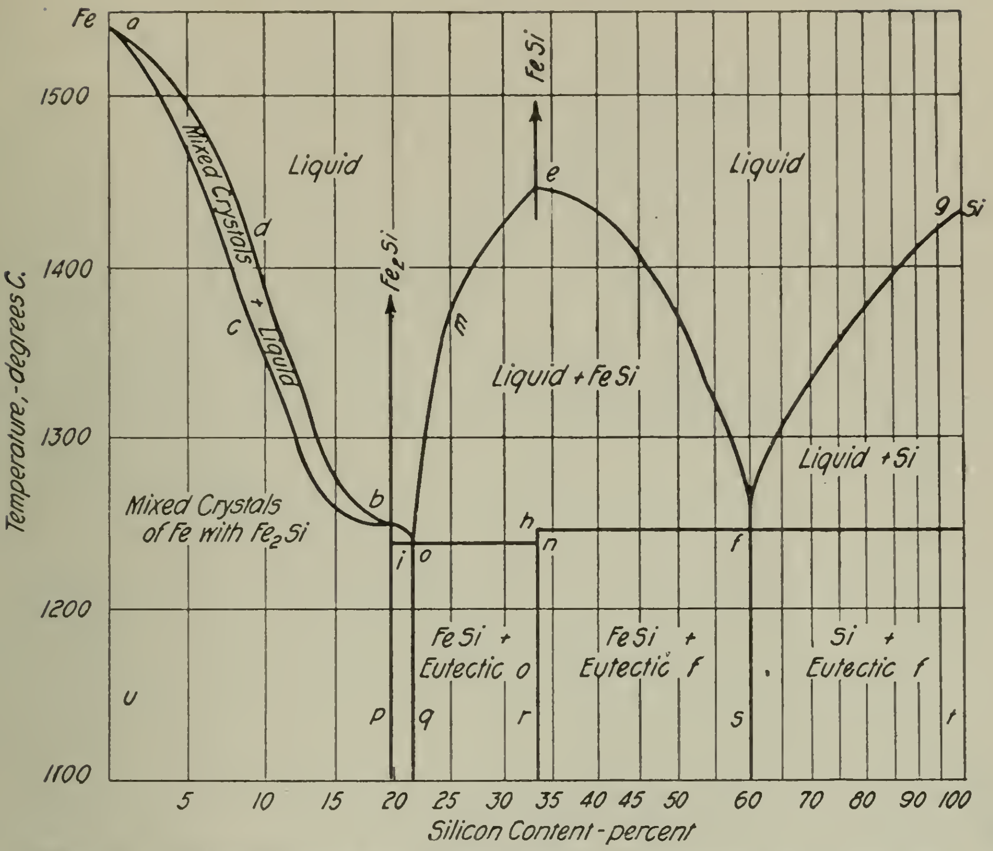

FIG. 3. PHASE DIAGRAM OF IRON-SILICON ACCORDING TO GUERTLER AND TAMMANN.

With regard to the exact manner in which the precipitation of carbon takes place in the presence of silicon, the writer has found no definite explanation. Wüst and Petersen* and Gontermann have made a thorough study of the ternary system, iron-silicon-carbon. The latter suggests that graphite is partly formed by a separation out of supersaturated silico-austenite and partly by the dissociation of silico-cementite.

5. *Summary of Historical Review.*—The information gathered from this review as to the action of silicon upon iron may be summarized as follows:

(a) Silicon is soluble in quantities up to about 15 per cent in all the allotropic modifications of iron. In the presence of silicon, carbon is readily precipitated, and if sufficient silicon is present,—above 4 per cent,—carbon may be completely precipitated as graphite, even though the solution should be rapidly cooled from the molten state.

(b) It has been shown that carbon in the form of graphite has a much less damaging effect upon the magnetic properties of iron than it has in the dissolved form or in the form of cementite. As silicon

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*Metallurgie, Vol. 3, p. 811, 1906.

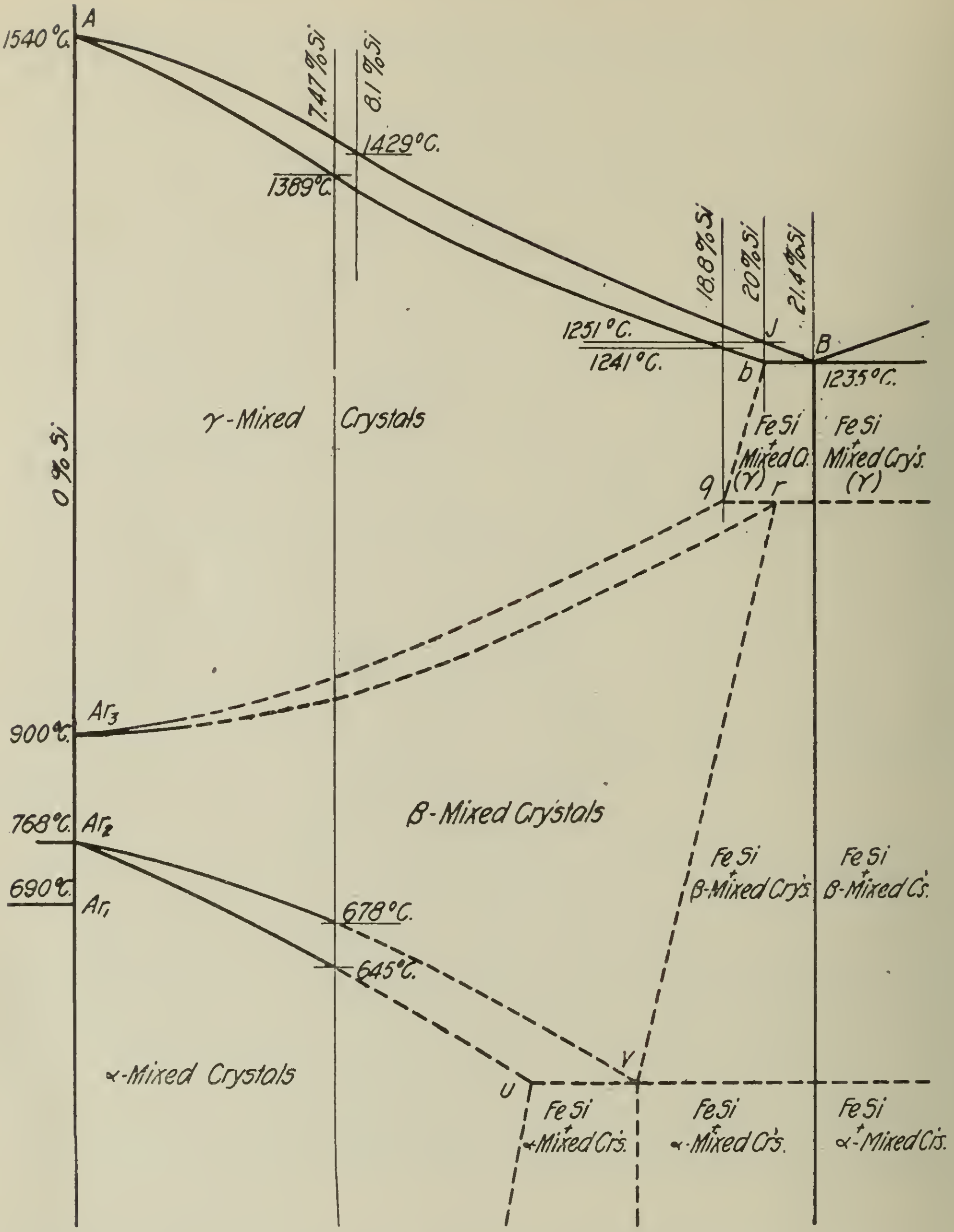

FIG. 4. EQUILIBRIUM DIAGRAM FOR IRON-SILICON ALLOYS ACCORDING TO GONTERMANN.

precipitates carbon as graphite, it is readily seen why silicon indirectly should have a beneficial effect upon the magnetic properties of iron, since even the purest iron contains small amounts of carbon. The most favorable silicon content varies, according to the various investigators, from 2 to 4 per cent.

(c). Silicon increases the electrical resistance of iron by 10 to 12 microhms per cu. cm. for each per cent added. As the eddycurrent loss decreases with an increase in resistance, it follows that the silicon content should be made as high as possible without loss of the most favorable magnetic properties.

(d). Silicon in quantities up to 4 per cent increases the strength of iron in proportion to the amount added. If more than 4 per cent is added the strength decreases rapidly probably due to the formation of graphite, and the yield point coincides with the breaking point. The ductility is but little affected by the silicon below 2.5 per cent; above this amount the alloys are brittle, and with 4 per cent or more the elongation and reduction of area are nil. With 7 per cent or more silicon, the alloys are not forgeable. Unlike carbon, silicon does not confer upon iron the property of becoming hardened when water-quenched.

## III. Material, Apparatus and Methods

Although details with regard to the methods of testing were given in Bulletin No. 72, so many changes have been made since its appearance, that it seems desirable to describe these changes as well as to review briefly the apparatus and methods as a whole. Where details are lacking the reader is referred to Bulletin No. 72.

6. *Iron*—The iron used as the basis of the investigation consisted of doubly refined electrolytic iron, containing 0.006 to 0.01 per cent carbon and 0.01 per cent silicon. Before being used, the iron was thoroughly cleaned with HCl, distilled water and alcohol and then dried by means of ether in vacuo.

7. *Silicon.*—The analysis of the silicon used as the alloying element gave the following result:

| | |
|---|---|
| Si | 92.23 % |
| Fe | 8.50 ” |
| C | .016 ” |
| | 100.746 % |

8. *Melting.*—About 600 grams of iron and silicon in the desired proportion were placed in a crucible, made from fused magnesia, and covered by means of a magnesia cover. The melting was done in an Arsem type vacuum furnace, Fig. 5, under a finishing pressure of 0.5 mm. of mercury, measured by means of a McLeod gage. To obtain this vacuum it was necessary, after the preliminary low heating, to bring the temperature of the carbon heating element up to as high a temperature as possible before the iron should have time to melt and begin to vaporize. This was accomplished by an input of about 14 KW. By this method large amounts of occluded gases were driven

out of the carbon, that otherwise would have prevented a good vacuum from being reached within a reasonable time. In about 15 minutes the iron would begin to vaporize from the hottest parts of the crucible, and, if the input was maintained, iron vapor arcs were formed that would practically cause a short-circuit of the furnace. The input was, therefore, decreased from 14 to 12 KW and maintained there for about 1 hour. At the beginning of this period the iron would be molten and the pressure would be about 2.0 mm, while at the end of the period the pressure would have decreased to 1.0 mm. To further reduce the pressure the input was decreased to 11 KW, this being just sufficient to keep pure iron molten. The carbon, having been allowed to cool off, would absorb some of the gases in the furnace and at the end of another one-hour period a pressure of 0.5 mm. was obtained. With the high silicon alloys the input could be reduced still further before the iron started to freeze, and consequently lower pressures—as low as 0.3 mm—could be obtained. The ingot was then allowed to cool in the furnace so as to prevent oxydation. When removed from the furnace the tops of the ingots would be perfectly bright, occasionally having the appearance shown in Fig. 6, due to a thin film of slag, probabaly $SiO_2$.

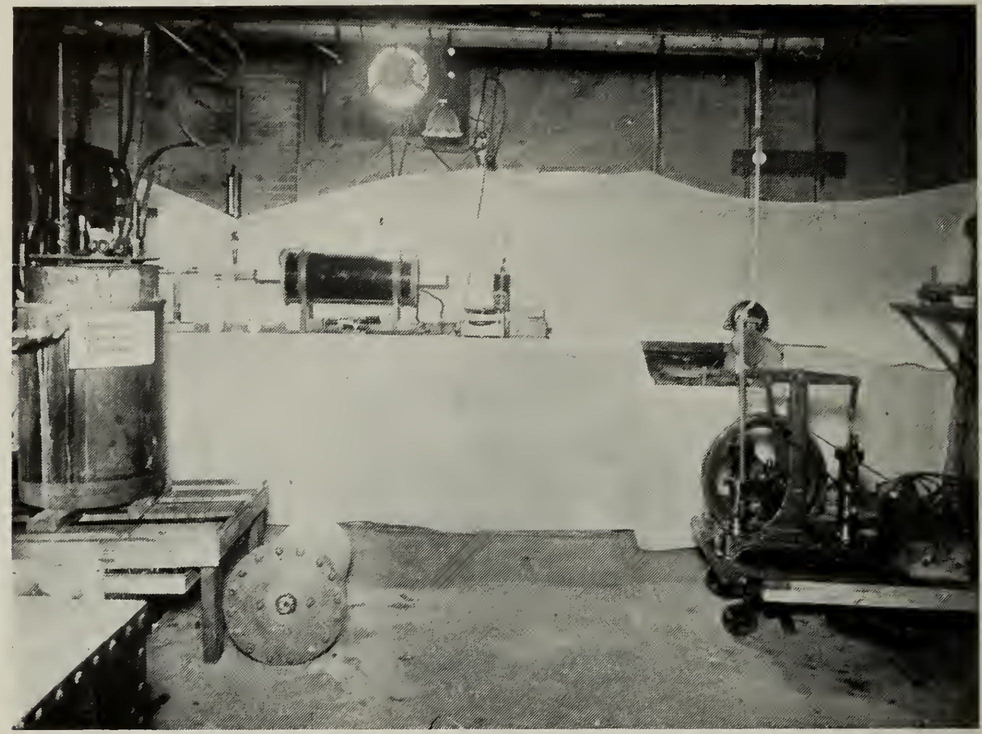

FIG. 5. FURNACE ROOM, SHOWING VACUUM FURNACES.

9. *Forging.*—The forging was done by heating the ingots in an ordinary coke forge to a cherry red, and drawing them out into rods

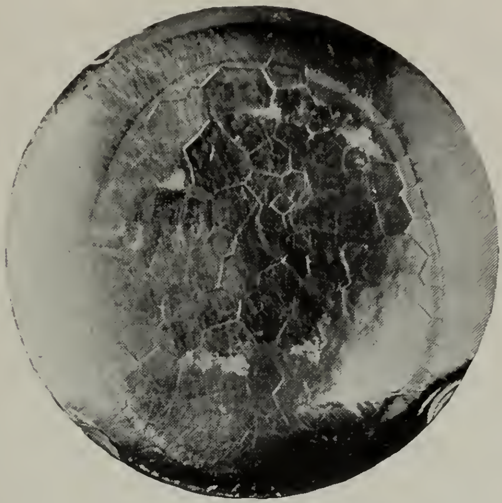

Fig. 6. Top Surface of a Pure Iron Ingot as Removed from Furnace.

about ½″ (1.25 cm) x 20″ (50 cm) under a steam or other power driven hammer. No difficulty was experienced in the forging of the alloys with exception of those containing 2.55, and 8.55 per cent silicon, both of which fell to pieces by the first blows of the hammer. That the latter should not forge was not surprising, but that an alloy containing 2.55 per cent silicon was non-forgeable was quite new. On that account a second ingot was made, that analyzed 2.57 per cent silicon. It, too, broke down into a mass of large crystals just like the previous ingot. The pieces from the 2.57 per cent alloy are shown in Fig. 30$a_2$ showing crystals varying in size from ⅛″ (3.2 mm) to ¼″ (6.4 mm) across. The 8.55 per cent alloy is shown in Fig. 39, exhibiting crystals considerably smaller than the 2.57 per cent alloy. The rest of the alloys forged very well, but as the silicon content increased above 3 per cent the alloys became increasingly harder, and the 6.57 per cent ingot had to be heated repeatedly to a high temperature before it could be drawn out.

10. *Testpieces.*—From the forged rods the following testpieces were prepared:

(a). One testpiece for the magnetic and electrical tests, 0.392″ (0.996 cm) in diameter and 14″ (35.5 cm) long.

(b). Two testpieces for the mechanical tests, having a middle section 0.3″ (0.76 cm) in diameter and 1.5″ (3.8 cm) long, with threaded ends 0.5″ x 0.5″ (1.25 cm). (See Fig. 13.)

(c). One small sample for the metallographic investigation.

The shavings obtained by the process of preparing these testpieces were collected, after first removing the exposed parts of the rods, and used for chemical analysis.

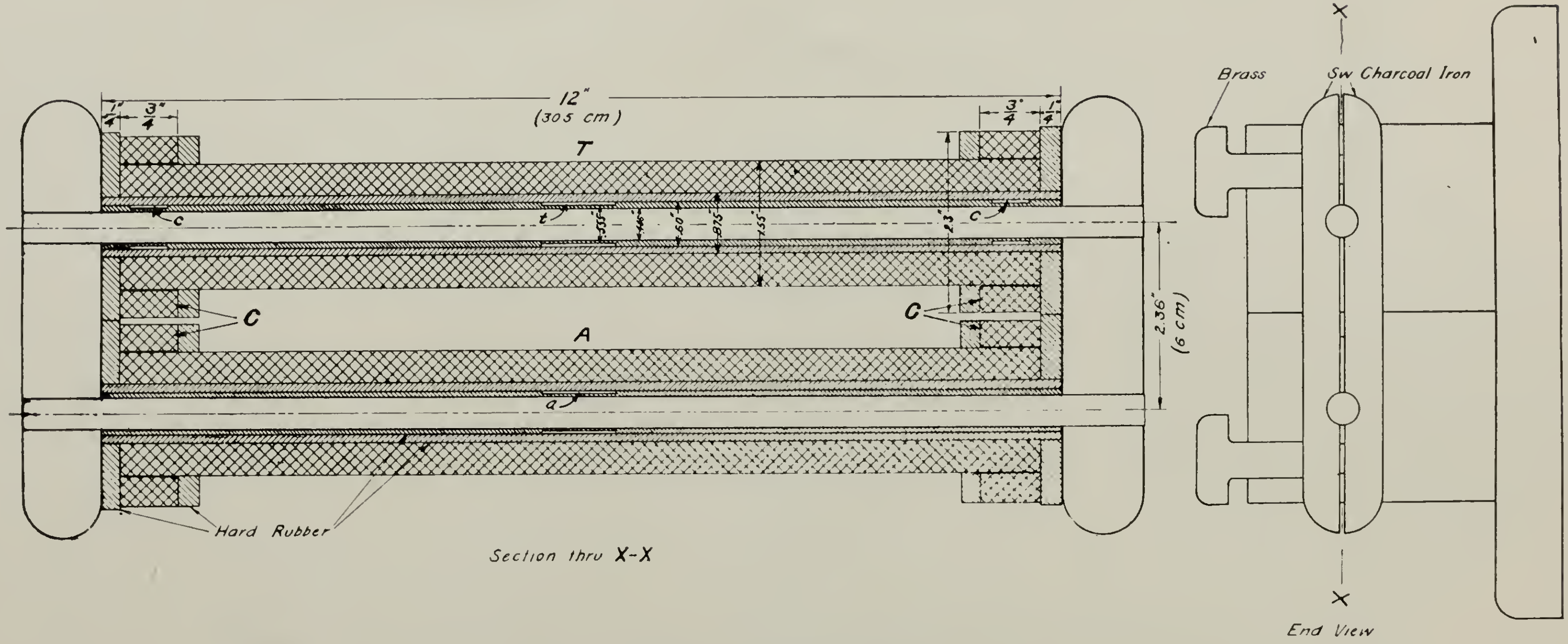

FIG. 7 PERMEAMETER.

Coil Data.

*T*, *A* and *C* coils—10 layers No. 18 B. & S. enameled copper wire, 20 turns per inch per layer (7.875 turns per cm.) *t* and *a* coils—64 turns each of No. 30 B. & S. d. s. c. copper wire; *c* coils 32 turns each of No. 30 B. & S. d. s. c. copper wire—connected in series.

All the rods machined without any particular difficulty, with the exception of the 6.57 per cent rod. This rod broke repeatedly in the lathe in spite of the care exercised by the machinist. The vibration was evidently sufficient to break it. No magnetic testpiece was therefore obtained from this rod, and mechanical testpieces were secured only by omitting the threads at the ends. These testpieces, therefore, had to be tested between flat grips, instead of screw sockets.

11. *Magnetic Testing.*—The magnetic testing was done by means of the Burrows compensated double bar and yoke method,* now generally adopted wherever accurate testing is desired. The apparatus used in the previous parts of the investigation was superseded by one requiring a 14″ (35.5 cm) rod, instead of a 12″ (30.5 cm) rod as previously used, and the distance between the rods was shortened. These changes were made in order to decrease the errors due to the ends of the various coils. With the new apparatus, shown in Fig. 7, and Fig. 8, the error in H, as measured by the magnetizing current in the main coil, and with the same current in all three coils, has been calculated according to the method described in Bull. No. 72, and found to be –0.02 per cent. With the highest permeability rods, however, it

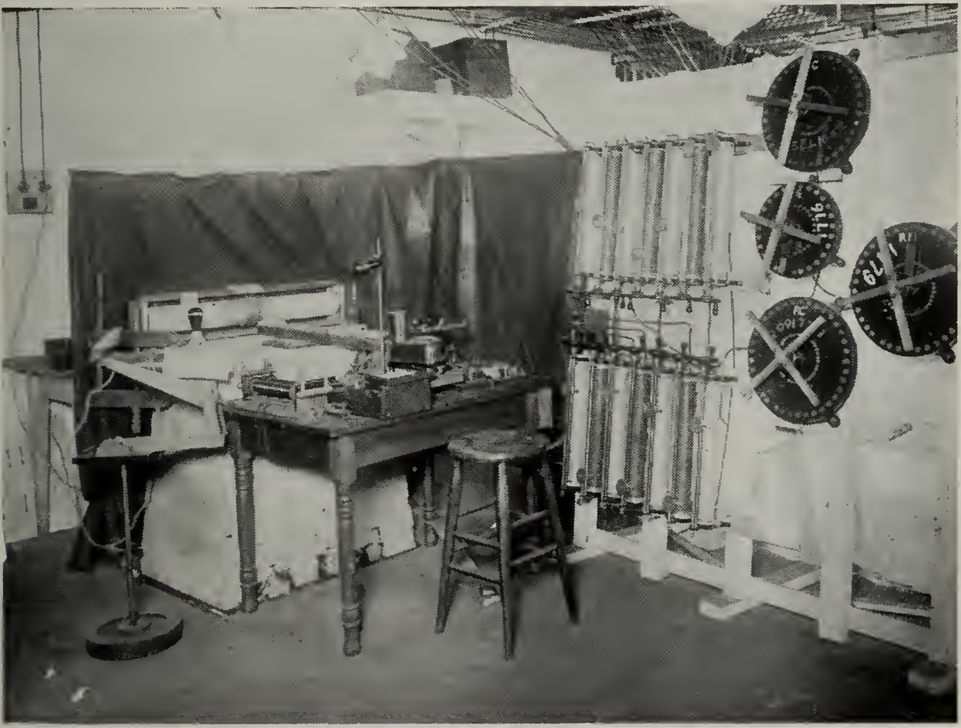

FIG. 8. MAGNETIC TESTING APPARATUS.

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*Bull. Bureau of Standards. Vol. 6. No. 1. Reprint No. 117.

was found that the current in the compensating coils in exceptional cases had to be 30 times the current in the main and auxiliary coils. Consequently, the maximum error in H is found to be 2.30 per cent, that is, H as measured should be increased by 2.30 per cent. Corrections have been made in the results according to these theoretical calculations, but as will be shown in section III, it is probable that the errors are larger than these would lead to.

A Grassot fluxmeter was used for the measurement of B. In this instrument the deflection is independent of the duration of the magnetic change, a feature which is essential in testing iron of high permeability and low resistance. With the high resistance silicon alloys a long period ballistic galvanometer would probably serve the purpose.

The apparatus was calibrated from time to time by means of an aircoil, and also by means of rods submitted to the Bureau of Standards for standardization. The results obtained by the writer check very well with those obtained by the Bureau and also with results obtained by the calibration laboratories of two of the large manufacturing companies. As will be shown in section III, care has to be exercised in clamping the rods between the yokes, as a very slight strain due to bending has a considerable influence upon the permeability of the high permeability rods.

12. *Heat-treatments.*—The rods were tested magnetically and electrically and photomicrographs were obtained after the following treatments:

(a). As forged
(b). Annealed at 900° C. Cooled at a rate of 30° per hour
(c). Annealed at 1100° C. Cooled at a rate of 30° per hour

The annealing was done in vacuo, by means of the furnace shown in Fig. 5 and Fig. 9. This is a special type Hoskins resistor furnace, fitted with two electroquartz tubes, one inside the other. The test-pieces were placed in the center of the inner tube around the pyrometer protecting tube in such a way that the end of the pyrometer was located at the center of the rods. The containing tube was then filled with magnesia and the ends closed by means of rubber stoppers and sealed with mercury. A vacuum of 0.2 mm to 0.1 mm was maintained during the entire heat treatment. Accidents happened occasionally through the breaking of the tube or leak in the ends, destroying the rods in one case, but ordinarily the rods after removal from the furnace needed very little cleaning. A trial experiment was made using nitrogen by first evacuating the furnace, and then filling it with nitrogen obtained by drawing air through four gas wash bottles filled with pyrogallic acid and one bottle filled with concentrated sulphuric acid. This experiment gave rods practically free from scale and may prove a convenient method for annealing, but the rods used in this trial were discarded as far as this report is concerned. Practically uniform cooling of the rods was obtained by means of a regulator that automatically inserted resistance in the circuit every half hour.

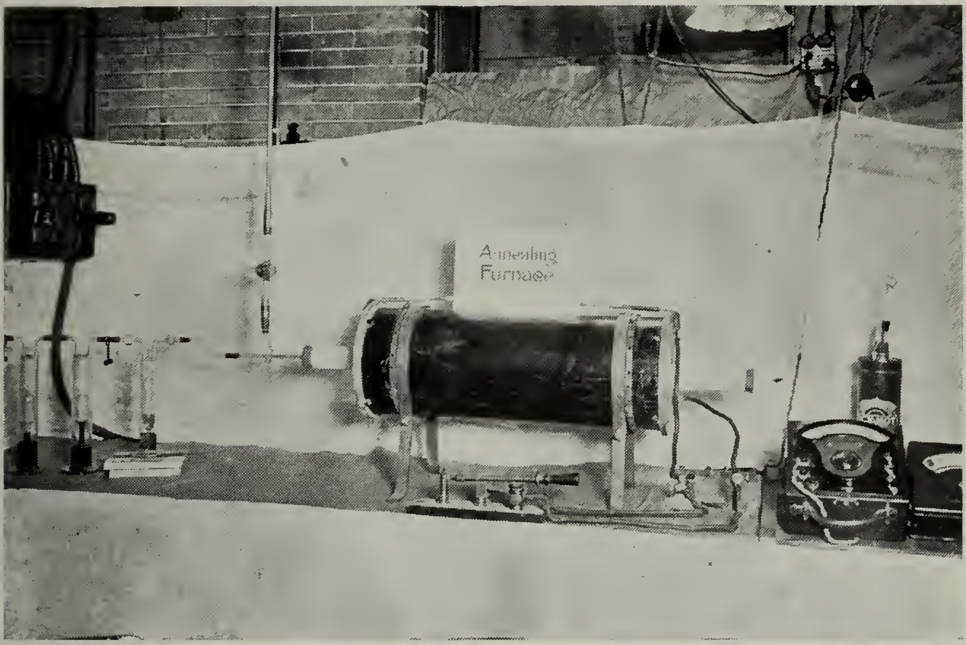

FIG. 9. VACUUM ANNEALING FURNACE.

13. *Photomicrographs.*—These were obtained by polishing in the usual way using jewelers' rouge for the final polishing. Great care had to be used with the higher silicon alloys. Numerous small spots would appear on the surface of some of the specimens, too numerous to be caused by graphite, as the carbon content amounted to only 0.01 to 0.02 per cent. Fig. 10 shows a peculiar formation on one of these specimens before etching. By careful polishing the spots were in most cases removed, as seen by the photomicrographs shown in section IV.

The etching was generally done by means of saturated picric acid in alcohol. In a few cases, however, a 10% solution of nitric acid in alcohol was found to give more satisfactory results.

The highest silicon alloys, containing 4.92 and 6.57 per cent respectively, had to be washed with dilute hydrofluoric acid to remove

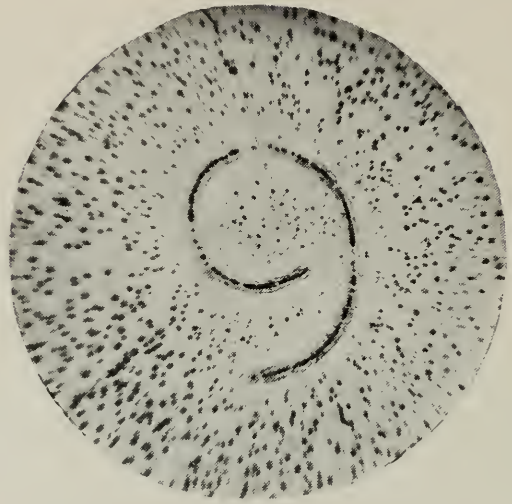

FIG. 10. PECULIAR FORMATION.

Black spots would appear in polishing some of the high silicon specimens, too numerous to be caused by graphite. This photomicrograph shows a peculiar formation of these black spots. By careful and prolonged polishing the spots would nearly all disappear.

Alloy No. 3Si27. 2.73% Si.
20 Diam. Unetched.

the thin film of a silicon composition that invariably formed on the surface after etching.*

14. *Mechanical Testing.*—The mechanical testing consisted in obtaining the yield point, ultimate strength, the elongation before the specimen had commenced to "neck," the ultimate elongation, and the reduction of area at the point of fracture. The testing was done on an Olsen 10000-pound testing machine, in which the test pieces were secured between screw sockets. The load was applied uniformly by means of an electric motor, and in most cases the yield point was detected with certainty. As stated before, the testpieces from the 6.57 per cent alloy could not be threaded on account of their brittleness and had to be tested between flat grips.

## IV. MAGNETIC TESTING. EFFECT OF STRAIN AND COMPENSATING CURRENT.

While it is well known that mechanical stress affects the magnetic properties of iron, it was at first considered safe to clamp the magnetic test rods in the permeameter without regard to the method of clamping, on the assumption that the strain due to the clamping would be negligible. It was, however, noticed that in certain cases the permeability increased materially by turning the rods from one position to another and reclamping, while in other cases the opposite took place. It soon became evident that this change of permeability was due to mechanical strain, in such a way that the least strain gave the highest permeability.* In the construction of the permeameter used in the present investigation, particular attention was given to the yokes to be sure that the holes for receiving the rods were perfectly parallel. However, even with these yokes, inconsistencies were observed that finally were traced to strain. Table 5 gives the results obtained by different methods of clamping. It shows (Table 5a) that by tight clamping the values of B, for $H = 0.5$, may vary from 8000 to 11000 for the same rod, that is, about 30 per cent, and that it seems to make little difference which rod is used as auxiliary. With the yokes clamped so as to give good contact and yet not strain the rods, the results vary only about 1 per cent, and this may be due to influences other than mechanical strain. The figures for tight yokes show that it is possible to have the yokes tight and yet have little strain on the rods, but there is no way to ascertain whether the rods are strained, if the yokes are tight.

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*L. Guillet: Rev. de Metallurgie, Memoirs, Vol. 1, p. 46, 1904.

Table 5b shows, perhaps even more convincingly, that the inconsistent results shown in 5a, are due to strain. Starting with no strain on the rods and the circuits balanced, the value for B was found to be 11460, which is considered to be the correct value for the rod used. Without changing any of the magnetizing currents the yokes were then tightened. Under this new condition the contact between the yokes and the rods were better than under the first condition and should require a smaller compensating current to balance the magnectic circuit. In spite of the too high compensating current, B was apparently decreased to 11300. Balancing the magnetic circuit shows the true value of B under this condition to be 10550. Leaving the magnetizing currents unchanged, the yokes were then loosened (condition 4). The compensating current under this condition was naturally too small, as the contact was not as good as before, and yet the apparent value of B was increased to 11460. This is probably the most forceful evidence in favor of the contention. Balancing the circuit (condition 5) shows the true value to be 11460, the same as under condition 4. This is evidently due to the fact that the auxiliary rod had been strained more than the main rod, shown by the necessity of decreasing $H_{aux}$ from 0.8 to 0.7.

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*Bulletin No. 72 Eng. Exp. Sta. p. 40.

## TABLE 5.

## EFFECT OF STRAIN AND COMPENSATING CURRENT ON MAGNETIC TESTING.

a. TEST ROD NO. 3Si14C WITH VARIOUS AUXILLIARY RODS.
INDUCTION B, FOR $H = 0.5$.

| Aux. rod used | Rods more or less strained, due to tight yokes. | | | Yokes loose, no strain |
|---|---|---|---|---|
| | Series 1 | Series 2 after reclamping | Series 3 after reclamping and cleaning ends of rods | Series 4 |
| 3–54B | 9330 | 9800 | 9250 | 11700 |
| 3Si09C | 9900 | 8240 | 8040 | 11600 |
| 3Si10C | 10110 | 10250 | 8940 | 11800 |
| 3Si11C | 11060 | 10250 | 11160 | 11800 |
| 3Si12C | 9650 | 10670 | 8290 | 11700 |
| 3Si13C | 10850 | 9230 | 8600 | 11700 |

b. TEST ROD NO. 3Si10C WITH AUXILLIARY ROD NO. 3Si09C.
INDUCTION B, FOR $H = 0.5$.

| Cond. No. | Method of Clamping. | $H_{aux}$ for 3Si09C | Compensating Current, $I_c$ | B for 3Si10C |
|---|---|---|---|---|
| 1 | Yokes loose, no strain on rods, no air gap between rods and yokes | 0.7 | .059 | 11460 |
| 2 | Yokes tight, rods strained, magnetizing currents same as above | same | same | 11300[x] |
| 3 | Yokes tight, rods strained, magnetic circuit balanced | 0.8 | .042 | 10550 |
| 4 | Yokes loose, no strain on rods, magnetizing currents same as above | same | same | 11460[t] |
| 5 | Yokes loose, no strain on rods, magnetic circuit balanced | 0.7 | .051 | 11460 |
| 6 | Yokes loose, no strain on rods, small air gap between rods and yokes | 0.7 | .105 | 12460 |
| 7 | Yokes tight, rods strained, small air gap between rods and yokes | 0.8 | .058 | 10890 |
| 8 | Yokes loose, no strain, no air gap between rods and yokes | 0.7 | .059 | 11460 |

[x] Magnetic circuit not balanced; on account of better contact between rods and yokes, $I_c$ is too large, and yet B is reduced, showing that strain decreases permeability.

[t] Magnetic circuit not balanced; on account of poorer contact between rods and yokes, $I_c$ is too small, and yet B is increased, showing that removal of strain restores the rod to its normal condition.

Under condition 6 a small airgap was produced between the rods and the yokes by means of tissue paper, necessitating a doubling of $I_c$. The value obtained for B in this case was 12460, an increase of 1000 over the value obtained with no airgap. According to theoretical calculations, only one-tenth of this increase can be attributed to the increase in $I_c$, as a compensating current of 0.105 (which is 21 times the main magnetizing current) should increase H as measured, only by about 2 per cent. It is possible that the rest of the increase may be due to the removal of a slight strain that existed with no airgap, but it seems more probable that the compensating current has a larger effect than is shown by theoretical considerations. If it be assumed that the total increase is due to $I_c$, H as measured is evidently increased 8 per cent due to a compensating current equal to 10 times the main magnetizing current, and correspondingly for other ratios. Should this assumption be correct, the results recorded in this bulletin are somewhat exaggerated, as corrections have been made according to theoretical considerations only. More experimental evidence, is, however, necessary before the final judgment can be passed, and in the meantime the reader, in studying the results, should keep this matter in mind, remembering at the same time that whatever the effect of the compensating current may be, the results are strictly comparable.

Under condition 7 the rods were strained, bringing the value of B down to 10890, nearly the same as under 3. Removing the airgap and testing with no apparent strain, the value found for B was 11460, the same as under similar conditions before. It is interesting to note that $I_c$ under the last two conditions was practically the same, while B with the rod strained was about 600 gausses less than with no strain, another forceful argument showing that the permeability is decreased by a slight strain.

The condition of no strain can only seldom be applied in practice, but it is necessary in an investigation of this kind to make the tests under conditions that can be duplicated and standardized, in order to obtain results that are comparable, and Table 5 shows very plainly that the ''no strain'' condition is the only one that fulfills this requirement. Another argument for the adoption of the ''no strain'' condition is that this is the condition that exists in the only other reliable method of magnetic testing that the writer knows of, namely the Rowland ring method.* In the appendix will be found some results obtained with ring specimens together with a discussion of the same.

## V. RESULTS

15. *Chemical Properties.*—Table 6 gives a complete list of all the alloys made, together with the chemical analysis for silicon. From the amount of silicon added, the percentage lost in terms of the weight of the ingot has been calculated and listed in the fourth column. The only elements that are present in the alloys as impurities in a measurable quantity are carbon, amounting to about 0.01 per cent, and oxygen.

In bulletins No. 72 and No. 77 of the Engineering Experiment Station it was shown that the electrolytic iron, even after the thorough cleaning to which it was subjected before melting, contained about 0.4 per cent oxygen in the form of some oxide of iron. It was shown that while carbon will reduce the iron oxide before commencing to combine with the iron, boron will combine with the iron before all the iron oxide is reduced, its affinity for oxygen being about twice its affinity for iron. From Table 6 it is seen that silicon in this respect acts like boron, with the difference, that its affinity for iron is much stronger than its affinity for oxygen. The percentage silicon lost, that is, the percentage that has been oxidized and changed into slag, increases, somewhat irregularly with the silicon added, but reaches a

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*Most of the inconsistencies that appeared in Bulletin No. 72 on the properties of pure iron can no doubt be attributed to strain in view of the results shown here.

### TABLE 6.

### LIST OF ALLOYS.

| Specimen No. | Silicon Added per cent | Silicon Content as per Chem. Anal. per cent | Silicon Lost per cent | Remarks |
|---|---|---|---|---|
| 3–51 | .000 | ab't. .001 | | These rods were discarded after annealing at 1100° C on account of oxidation due to a leak in the furnace. |
| 3–52 | .000 | ” .001 | | |
| 3–53 | .000 | ” .001 | | |
| 3–54 | .000 | ” .001 | | |
| 3–55 | .000 | ” .001 | | |
| 3Si05 | .092 | .068 | .024 | |
| 3Si06 | .185 | .148 | .037 | |
| 3Si07 | .276 | .242 | .034 | The magnetic data for these rods, after annealing at 900° C. were taken before the effect of strain due to tight clamping of the yokes was noticed. It was impossible to re-test these rods annealed at 900° because they were already annealed at 1100°. Data for the 1100° annealing were taken without strain. |
| 3Si08 | .368 | .309 | .059 | |
| 3Si09 | .460 | .400 | .060 | |
| 3Si10 | .551 | .472 | .079 | |
| 3Si11 | .643 | .563 | .080 | |
| 3Si12 | .734 | .673 | .061 | |
| 3Si13 | .825 | .698 | .127 | |
| 3Si14 | .916 | .822 | .094 | |
| 3Si15 | .138 | .064 | .074 | |
| 3Si16 | .046 | .010 | .036 | |
| 3Si17 | .230 | .230 | .000 | |
| 3Si18 | 1.810 | 1.741 | .069 | |
| 3Si19 | 2.690 | 2.550 | .140 | Not forgeable. Crushed into mass of crystals. |
| 3Si20 | 3.560 | 3.580 | –.020 | Flaw in center of rod after forging. |
| 3Si21 | .092 | .048 | .044 | |
| 3Si22 | .185 | .091 | .094 | |
| 3Si23 | .276 | .205 | .071 | |
| 3Si24 | 2.690 | 2.570 | .120 | Not forgeable. Same as 3 Si 19. |
| 3Si25 | 3.560 | 3.400 | .160 | |
| 3Si26 | 2.260 | 2.180 | .080 | Contaminated in melting. |
| 3Si27 | 3.125 | 2.730 | .395 | |
| 3Si28 | 4.410 | 4.440 | –.030 | |
| 3Si29 | 5.230 | 4.920 | .310 | |
| 3Si30 | 8.420 | 8.550 | –.130 | Not forgeable. |
| 3Si31 | 2.260 | 1.710 | .550 | |
| 3Si32 | 6.850 | 6.570 | .280 | |
| 3Si33 | .459 | .420 | .039 | Not used. Made from Electrolytic Iron that proved to be impure. |
| 3Si34 | .915 | .700 | .215 | |
| 3Si35 | .276 | .193 | .083 | |
| 3Si36 | 3.980 | 3.550 | .430 | |
| 3Si37 | 4.810 | 4.390 | .420 | |
| 3Si38 | 2.460 | 2.260 | .200 | Used for mechanical tests only. |
| 3Si39 | 3.560 | 3.185 | .375 | Used for rings for comparative magnetic testing. |
| 3Si40 | 3.560 | 2.960 | .600 | |
| 3Si41 | 2.690 | 2.360 | .330 | Used for mechanical tests only. |
| 3Si42 | 2.690 | 2.700 | –.010 | |
| 3Si43 | 2.770 | 2.550 | .220 | |
| 3Si46 | 2.600 | 2.530 | .070 | |
| 3Si47 | 2.510 | 2.410 | .100 | |
| 3Si51 | 2.660 | 2.600 | .060 | |

maximum of about 0.5 per cent. As silicon oxidizes to $SiO_2$ the maximum amount of oxygen absorbed is 0.44 per cent, or the same amount that was found in the two previous investigations, using carbon or boron.

16. *Mechanical Properties.*—The results of the mechanical tests are shown in Tables 7 and 8 and graphically in Figs. 11 and 12. A comparison between these results and those obtained by previous investigators was given in Figs. 1 and 2. Fig. 13 is a photograph of some of the test pieces after being tested, showing very clearly the variation in the elongation and reduction of area caused by silicon.

From the data thus presented it may be stated in general that silicon increases the strength of iron in almost direct proportion to the amount added, until the maximum strength is reached with a silicon content of about 4.5 per cent. From this point on, the elastic limit coincides with the ultimate strength, and both decrease very rapidly. The limit of forgeability lies at about 7 per cent. The values for the forged condition are considerably higher than for the annealed condition, the difference varying between 10,000 and 20,000 lb. per

TABLE 7.

RESULTS OF MECHANICAL TESTS.

AS FORGED.

| Number of Specimen | Silicon Content, % | Yield Point, lb. per sq. in. | Ultimate Strength, lb. per sq. in. | Elongation % | | Reduction of Area, % | Remarks |
|---|---|---|---|---|---|---|---|
| | | | | Before "Necking" | Ultimate | | |
| 3–39 | .001 | 35,800 | 44,700 | . . | 39 | 80.4 | |
| 3–53 | .001 | 44,400 | 46,500 | 3 | 24 | 53.8 | |
| 3–55 | .001 | 38,000 | 40,800 | 8 | 40 | 88.5 | |
| 3Si16 | .010 | 41,800 | 45,200 | 11 | 35 | 78.0 | |
| 3Si21 | .048 | 42,850 | 46,900 | 5 | 25 | 91.6 | Failed near Punch Mark |
| 3Si05 | .068 | 36,800 | 43,800 | 10 | 37 | 92.0 | |
| 3Si22 | .091 | 35,600 | 43,750 | 12 | 36 | 91.7 | |
| 3Si06 | .148 | 38,600 | 45.000 | 11 | 42 | 94.8 | |
| 3Si23 | .205 | 42,500 | 49,700 | 10 | 39 | 93.4 | |
| 3Si17 | .230 | 41,300 | 47,500 | 10 | 45 | 89.7 | |
| 3Si11 | .563 | 40,750 | 51,000 | 12 | 41 | 92.6 | |
| 3Si12 | .673 | | 58,000 | 9 | 30 | 91.4 | |
| 3Si14 | .822 | 45,200 | 55,800 | 11 | 36 | 93.1 | |
| 3Si31 | 1.71 | 68,100 | 76,300 | 6 | 29 | 87.2 | |
| 3Si38 | 2.25 | 66,550 | 77,750 | 12 | 37 | 82 | |
| 3Si41 | 2.36 | 77,200 | 86,450 | 8 | 24 | 66 | |
| 3Si47 | 2.41 | 63,500 | 75,800 | 9 | 23 | 65 | |
| 3Si46 | 2.54 | 60,300 | 73,400 | 14 | 16 | 20 | |
| 3Si51 | 2.63 | 68,700 | 78,500 | 13 | 24 | 43 | |
| 3Si43 | 2.65 | 59,000 | 74,700 | 14 | 34 | 72 | |
| 3Si42 | 2.78 | 68,250 | 78,000 | 6 | 21 | 74 | Slight Flaw. |
| 3Si25 | 3.40 | 74,500 | 86,300 | 10 | 31 | 74.7 | |
| 3Si36 | 3.55 | 83,400 | 99,300 | 12 | 23 | 41.3 | |
| 3Si37 | 4.39 | 94,000 | 105,000 | 6 | 6 | 7.5 | |
| 3Si29 | 4.92 | | 50,250 | Nil | Nil | Nil | Failed at Base of Head |
| 3Si32 | 6.57 | 5,120 | 5,120 | Nil | Nil | Nil | Tested between grips without being machined |

sq. in. (7 to 14 kg. per sq. mm.). For the 4.5 per cent alloy "as forged" the ultimate strength is 105,000 lb. per sq. in. (73.5 kg. per sq. mm.) about 8000 lb. (5.6 kg.) higher than the maximum obtained by previous investigators. The practical absence of carbon in the vacuum iron causes the low-silicon alloys to be weaker than the corresponding alloys tested by previous investigators, but this same absence of carbon is evidently a cause for added strength in the 4.5 per cent alloy, in which the carbon exists in the form of graphite.

### TABLE 8.
### RESULTS OF MECHANICAL TESTS.
### ANNEALED.

| Number of Specimen | Silicon Content, % | Yield Point, lb. per sq. in. | Ultimate Strength, lb. per sq. in. | Elongation % | | Reduction of Area, % | Remarks |
|---|---|---|---|---|---|---|---|
| | | | | Before "Necking" | Ultimate | | |
| Annealed at 970° C. | | | | | | | |
| 3–39 | .001 | 16,400 | 36,100 | .. | 61 | 80.9 | |
| 3Si16 | .010 | 16,050 | 34,900 | 25 | 53 | 81.5 | |
| 3Si21 | .048 | 20,100 | 35,000 | 23 | 48 | 89.3 | |
| 3Si15 | .064 | 14,750 | 34,100 | 27 | 29 | 45.1 | Failed near Head Flaw. |
| 3Si05 | .068 | 20,400 | 34,900 | 28 | 64 | 94.8 | |
| 3Si22 | .091 | 14,290 | 35,400 | 26 | 64 | 91.8 | |
| 3Si06 | .148 | 15,890 | 35,200 | 29 | 48 | 67.0 | |
| 3Si23 | .205 | 25,075 | 38,650 | 19 | 50 | 89.4 | |
| 3Si19 | .230 | 14,910 | 35,500 | 30 | 60 | 84.7 | Broke near Head. Flaw |
| 3Si07 | .242 | 18,100 | 38,400 | 25 | 60 | 91.3 | |
| 3Si08 | .309 | 21,650 | 40,400 | 25 | 55 | 90.0 | |
| 3Si09 | .400 | 26,000 | 42,000 | 20 | 55 | 91.0 | |
| 3Si10 | .472 | 17,340 | 42,750 | 26 | 54 | 91.4 | |
| 3Si11 | .563 | 25,700 | 41,200 | 08 | .. | ... | Was not Stressed to Failure. Flaw |
| 3Si12 | .673 | 26,550 | 45,230 | 21 | 45 | 88.2 | |
| 3Si13 | .698 | 23,100 | 43,000 | 25 | 57 | 89.0 | |
| 3Si14 | .822 | 26,200 | 45,150 | 28 | 50 | 91.6 | |
| 3Si31 | 1.710 | 35,800 | 54,250 | 25 | 50 | 90.6 | |
| 3Si18 | 1.741 | 45,750 | 55,000 | 14 | .. | 84.7 | Failed at Punch Mark. |
| 3Si38 | 2.28 | 42,200 | 63,500 | 23 | 50 | 85.0 | |
| 3Si43 | 2.36 | 46,200 | 63,600 | 17 | 29 | 51.0 | |
| 3Si41 | 2.38 | 43,000 | 64,200 | 26 | 50 | 84.5 | |
| 3Si47 | 2.38 | 47,500 | 68,700 | 22 | 45 | 78.5 | |
| 3Si46 | 2.56 | 47,250 | 64,200 | 14 | 18 | 31.5 | |
| 3Si51 | 2.59 | 42,500 | 68,200 | 22 | 42 | 82.5 | |
| 3Si42 | 2.70 | 48,500 | 61,800 | 6 | 6 | 11.3 | |
| 3Si27 | 2.73 | 49,600 | 67,800 | 18 | 19 | 15.5 | |
| 3Si25 | 3.40 | 57,100 | 77,400 | 15 | 21 | 28.7 | |
| 3Si37 | 4.39 | 85,000 | 85,000 | Nil | Nil | 1.2 | Annealed at 1030° C in Nitrogen |
| 3Si28 | 4.44 | 72,900 | 91,600 | 14 | 24 | 25.1 | |
| 3Si29 | 4.92 | 47,700 | 47,700 | Nil | Nil | Nil | Head failed |
| 3Si32 | 6.57 | 13,000 | 13,000 | Nil | Nil | Nil | Tested between Grips. |
| Annealed at 1100° C. | | | | | | | |
| 3Si41 | 2.35 | 39,400 | 58,300 | 8 | 10 | 10 | |
| 3Si47 | 2.43 | 41,700 | 63,100 | 24 | 45 | 67 | |
| 3Si46 | 2.48 | 45,200 | 66,300 | 21 | 47 | 75 | |
| 3Si43 | 2.53 | 43,300 | 62,500 | 20 | 40 | 71 | |
| 3Si42 | 2.61 | 38,900 | 59,500 | 17 | 32 | 61 | Slight Flaw |
| 3Si51 | 2.63 | 43,300 | 45,600 | 1 | 2 | Nil | |

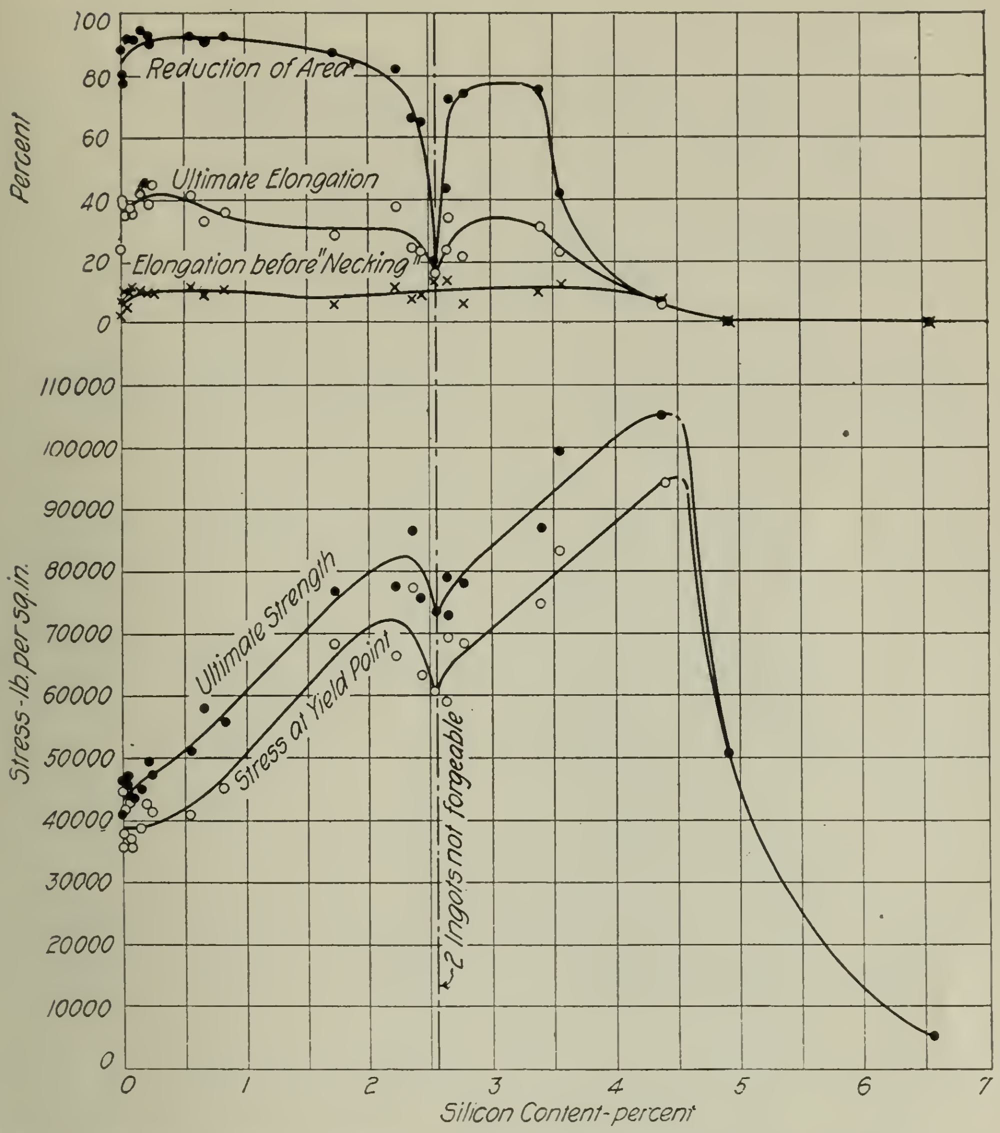


FIG. 11. MECHANICAL PROPERTIES OF IRON-SILICON ALLOYS, MELTED IN VACUO. AS FORGED.

With regard to the elongation and reduction of area, the results in general confirm those obtained by Hadfield, Baker and Paglianti concerning the effect of silicon. However, the vacuum alloys again show the effect of the lack of carbon in being much tougher in the region of low silicon as well as in the region between 3 and 5 per cent. The latter is particularly significant, as it is in this region that the maximum strength occurs. While the strength maximum for alloys containing small amounts of carbon corresponds to zero elongation and reduction of area, the strongest vacuum alloy in the forged condition has a reduction of area of 8 per cent, and an elongation of 7 per cent.

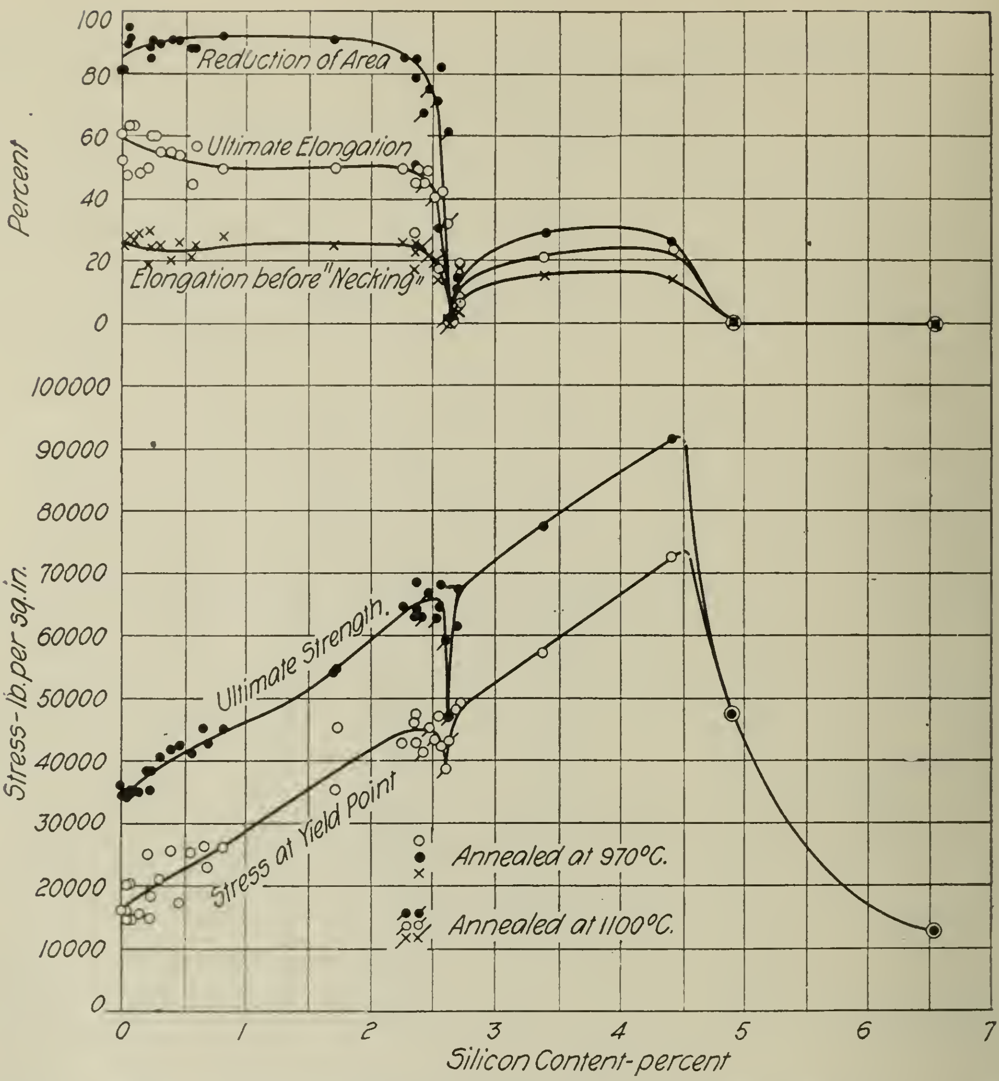
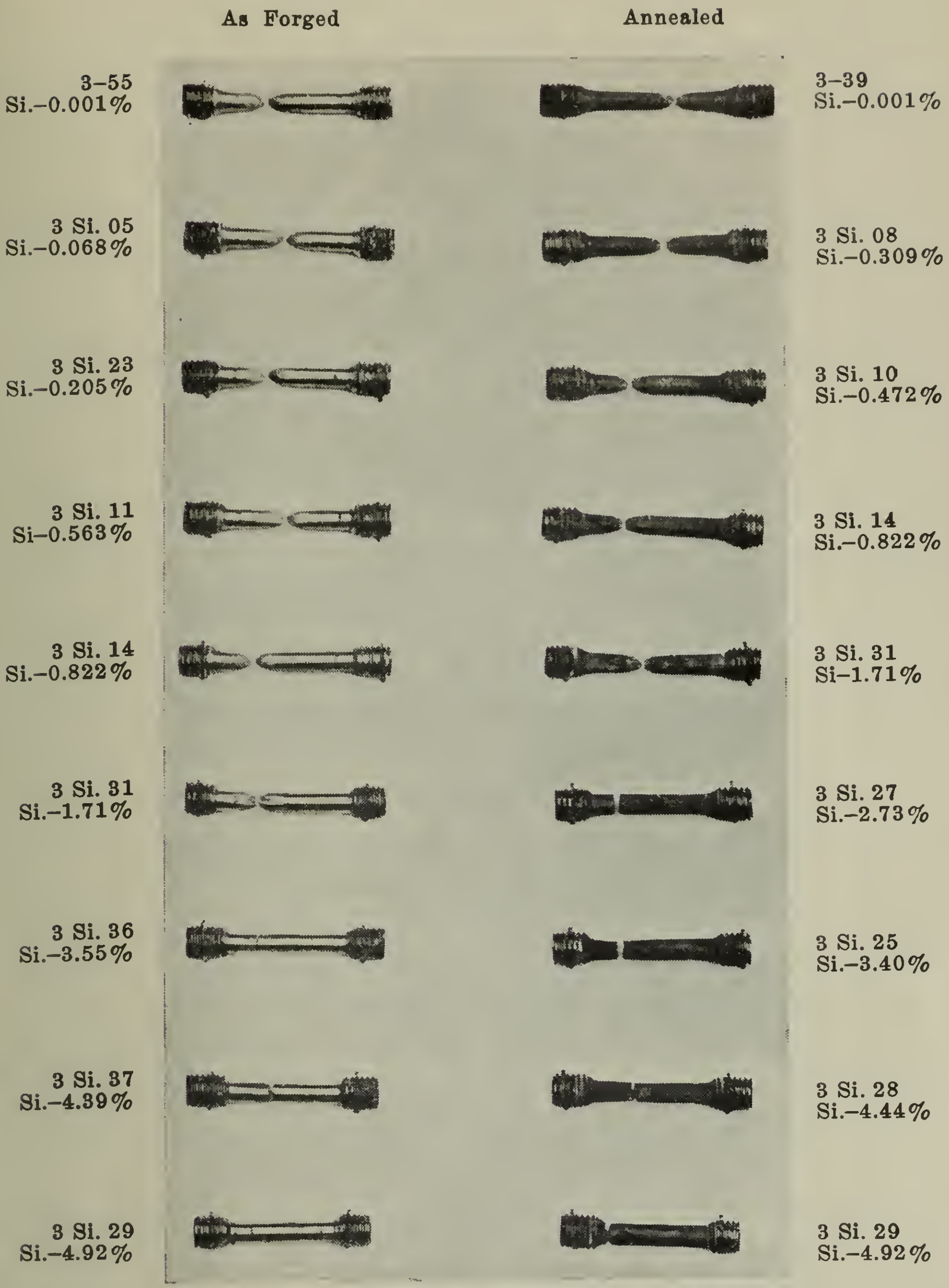


Fig. 12. Mechanical Properties of Iron-Silicon Alloys, Melted in Vacuo. Annealed.

In the annealed state the corresponding figures for the same alloy are 24 and 22 per cent.

A very interesting feature, as shown by the curves of Figs. 11 and 12, is the critical point that occurs with about 2.60 per cent silicon, the characteristic of the alloys in this region being their comparative brittleness. This point was first noticed by the fact that two ingots, containing 2.55 and 2.57 per cent silicon respectively, were not forgeable, but fell into a mass of crystals that apparently had no adhesive strength. One of these alloys is shown in Fig. 30. That this occurrence was not accidental is shown by the fact that the two alloys were made at different times, and they were subjected to forging on different days, in company with other alloys that forged perfectly. The



[YENSEN]

Fig. 13. Some of the Mechanical Test Pieces after being Tested.

structure of the two alloys is identical, consisting of large allotriomorphic crystals ⅛ in. (3 mm.) to ¼ in. (6 mm.) across. Since his first report* upon the iron-silicon alloys, where this critical point was mentioned, the writer has obtained additional experimental data in the region in which this point occurs. These data are included in the present report, making it possible to draw definite curves through the critical range. The critical point appears in all the characteristic curves for the mechanical properties, both for the unannealed and for the annealed alloys. For the unannealed specimens there is a sudden drop in the reduction of area and ultimate elongation at 2.25 per cent silicon, the curves reaching their minima at about 2.56 per cent, which is the silicon content of the two non-forgeable alloys. With increasing silicon contents these curves again rise as suddenly as they previously dropped, attaining about the same values as before the critical point was reached. At 3.50 per cent the second and final drop begins for these curves, reaching zero at about 5 per cent silicon.

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*A. I. E. E. Proceedings, Vol. 34, p. 2455, Oct., 1915.

For the annealed specimens the critical point is even more marked, but the reduction of area and the elongation do not recover as completely after the critical point is passed as was the case with the unannealed specimens. The critical point for the annealed alloys occurs with a silicon content of about 2.65 per cent, slightly higher than for the unannealed alloys. The writer knows of no satisfactory explanation of this critical point, neither has he been able to find any such phenomenon reported by anybody else. From Figs. 1 and 2 it is seen, however, that both Hadfield and Paglianti, and to some extent Baker, indicate irregularities in some of their curves for a silicon content between 2 and 3 per cent. None of the above investigators, however, and, as far as the writer has been able to find out, no other investigator has recorded in the literature the properties of an iron-silicon alloy containing between 2.50 and 2.67 per cent silicon.

As critical points are usually associated with the formation of definite compounds of the elements present, it may be of some interest to note that a compound of the formula $Fe_{19}Si$ would contain 2.56 per cent silicon, and similarly that a compound of the formula $Fe_{19}Si_2$ would contain 4.99 per cent silicon. It was stated above that the first critical point in the present case occurs with a silicon content of 2.56 per cent, and also that there is a sudden change at about 5 per cent silicon. Whether this agreement is a mere coincidence, or whether these compounds, or others, actually exist, the writer is not prepared to say, as conclusive evidence, in the way of cooling curves for these particular alloys, are not available.

17. *Magnetic and Electrical Properties.*—The results of the magnetic and electrical tests are shown in Tables 9 and 10, and in Figs. 14 to 18 incl. Figs. 14 and 15 give at a glance the magnetic and electrical properties of the series, Fig. 14 after annealing at 900°C, and Fig. 15 after annealing at 1100°C. The properties in the forged state have not been thus plotted for the reason that they are of less

interest on account of their inferiority. However, in order to have them included in the report, Fig. 16 gives the flux densities for various magnetizing forces for the alloys as forged; Figs. 17 and 18 give the corresponding values after annealing at 900°C and 1100°C respectively, thus affording means for comparison as to the effect of the various heat treatments. From these three figures it is seen that for $H = 50$ or above the annealing at 1100° has the effect of decreasing B by 500 to 1000 gausses for low silicon alloys, while for $H = 20$ and below the forged condition is decidely inferior to the annealed conditions. Comparing the 900° and 1100° annealing it is seen that for $H = 20$ to $H = 0.5$ the 900° annealing is superior for decreasingly lower silicon contents only, and for $H = 0.3$ and below the 1100° annealing is superior for the whole range. In general, it may be said that for high magnetizing forces B decreases with increasing silicon content, no matter what the heat treatment has been. This was expected to be the case from previous researches on the saturation values of iron-silicon alloys. Thus Gumlich and Goerens found that the saturation value $4 \pi I_{max}$ is decreased by 500 gausses for each 1 per cent silicon.

### TABLE 9.

### RESULTS OF MAGNETIC AND ELECTRICAL TESTS
### RODS ANNEALED AT 900°C.

| Rod No. | Silicon Content, % | Maximum Permeability | Density for Max. Permeability gausses | Permeability for B = 10000 | Hysteresis Loss, ergs per cc. per cycle | | Retentivity gausses | | Coercive Force gilberts per cm | | Spec. Elec. Res. at 20° C.-microhms | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | for $B_{max} = 10000$ | for $B_{max} = 15000$ | for $B_{max} = 10000$ | for $B_{max} = 15000$ | for $B_{max} = 10000$ | for $B_{max} = 15000$ | | |
| 3–54 | .001 | 23100 | 8500 | 21800 | 764 | 1610 | 9400 | 14200 | .25 | .30 | 9.85 | |
| 3–55 | .001 | 22500 | 11000 | 21300 | 875 | 1790 | 9100 | 13800 | .29 | .36 | 9.82 | |
| 3Si16 | .010 | 25000 | 10000 | 25000 | 795 | 1770 | 9480 | 14600 | .28 | .35 | 9.89 | |
| 3Si15 | .064 | 22800 | 10000 | 21700 | 782 | 1738 | 9100 | 14500 | .26 | .35 | 10.65 | |
| 3Si05 | .068 | 37500 | 9000 | 36300 | 405 | 1210 | 9480 | 14520 | .12 | .23 | 10.75 | |
| 3Si06 | .148 | 47000 | 8000 | 42500 | 396 | 965 | 9300 | 14100 | .12 | .19 | 11.8 | |
| 3Si17 | .230 | 30000 | 6000 | 26300 | 496 | 1311 | 9000 | 13600 | .15 | .23 | 11.60 | |
| 3Si10 | .472 | 14000 | 7000 | 12700 | 960 | 1863 | 8900 | 11700 | .30 | .33 | 16.2 | Rod Strained in Testing |
| 3Si14 | .822 | 13500 | 9000 | 13300 | 1215 | 2432 | 9100 | 12400 | .42 | .53 | 21.3 | Rod Strained in Testing |
| 3Si31 | 1.71 | 18000 | 7000 | 15900 | 800 | 1541 | 8400 | 11540 | .26 | .32 | 33.2 | |
| 3Si18 | 1.740 | 14300 | 8000 | 14100 | 935 | 2162 | 9000 | 12600 | .29 | .42 | 31.2 | |
| 3Si27 | 2.73 | 16800 | 6000 | 13300 | 821 | 1779 | 8270 | 10500 | .27 | .34 | 41.8 | |
| 3Si25 | 3.40 | 20000 | 6000 | 15900 | 560 | 1390 | 7900 | 9570 | .20 | .23 | 48.5 | |
| 3Si36 | 3.55 | 14000 | 4500 | 8850 | 802.5 | 1812 | 6900 | 8100 | .28 | .34 | 51.5 | Rods Contaminated by Impure Rods. |
| 3Si37 | 4.39 | 12750 | 4500 | 7500 | 846.0 | 1956 | 6600 | 7700 | .29 | .31 | 58.8 | |
| 3Si28 | 4.44 | 16100 | 4800 | 10100 | 623 | 1575 | 7000 | 8370 | 16 | .24 | 57.7 | |
| 3Si29 | 4.92 | 9100 | 4500 | 5330 | 776 | 2006 | 4500 | 5300 | .27 | .36 | 66.5 | |

## TABLE 10.

## RESULTS OF MAGNETIC AND ELECTRICAL TESTS RODS ANNEALED AT 1100°C.

| Rod No. | Silicon Content, % | Maximum Permeability | Density for Max. Permeability gausses | Permeability for B = 10000 | Hysteresis Loss, ergs per cc. per cycle | | Retentivity gausses | | Coercive Force gilberts per cm. | | Spec. Elec. Resist. at 20° C.-microhms | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | for $B_{max} = 10000$ | for $B_{max} = 15000$ | for $B_{max} = 10000$ | for $B_{max} = 15000$ | for $B_{max} = 10000$ | for $B_{max} = 15000$ | | |
| 3–54 | .001 | 22800 | 8000 | 21300 | 665 | 1860 | 9300 | 13300 | .20 | .24 | 9.84 | |
| 3–55 | .001 | 25800 | 9000 | 25600 | 707 | 1451 | 9300 | 12700 | .23 | .28 | 9.85 | |
| 3Si16 | .01 | 29000 | 9000 | 28670 | 707 | 1604 | 9600 | 14300 | .21 | .31 | 9.90 | |
| 3Si21 | .040 | 27000 | 10000 | 27000 | 700 | 1660 | 9440 | 14480 | .23 | .32 | 10.50 | |
| 3Si15 | .064 | 36800 | 9000 | 36300 | 502 | 1336 | 9500 | 14300 | .16 | .25 | 10.67 | |
| 3Si05 | .068 | 44200 | 9000 | 43500 | 407 | 1214 | 9480 | 14200 | .13 | .22 | 10.78 | |
| 3Si22 | .091 | 45250 | 9000 | 43500 | 394 | 929 | 9500 | 14300 | .13 | .17 | 10.96 | |
| 3Si06 | .148 | 66500 | 6500 | 41700 | 286 | 916 | 9080 | 12000 | .09 | .16 | 11.80 | |
| 3Si23 | .205 | 30200 | 9000 | 29500 | 649 | 1526 | 9300 | 14480 | .20 | .27 | 12.50 | |
| 3Si17 | | | | | | | | | | | | Contaminated by impure Rods |
| 3Si07 | .242 | 36500 | 7500 | 33000 | 436 | 1346 | 9700 | 14500 | .13 | .21 | 13.40 | |
| 3Si08 | .309 | 44800 | 9000 | 43500 | 445 | 1412 | 9600 | 14500 | .13 | .24 | 14.40 | |
| 3Si09 | .400 | 22500 | 9000 | 22000 | 725 | 1820 | 9440 | 14480 | .21 | .32 | 15.30 | |
| 3Si10 | .472 | 31150 | 6200 | 25000 | 535 | 1358 | 9300 | 14200 | .16 | .21 | 16.57 | |
| 3Si11 | .563 | 25000 | 9000 | 25000 | 601 | 1624 | 9200 | 14320 | .20 | .28 | 17.50 | |
| 3Si12 | .673 | 28000 | 7000 | 24500 | 468 | 1636 | 9200 | 13670 | .13 | .23 | 19.10 | |
| 3Si13 | .698 | 20350 | 8000 | 19600 | 780 | 2220 | 9300 | 14400 | .25 | .40 | 19.60 | |
| 3Si14 | .822 | 30800 | 9500 | 30300 | 542 | 1765 | 9200 | 14100 | .18 | .35 | 21.25 | |
| 3Si31 | 1.71 | 30150 | 6500 | 24700 | 440 | 1292 | 8700 | 12000 | .12 | .22 | 33.25 | |
| 3Si18 | 1.740 | 33000 | 7000 | 26300 | 416 | 1112 | 9200 | 12600 | .13 | .19 | 31.00 | |
| 3Si27 | 2.73 | 46800 | 9500 | 46000 | 404 | 1260 | 9100 | 13300 | .13 | .23 | 42.00 | |
| 3Si25 | 3.40 | 63300 | 6500 | 46500 | 280 | 1025 | 9100 | 12400 | .08 | .15 | 48.50 | |
| 3Si36 | 3.55 | 36000 | 7500 | 29500 | 419 | 1157 | 8920 | 12000 | .13 | .21 | 48.50 | Rods annealed in Nitrogen |
| 3Si37 | 4.39 | 25700 | 6000 | 15400 | 591 | 1819 | 8300 | 10200 | .20 | .25 | 56.10 | |
| 3Si28 | 4.44 | 30200 | 3000 | 15900 | 405 | 1171 | 7000 | 8000 | .12 | .15 | 57.40 | |
| 3Si29 | 4.92 | 12200 | 5000 | 7040 | 780 | 2620 | 6300 | 7100 | .26 | .35 | 66.20 | |

Turning now to Figs. 14 and 15 two maxima appear very distinctly in the curves for maximum permeability, corresponding to two minima in the curves for hysteresis loss and coercive force. The first of these points occurs at a silicon content of 0.15 per cent, the second at a silicon content of about 3.5 per cent for the permeability and about 4.0 per cent for the hysteresis and coercive force. The reason that the second maximum for permeability and minimum for hysteresis loss do not occur at the same silicon content appears very clearly from the retentivity and coercive force curves, when it is remembered that the hysteresis loss primarily depends upon the coercive force and the retentivity and only to a less extent upon the maximum permeability.

That a maximum—or minimum—should occur for a low silicon content was not surprising in view of the results previously obtained

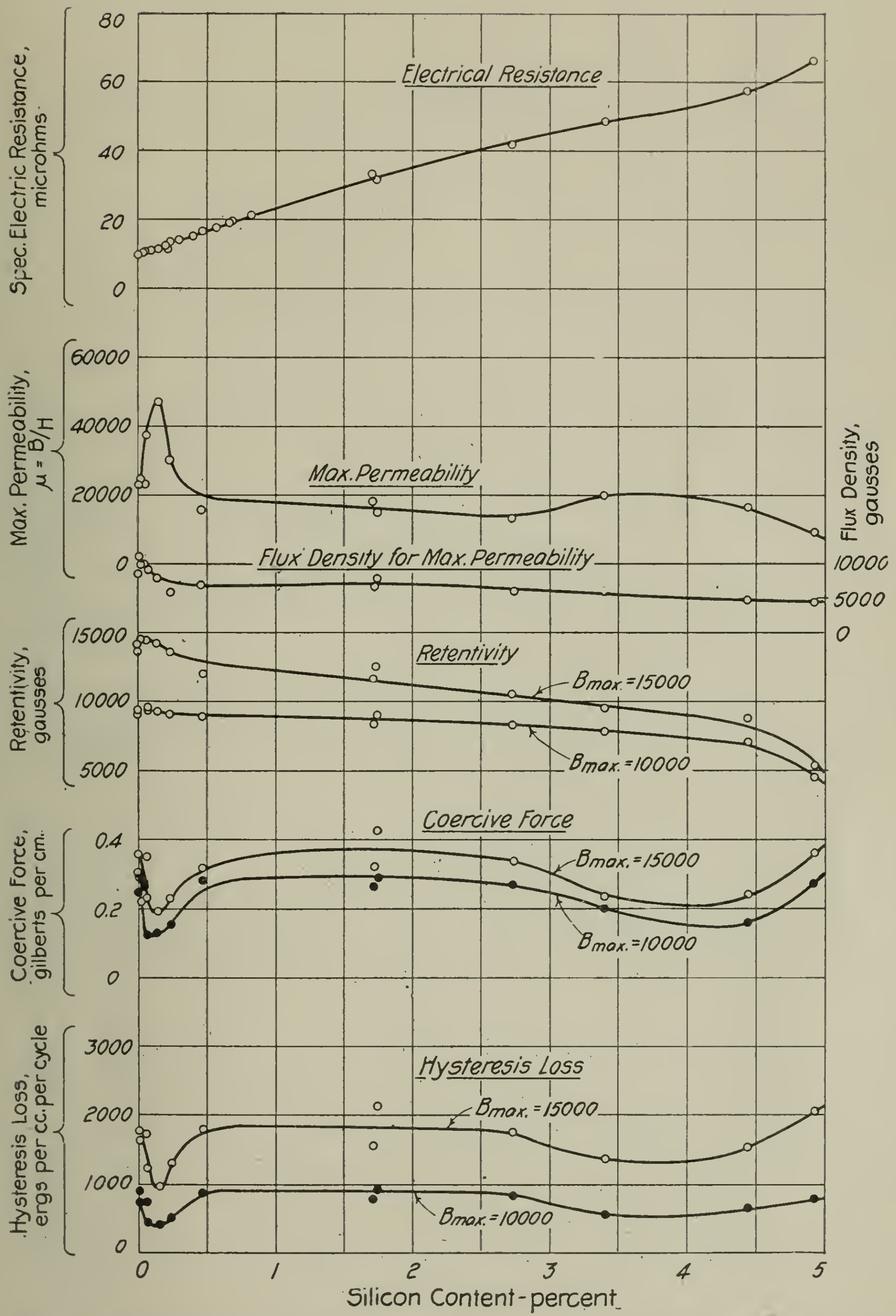

FIG. 14. MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-SILICON ALLOYS MELTED IN VACUO. ANNEALED AT 900° C.

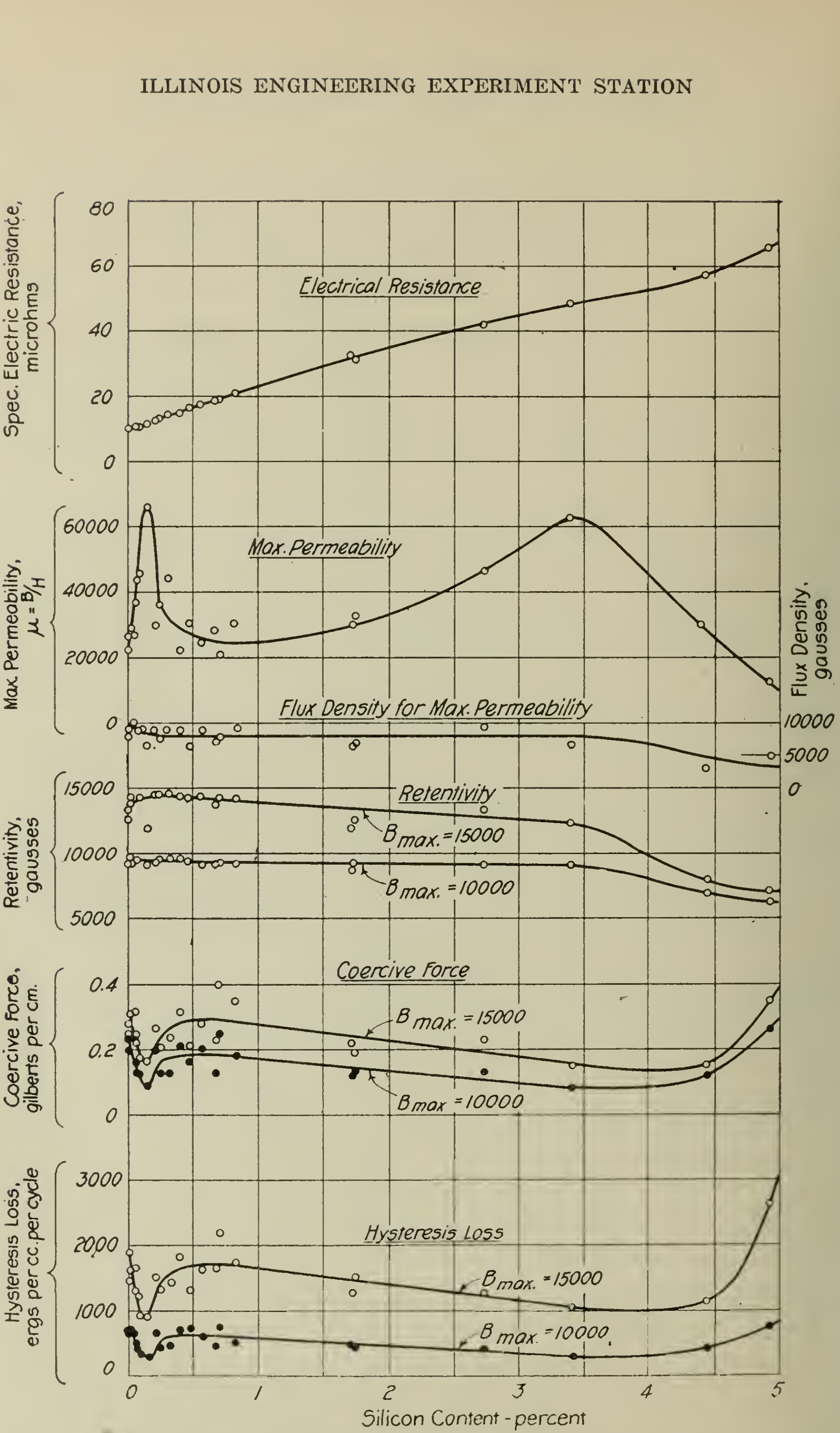

FIG. 15. MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-SILICON ALLOYS MELTED IN VACUO. ANNEALED AT 1100° C.

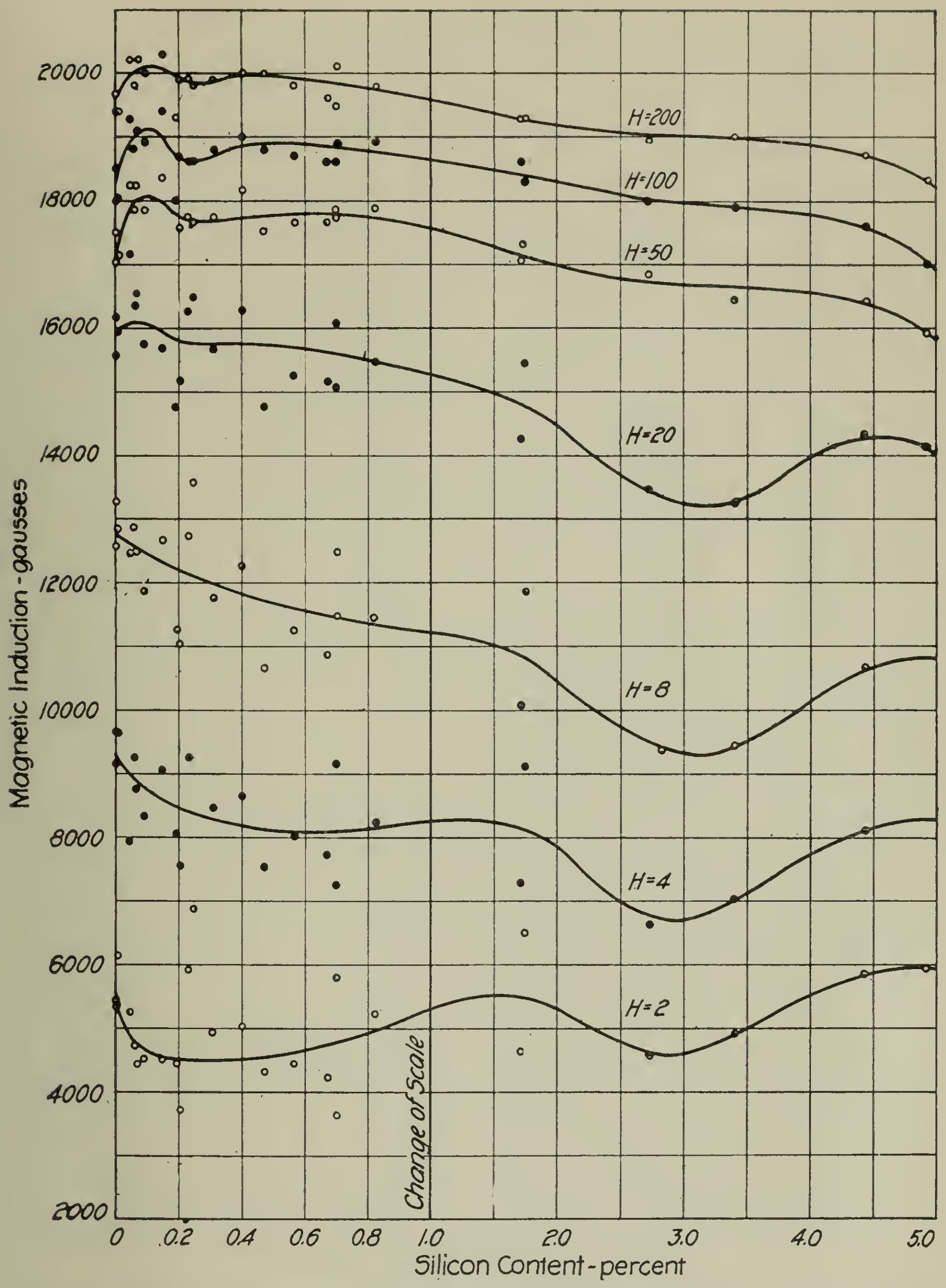

FIG. 16. FLUX DENSITY FOR VARIOUS MAGNETIZING FORCES. AS FORGED.

with pure iron, iron-carbon, and iron-boron alloys. In the latter case a maximum was obtained with a trace of boron, evidently on account of a slight purification of the iron, but as soon as the boron content became measurable the magnetic properties immediately depreciated. The first maximum in the present case can, no doubt, be accounted for in the same way, and consequently this point may be regarded as

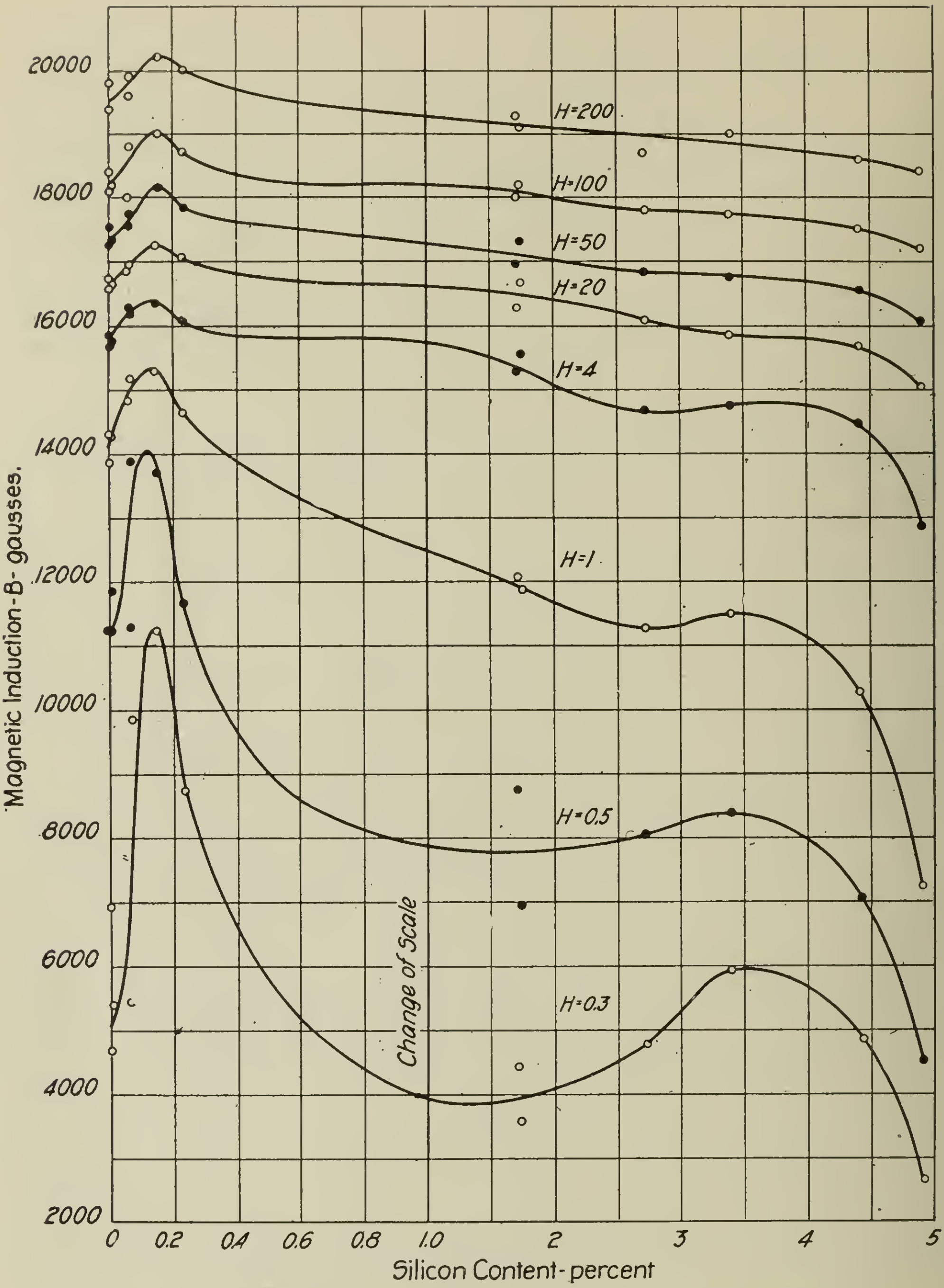

Fig. 17. Flux Density for Various Magnetizing Forces. Annealed at 900° C

characteristic of the purest iron obtainable under the present conditions, containing 0.15 per cent silicon and a small amount of oxygen in the form of iron oxide. The slight uncertainty existing for silicon contents below 1.0 per cent, as shown by the distribution of the points, is probably due to the varying amounts of oxide left in the iron.

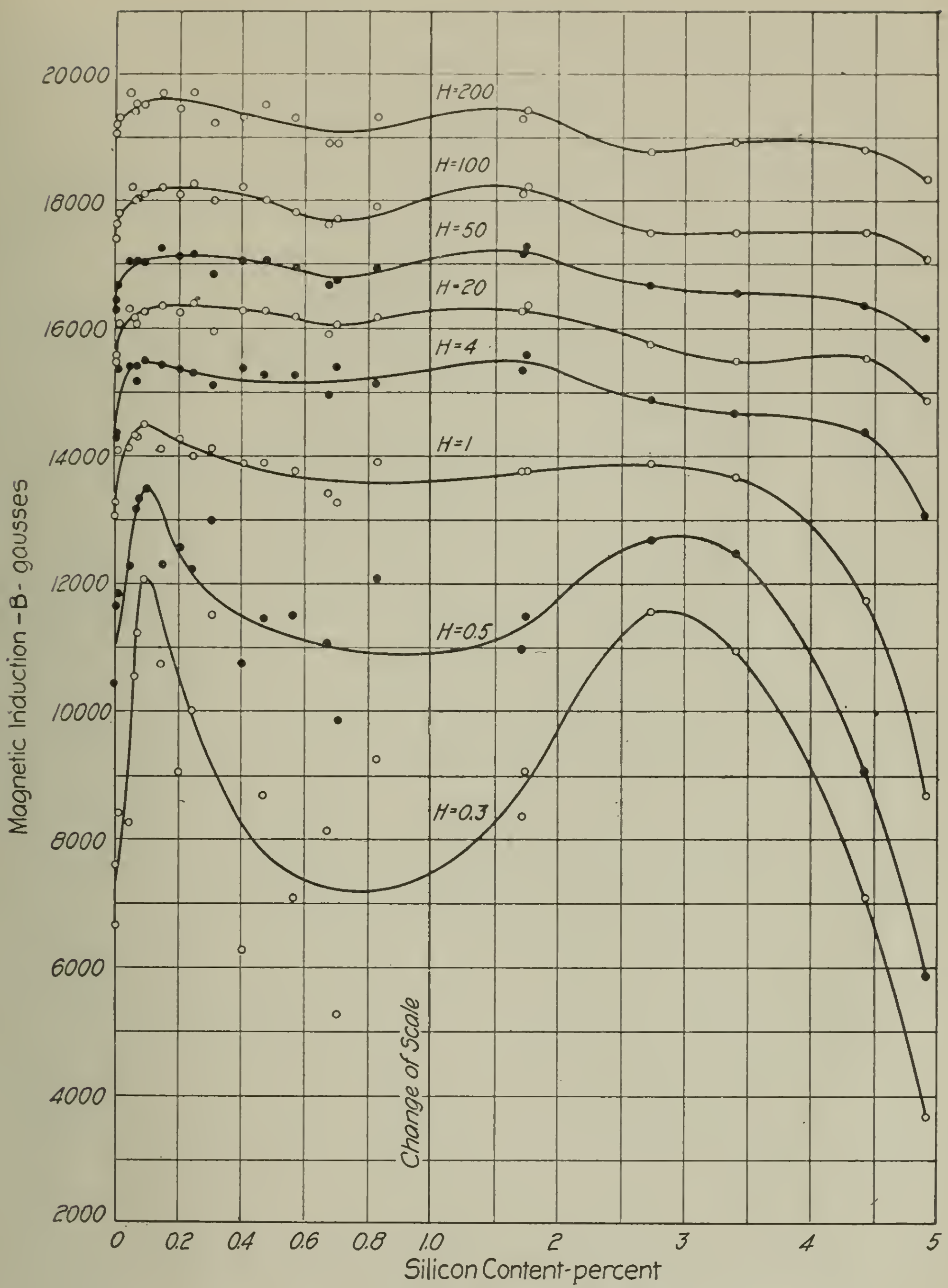

FIG. 18. FLUX DENSITIES FOR VARIOUS MAGNETIZING FORCES. ANNEALED AT 1100° C.

That the oxide in the iron before melting varied to some extent is apparent from Table 6, judging from the amount of silicon oxydized in the high alloys, where probably all the oxide was reduced. This reasoning would consequently also account for the uniform results obtained for the high silicon contents.

The second maximum—or minimum—was wholly unexpected as strength and brittleness are not generally associated with high magnetic quality. It is true that previous investigators have found a maximum between 2.5 to 4.0 per cent silicon, but as was pointed out on pp. 12 and 16 this was thought to be due to the neutralizing effect of the silicon upon the relatively large amounts of impurities, chiefly carbon, present in the iron. In the present case the alloys contained only about 0.01 per cent of carbon, an amount considered too low to be of any consequence, and only minute traces of other impurities were present. Thus it seems improbable that the second maximum in this case can be attributed solely to the conversion of 0.01 per cent of combined carbon into graphite. It seems more probable that the improvements are due partly to this conversion and partly to the complete reduction of iron oxide. If this second maximum should be due entirely to the conversion of combined carbon into graphite, the current ideas regarding the influence of carbon upon the magnetic properties of iron will certainly have to be changed, and it will become desirable to remove from the iron the last trace of carbon. According to these hypotheses, the first maximum is due to pure iron in spite of small amounts of iron oxide and combined carbon, while the second maximum is due to pure iron in spite of a relatively large amount of dissolved silicon. If *none* of the above hypotheses is correct, the only other explanation remaining is that the second maximum is due directly to silicon dissolved in the iron. As an argument against such a theory Hadfield and Hopkinson* in 1910 and Gumlich† in 1912 brought out the fact that silicon reduces the saturation value of iron in direct proportion to the silicon dissolved in the iron, and consequently it did not seem probable that silicon could directly improve the permeability at lower densities. However, it is a curious coincidence that at the same meeting of the Faraday Society at which Dr. Gumlich made the above statement, Dr. P. Weiss‡ read a paper on iron-cobalt alloys, showing that the alloy $Fe_2Co$ has a saturation value 10 per cent higher than that of pure iron. The writer¶ in cooperation with Dr. E. H. Williams§ has recently shown that while the iron-cobalt alloy, $Fe_2Co$, melted in vacuo has a saturation value 13 per cent higher than that of pure iron melted under identical conditions, its permeability at low densities is much lower. Evidently both the high saturation value and the comparatively low permeability at lower densities of the $Fe_2Co$ alloy must be due to the combination between iron and cobalt, or, in other words, must be attributed directly to the cobalt. There is no reason, then, why the low saturation value and the high permeability at low densities of the 3.40 per cent iron-silicon alloy can not both be due directly to silicon. That is, no foundation exists any longer for assuming that there is a direct connection between the saturation value of a certain alloy and its properties at low and medium densities, and it is consequently possible that the second maximum occuring in the maximum permeability curve for the iron-silicon series may be due directly to silicon. More experimental evidence is needed, however, before it is safe to make a definite statement in this respect.

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*Inst. of Elect. Engrs. Vol. 46, p. 225, 1911.

†Trans. Faraday Soc. Vol. 8, p. 109, 1912.

‡Trans. Faraday Soc. Vol. 8, p. 149, 1912.

¶Gen. Elect. Rev. Oct., 1915, Vol. 18, p. 881, Sept., 1915.

§Phys. Rev. N.S. Vol. 6, p. 404, Nov. 1915.

While silicon is thus, directly or indirectly, engaged in improving the magnetic properties of iron, it serves a very useful purpose by increasing the electrical resistance of the iron enormously, giving the iron the exact characteristics desired for electromagnetic machinery.

The iron-silicon series thus offers two important alloys for electrical purposes, both having high permeability and low hysteresis loss, but differing in that one has a very low, while the other has a very high electrical resistance.

The values obtained are, undoubtedly, without precedent in the annals of the magnetic properties of iron and iron alloys, and it is only after a careful analysis of the apparatus used and the methods employed in testing that the writer feels justified in publishing them. As an extra precaution, however, he wishes to repeat the statement made on p. 27 that experimental evidence seems to point towards a larger percentage error due to the compensating current than theoretical considerations according to Burrows and others would lead to. But even if the maximum error in the results as given should amount to 20 per cent, their significance would not be altered appreciably. Whether the true maximum permeability attained is 66,500 or 53,200 is of little consequence at the present time, as long as it is reasonably certain that it lies in this neighborhood. Should it be definitely established that the value of the required compensating current imposes a limit upon the Burrows double bar and yoke method, beyond which the errors introduced are too large, a new method will undoubtedly be developed to meet this exigency.

It is only a few years ago that a permeability of 6000 was regarded as exceptionally high. This was gradually raised to 8000 and then Terry,* in 1910, obtained a value of 11000 for a ring of electrolytic iron as deposited and annealed. Gumlich, in 1912, also obtained this value for a high grade low silicon alloy in the form of sheets. Last year the writer published 19000 as the maximum found for pure iron melted in vacuo, and this value was regarded as remarkable. The step from 19000 to 66500 indeed seems a long one, but intermediate steps have been taken in the meantime in the laboratory. The values for the hysteresis loss have followed in a similar path, as seen from Table 11.

### TABLE 11.

### PROGRESS MADE IN RECENT YEARS.

| Year | Investigator | Kind of Material used | Maximum Permeability | Coercive Force gilberts per cm. | | Hysteresis Loss ergs per cc per cycle | |
|---|---|---|---|---|---|---|---|
| | | | | for $B_{max}$ = 1000 | for $B_{max}$ = 15000 | for $B_{max}$ = 10000 | for $B_{max}$ = 15000 |
| 1900 | Hadfield | SW. Char. Iron | 4000 | 0.920 | 1.00 | abt. 2700 | abt. 5500 |
| 1900 | Hadfield | 2½% Si–Iron | 5100 | 0.72 | 0.79 | ” 2200 | ” 4700 |
| 1901 | Gumlich & Schmidt | Wrought Iron | 8350 | | 0.60 | | |
| 1903 | Baker | 4.9% Si–Iron | | | 1.20 | | ” 6200 |
| 1910 | Terry | Electrolytic Iron | 11000 | | | | |
| 1912 | Gumlich & Goerens | 0.4% Si. Sheets | 11600 | | 0.54 | | |
| 1912 | Gumlich & Goerens | 4.0% Si. Sheets | 9400 | | | | |
| 1912 | Paglianti | 1.75% Si–Iron | | 0.60 | 0.75 | 1650 | ” 3500 |
| 1914 | Yensen | Pure Vacuum Iron | 19000 | | 0.29 | 813 | 1640 |
| 1915 | Yensen | 0.15% Si. ” Iron | 66500 | 0.09 | 0.16 | 286 | 916 |
| 1915 | Yensen | 3.40% Si. ” Iron | 63300 | 0.08 | 0.15 | 280 | 1025 |

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*Phys. Rev. Vol. 30, p. 133, 1910.

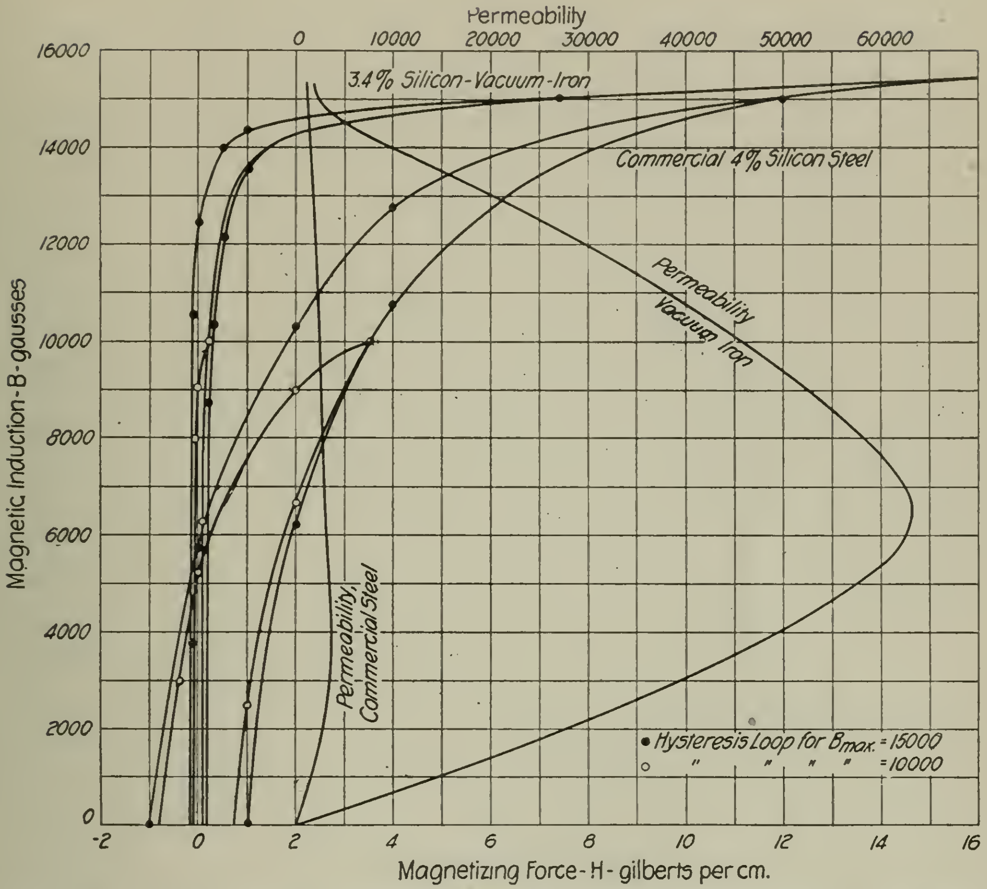

Fig. 19. Comparison between Vacuum Iron and Commercial Steel, both containing between 3 per cent and 4 per cent Silicon—Thoroughly Annealed.

| | Vacuum Iron | Commercial Steel |
|---|---|---|
| Hysteresis loss, for $B_{max}$ = 10,000; ergs per cu. cm. per cycle. . . . | 280 | 2160 |
| ,, ,, ,, ,, = 15,000; ,, ,, ,, ,, ,, ,, . . . . . . | 1025 | 4290 |
| Specific electrical resistance, microhms. . . . . . . . . . . . . . . . . . . . . . . . . . | 48.50 | 51.15 |

Fig. 19 illustrates the difference in magnetic quality between a silicon-vacuum-iron and a commercial silicon steel, both containing approximately the same amount of silicon. The maximum permeability is as 20 to 1, the hysteresis loss for $B_{max} = 10000$ as 10 to 1, and for $B_{max} = 15000$ as 3 to 1 in favor of the vacuum alloy. For $B = 15000$ the permeability of the latter is twice the permeability of the commercial iron. The electrical resistance is nearly the same for both alloys, or about 5 times that of pure iron.

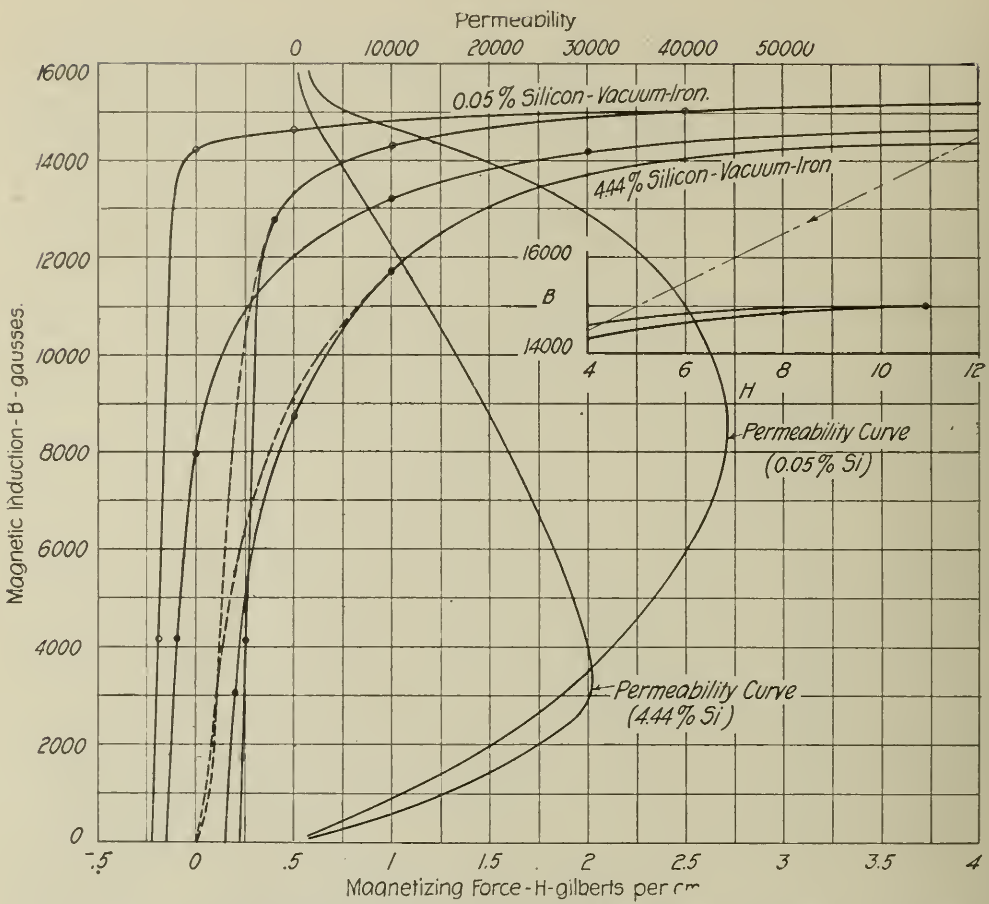

FIG. 20. COMPARISON BETWEEN A HIGH AND LOW SILICON VACUUM IRON—ANNEALED AT 1100°C.

| | 0.048% Si. | 4.44% Si. |
|---|---|---|
| Hysteresis loss, for $B_{max}$ = 10,000; ergs per cu. cm. per cycle.... | 407 | 405 |
| " " " " " = 15,000; " " " " " " ...... | 1214.5 | 1171 |
| Specific electrical resistance, microhms........................ | 10.78 | 57.40 |

Fig. 20 shows to a large scale the difference between a high and a low silicon-vacuum-iron, having about the same hysteresis loss. The characteristics of the low silicon alloy are seen to be: very high retentivity and high permeability for medium and high densities, while the high silicon alloys have a low retentivity, and the maximum permeability occuring at a low density. The chief advantage of the high silicon alloy lies in the high electrical resistance amounting to more than 5 times that of the low silicon alloy.

An attempt was made to obtain a few more points for the high silicon alloys, and two alloys, Nos. 3Si36 and 3Si37 were melted and testpieces prepared. Unfortunately, however, these rods were annealed first at 900° and then at 1100°, in company with three rods made from impure iron and were evidently contaminated by them.

This was discovered by the behavior of rod No. 3Si17 that had previously been annealed at 900° under normal conditions. After annealing at 1100° in company with the contaminated rods, its magnetic quality was sadly impaired, while the five other rods all had improved. As a matter of fact, it was the first occurence of a rod depreciating during the 1100° annealing, and the only cause that could be found was contamination by the impure rods. One rod being thus affected it seemed natural to conclude that Nos. 3Si36 and 3Si37 were similarly affected.* The results for these two alloys are shown in the tables, but the values have not been plotted in the figures, although the points would have deviated but little from the curves. Another reason for not plotting these values is that the 1100° annealing was done in an atmosphere of nitrogen. While the nitrogen did not seem to act differently from the vacuum, the method has not been tested sufficiently to warrant including the results thus obtained without question. However, these results, although not wholly satisfactory, serve as a check upon the results previously obtained.†

18. *Photomicrographs.*—In the following pages a large number of photomicrographs are reproduced representative of the alloys tested. They are arranged in order of their silicon content, the pure iron appearing first and the highest silicon alloy last. With only a few exceptions the ''As forged'' condition occupies the upper part of the pages, the 900° annealed condition the middle part, and the 1100° annealed condition occupies the lower part. The magnifications used are either 40 diameters or 10 diameters. In one case recourse was had to 7 diameters. For the high silicon alloys both the 40 and the 10 diameter magnifications are shown in most cases. The lower magnifications give a general view of the structures, as they cover about ⅛ of the surface of the specimens, while the higher magnifications give more detail. Even with the information thus at hand perfectly definite conclusions can not be made with regard to each alloy taken separately, as the specimen may not in every case be strictly representative of the particular alloy from which it was taken. Furthermore, in comparing the photomicrographs after the different heat treatments, it should be remembered, that the specimens had to be repolished after each treatment, and the photomicrographs do not, therefore, represent the same spot in each case. However, general conclusions can be drawn by considering the series as a whole.

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*That such contamination takes place during annealing was shown in Bulletin No. 72, in the case of five rods that were annealed in company with a rod that contained 0.18 per cent carbon.

†In the tests comparing the Burrows method and the ring method a rod was obtained, containing 3 per cent silicon, that according to the Burrows method of testing has a maximum permeability of 72000. (For details see Appendix.)

From these photomicrographs it is seen very clearly that the iron-silicon alloys, in the range here investigated, consist of only one kind of crystals, thus confirming the results obtained by previous investigators that iron and silicon, for silicon contents below about 15 per cent, form a solid solution everywhere between the freezing point and ordinary temperature.

Below 1 per cent silicon appears to have no marked effect upon the structure of the iron, either as forged or annealed. Annealing at 900°C does not seem to change the structure of the alloys in this range, but annealing at 1100° produces numerous small crystals inside the larger ones, giving an appearance of a very fine structure.*

There is no sign of foreign substances in these structures, other than those arising from imperfect polishing. After passing the 1 per cent mark silicon begins to show its effect. The crystals are generally larger than for the pure iron, and are readily polished in relief, showing that they are not of uniform hardness. There is no breaking up of the crystals by the 1100° annealing as in the case of the low alloys. The 1.71 per cent and the 2.73 per cent alloys, as seen in Figs. 29 and 31, show very irregular structures as forged, and after the 900° annealing, but these give way to structures of more regularity by the 1100° annealing. In the region between the last two alloys is found the nonforgeable alloys, Nos. 3Si19 and 3Si24, containing 2.55 to 2.57 per cent silicon respectively. The structure of these alloys is shown in Fig. 30, exhibiting very large, uniform crystals, measuring ⅛″ (3 mm.) to ¼″ (6 mm.) across. In the first specimen used for the 3.40 per cent alloy the crystals are of about the same size and shape as the ones for the nonforgeable alloys, as seen in Fig. 35. In order to investigate this singularity further, another specimen from the same alloy was prepared, and this showed a much more normal structure (Fig. 32). Specimens were also investigated for two other alloys with very nearly the same silicon content, and these also showed structures that were quite normal, (Figs. 33 and 34) so that the large crystals, shown in Fig. 35, are evidently freaks, caused by peculiar conditions. It may be that the specimen was taken at one end of the forged rod. Nevertheless, the occurence of these enormous crystals is very interesting, not only on account of the enormous size, but also because it shows that the size of the crystals in and of itself does not prevent the material from being forgeable. The remainder of the photomicrographs exhibit quite normal structures with no marked change caused by the two heat treatments.

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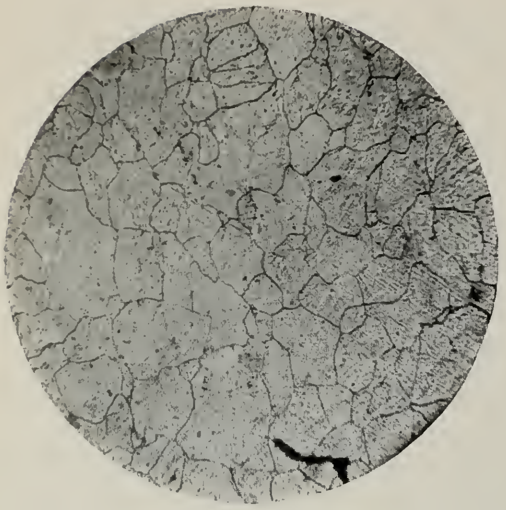
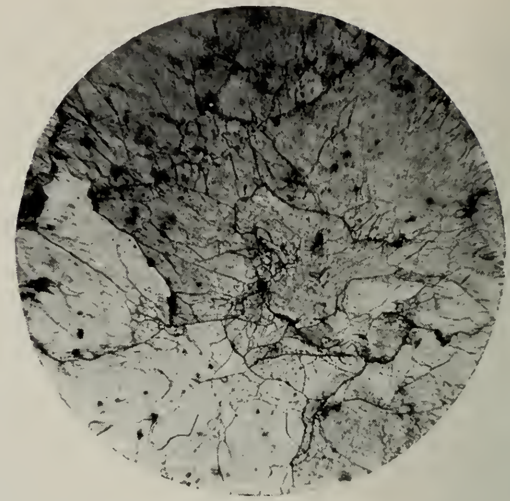
*This phenomenon is explained by Stead and Carpenter, Journ. Iron & Steel Inst. Sept., 1913, as follows: Upon heating the iron above the $Ac_3$ point (900° C for pure iron) and holding it in the region of the gamma modification sufficiently long, both the alpha crystals and their nuclei will be destroyed, giving place to gamma crystals. If now the iron is cooled, numerous alpha nuclei will be formed simultaneously on passing through $Ar_3$ and the resulting crystals are consequently small.

See also Storey: AM. Electroclem Soc. Apr. 18, 1914.

The principle difference between these photomicrographs and the ones published by previous investigators is the absence of foreign matter in the structure of the vacuum alloys. Even Baker, whose alloys contained only 0.04 per cent carbon, shows in his photomicrographs besides small amounts of pearlite or graphite, some other foreign substance that could not be explained at the time. It seems probable, in view of what has been said earlier in this report regarding the method used for melting and the mechanical properties of his alloys, that these other foreign substances may be oxides. The other investigators invariably show comparatively large amounts of pearlite for low alloys and patches or spots of graphite for high alloys. The size of crystals for the vacuum alloys, excluding the abnormal cases, is very much larger than for the less pure alloys. This is true both for low and high silicon contents.

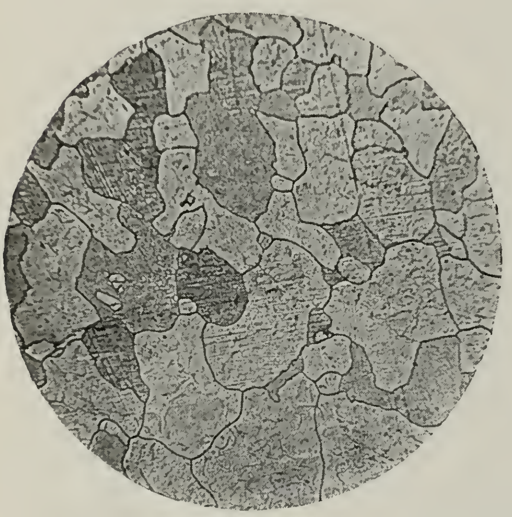


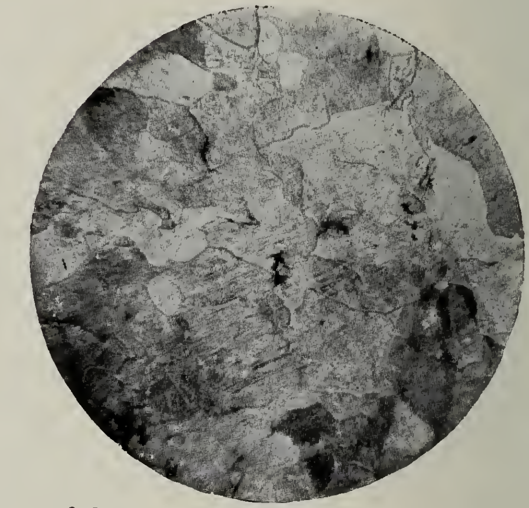


As Forged.

Fig. 21a. 40 Diam. Picric Acid.

Fig. 22a. 40 Diam. Picric Acid.

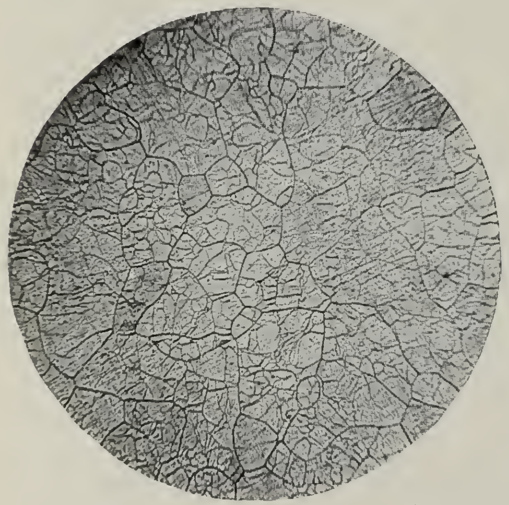


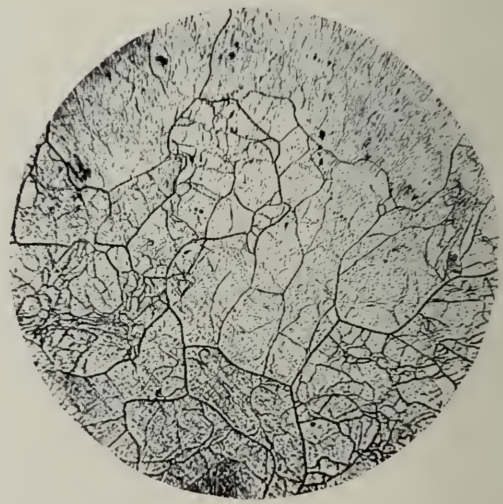


Annealed at 900° C.

Fig. 21b. 40 Diam. Picric Acid.

Fig. 22b. 40 Diam. Picric Acid.





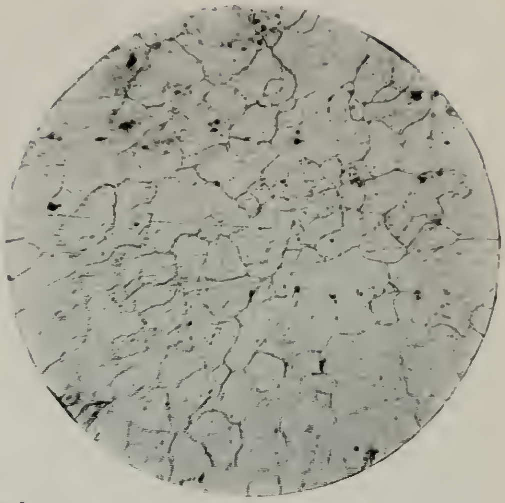
Annealed at 1100° C.

Fig. 21c. 40 Diam. Picric Acid.

Fig. 22c. 40 Diam. Picric Acid.

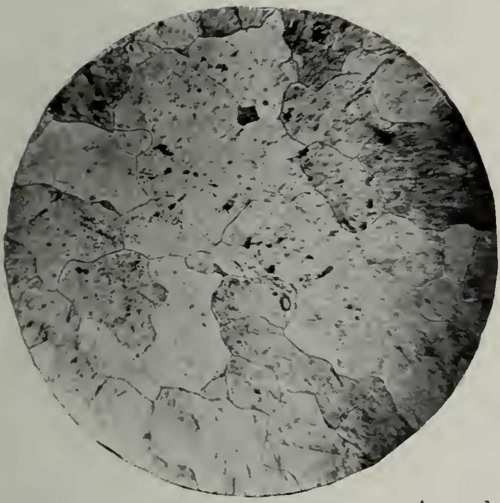
Alloy No. 3—39. 0.001% Si.

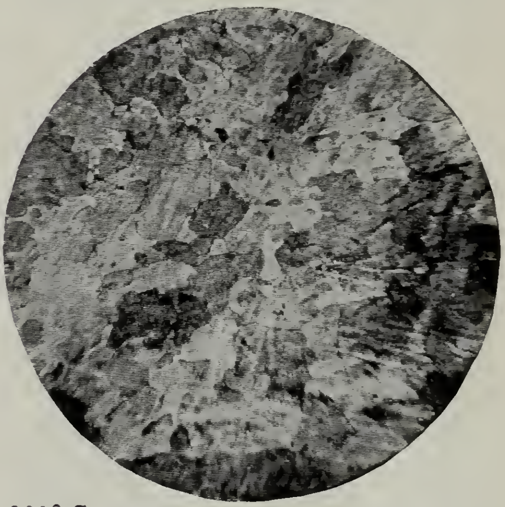
Alloy No. 3—54. 0.001% Si.



As Forged.

Fig. 24a. 40 Diam. Picric Acid.

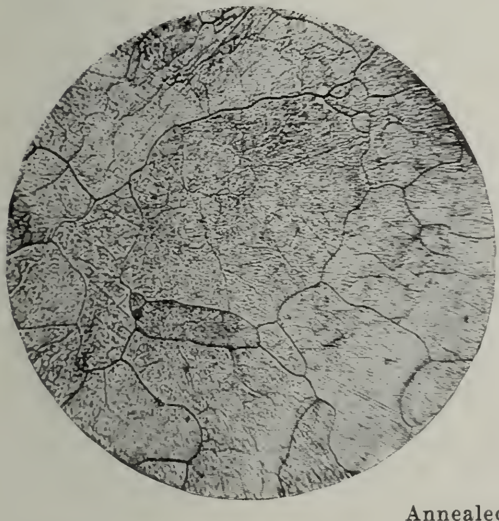


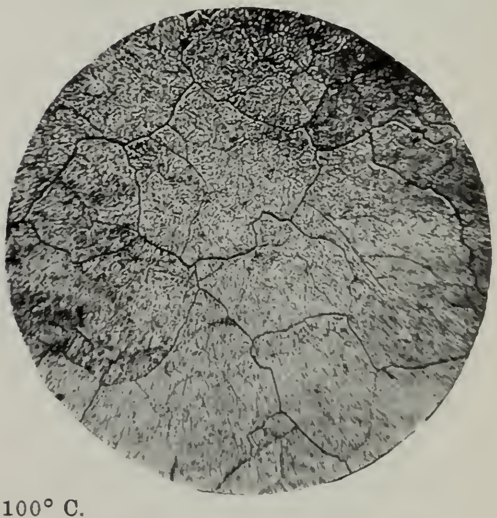


Annealed at 900° C.

Fig. 23b. 40 Diam. Picric Acid.

Fig. 24b. 40 Diam. Picric Acid.





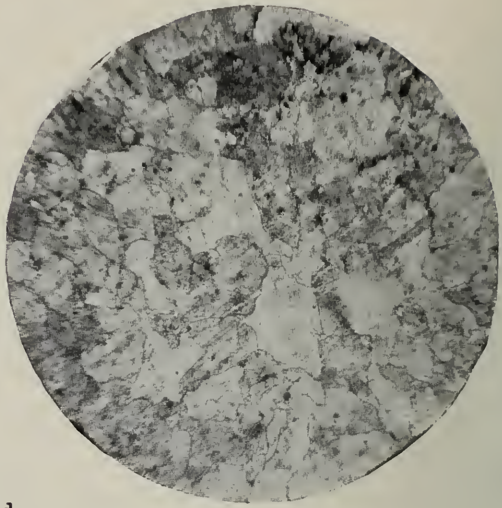
Annealed at 1100° C.

Fig. 23c. 40 Diam. Picric Acid.

Fig. 24c. 40 Diam. Picric Acid.

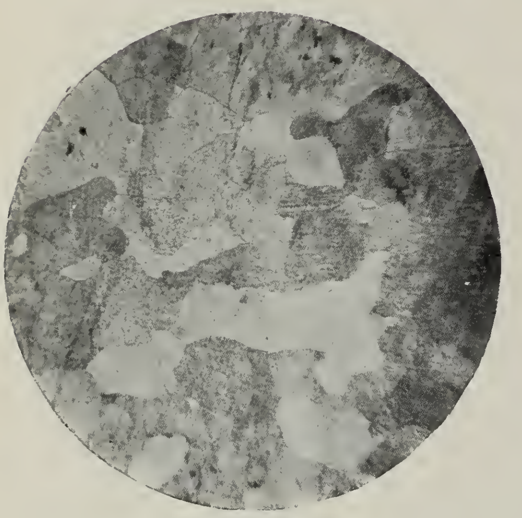
Alloy No. 3Si05. 0.068% Si.

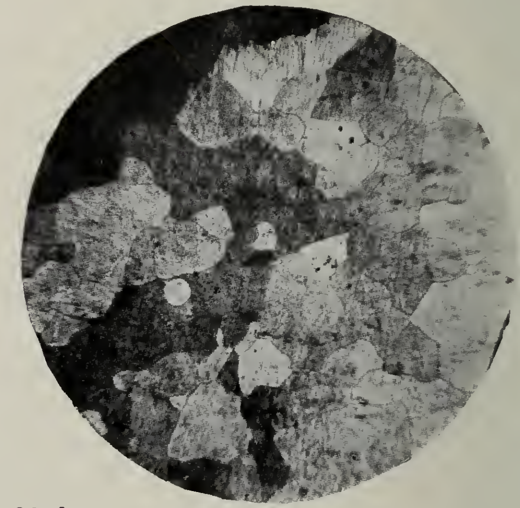
Alloy No. 3Si06. 0.148% Si.



As Forged.

FIG. 26a. 40 DIAM. PICRIC ACID.

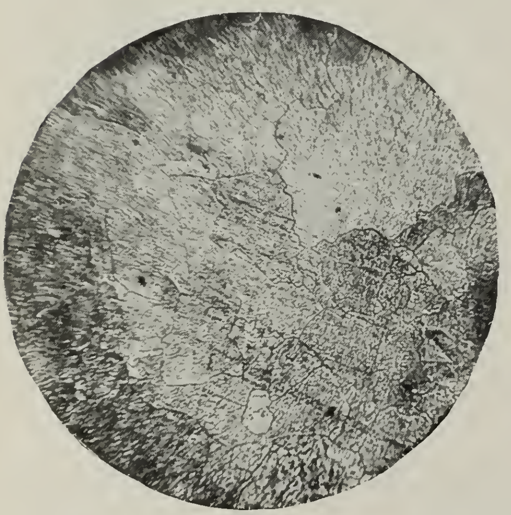


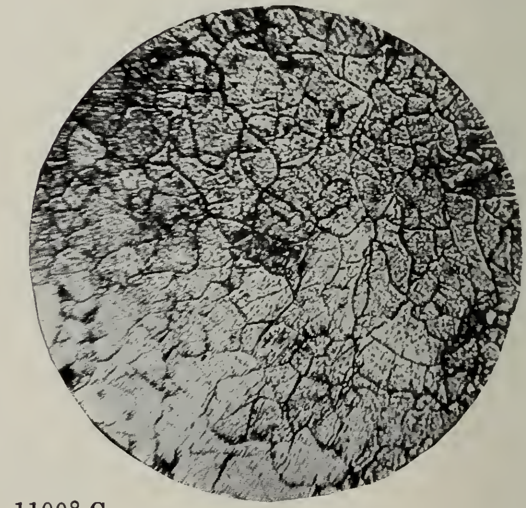


Annealed at 900° C.

FIG. 25b. 40 DIAM. PICRIC ACID.

FIG. 26b. 40 DIAM. PICRIC ACID.





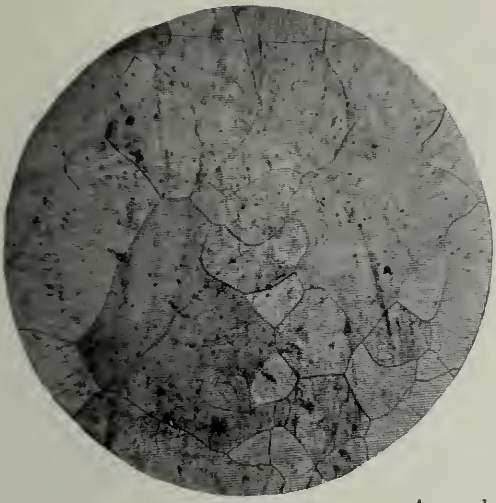
Annealed at 1100° C.

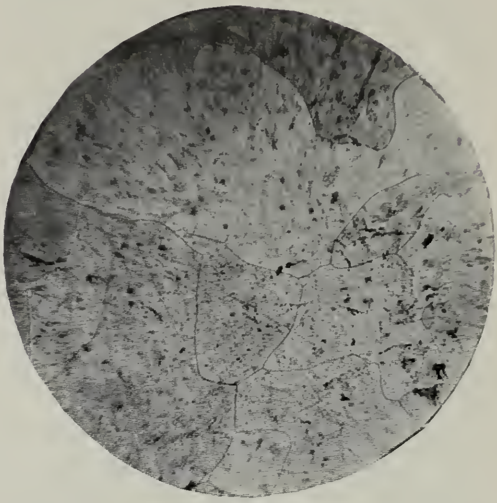
FIG. 25c. 40 DIAM. PICRIC ACID.

FIG. 26c. 40 DIAM. PICRIC ACID.

ALLOY NO. 3SI08. 0.309% SI.

ALLOY NO. 3SI10. 0.472% SI.

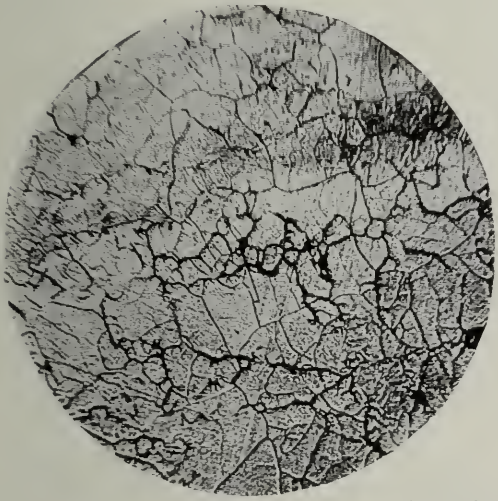


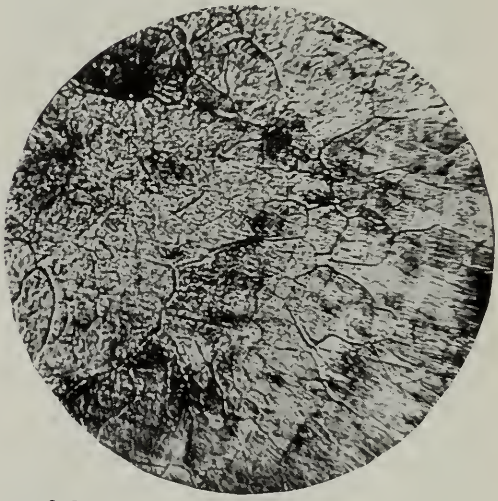


Annealed at 900° C.

FIG. 27b. 40 DIAM. PICRIC ACID.

FIG. 28b. 40 DIAM. PICRIC ACID.





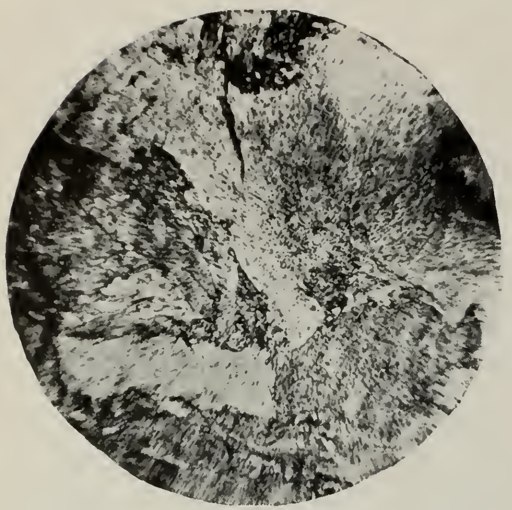
Annealed at 1100° C.

FIG. 27c. 40 DIAM. PICRIC ACID.

FIG. 28c. 40 DIAM. PICRIC ACID.

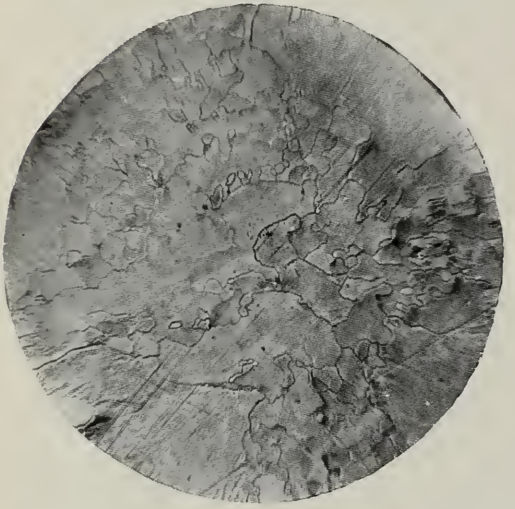
ALLOY NO. 3SI12. 0.673% SI.

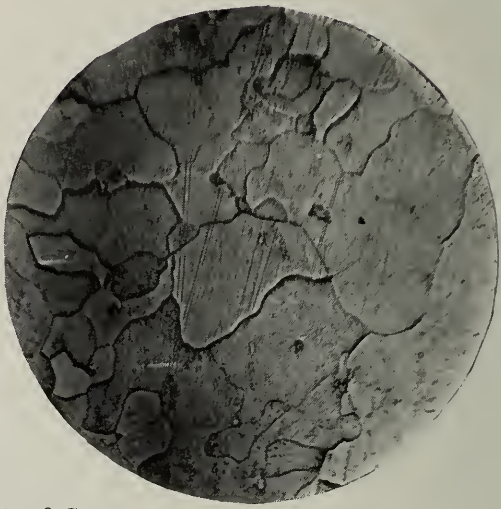
ALLOY NO. 3SI14.. 0.822% SI.



As Forged.

Fig. 29a. 10 Diam. Picric Acid.

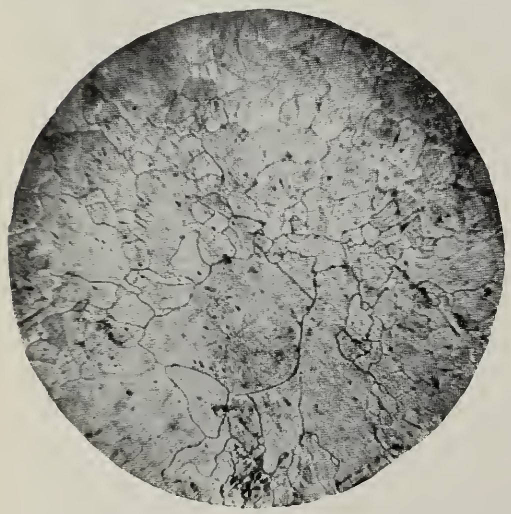


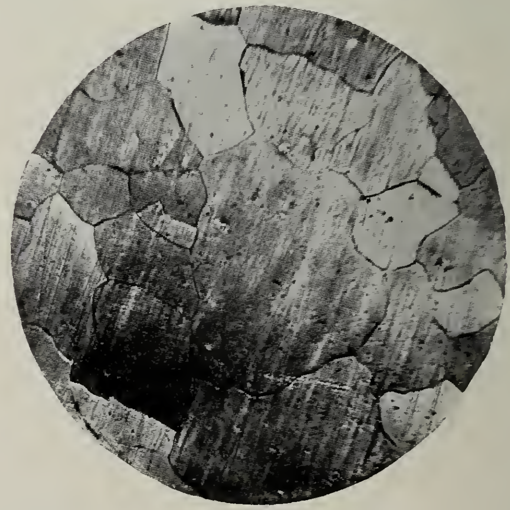


Annealed at 900° C.

Fig. $29b_1$. 10 Diam. Picric Acid.

Fig. $29b_2$. 40 Diam. Picric Acid.



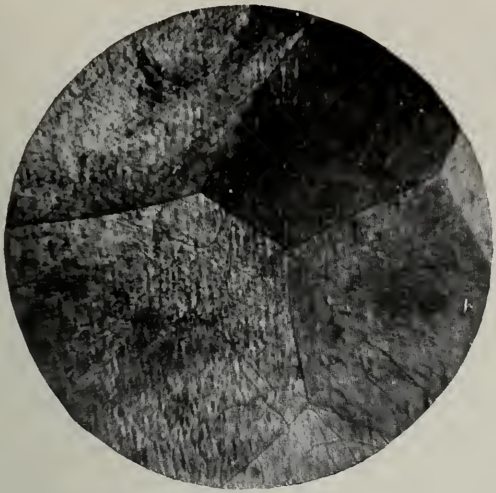


Annealed at 1100° C.

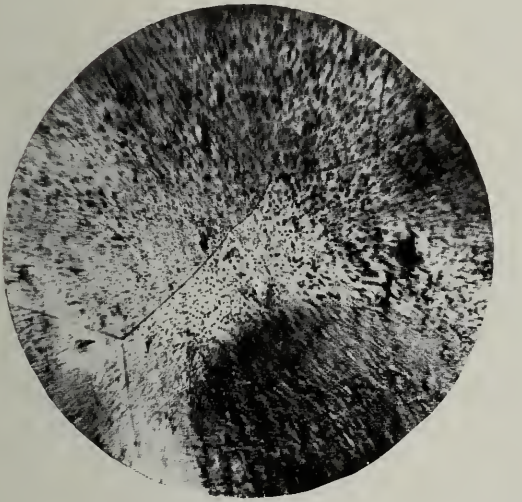
Fig. $29c_1$. 10 Diam. Picric Acid.

Fig. $29c_2$. 40 Diam. Picric Acid.

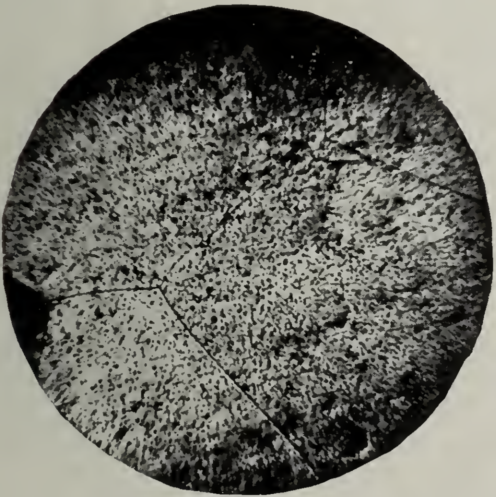
Alloy No. 3Si31. 1.71% Si.



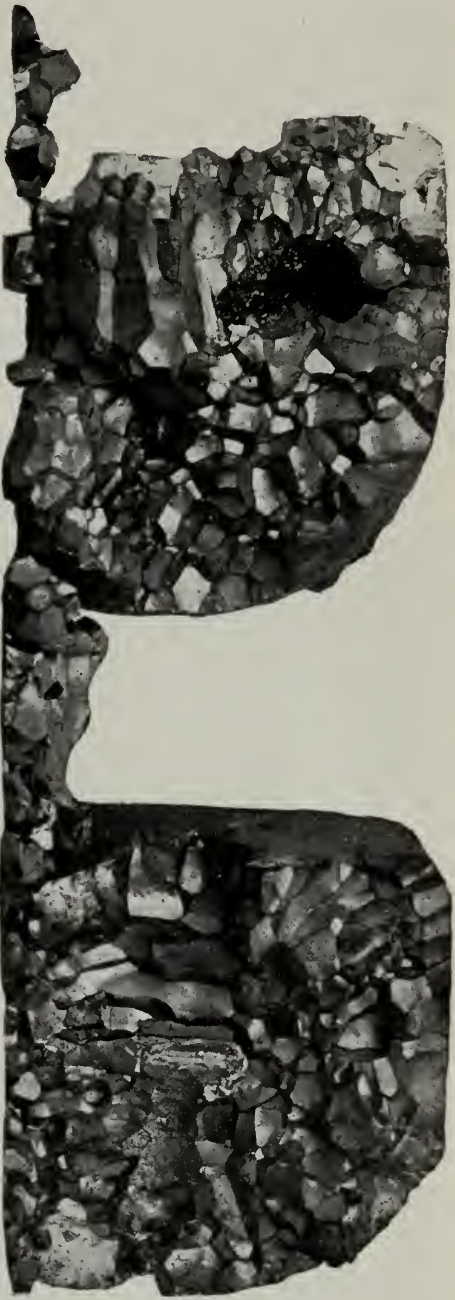
As cut from Ingot before Forging
FIG. $30a_1$. 10 DIAM. PICRIC ACID.



Annealed at 900° C.
FIG. 30b. 10 DIAM. PICRIC ACID.

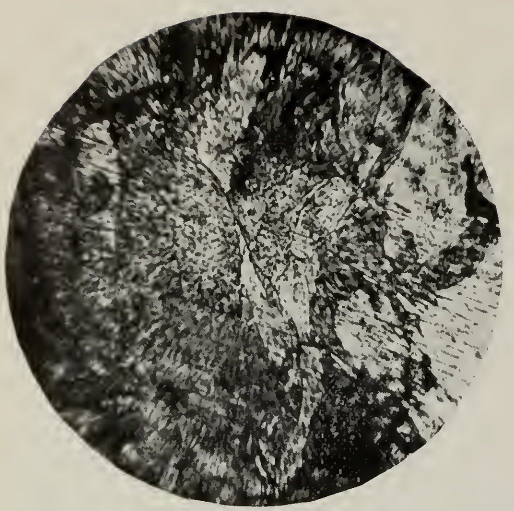


Annealed at 1100° C.
FIG. 30c. 10 DIAM. PICRIC ACID.



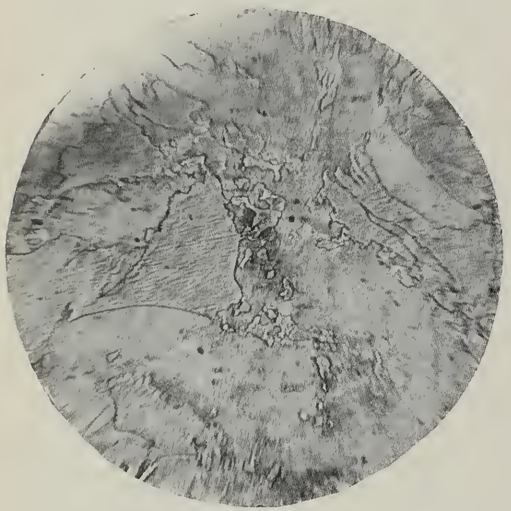
Ingot after Forging.
FIG. $30a_2$. NEARLY FULL SIZE.

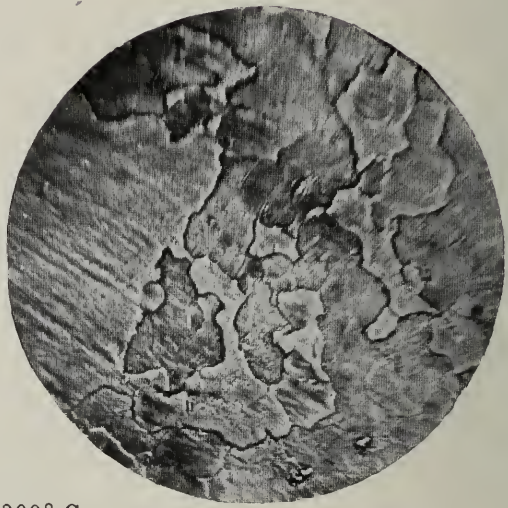
THE NON-FORGEABLE ALLOY NO. 3SI24. 2.57% SI.



As Forged.

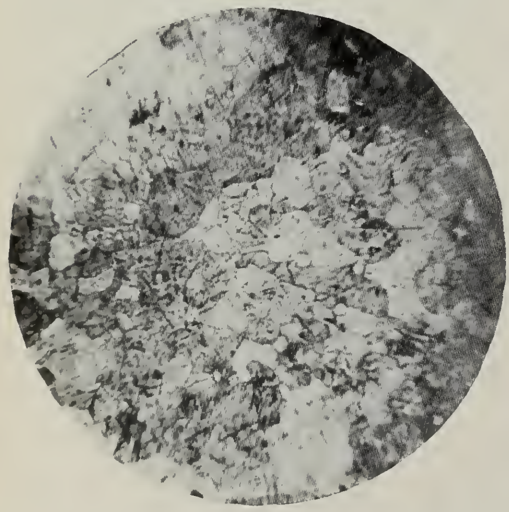
FIG. 31a. 10 DIAM. PICRIC ACID.

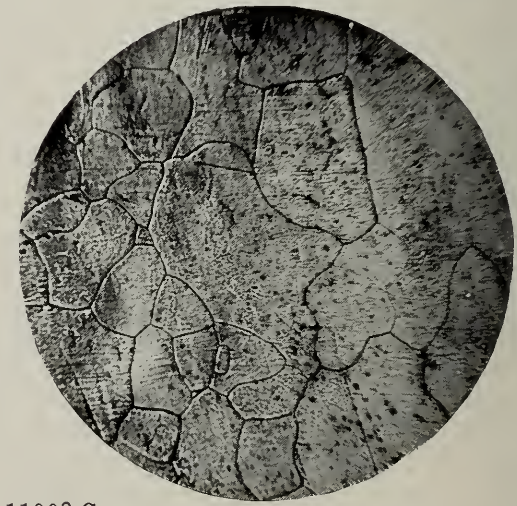




Annealed at 900° C.

FIG. $31b_1$. 10 DIAM. PICRIC ACID. FIG. $31b_2$. 40 DIAM. PICRIC ACID.

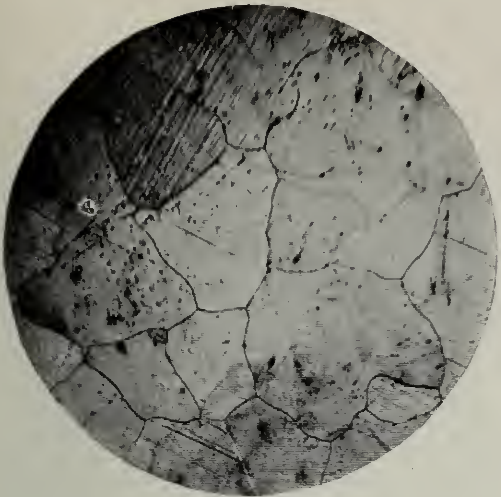




Annealed at 1100° C.

FIG. $31c_1$. 10 DIAM. PICRIC ACID. FIG. $31c_2$. 40 DIAM. PICRIC ACID.

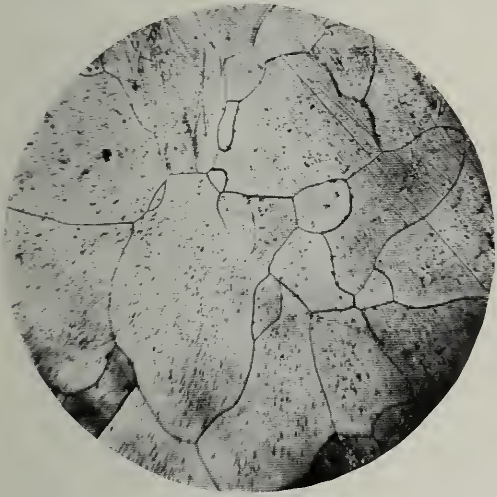
ALLOY NO. 3SI27. 2.73% SI.



**As Forged.**

Fig. 32. 40 Diam. Nitric Acid.

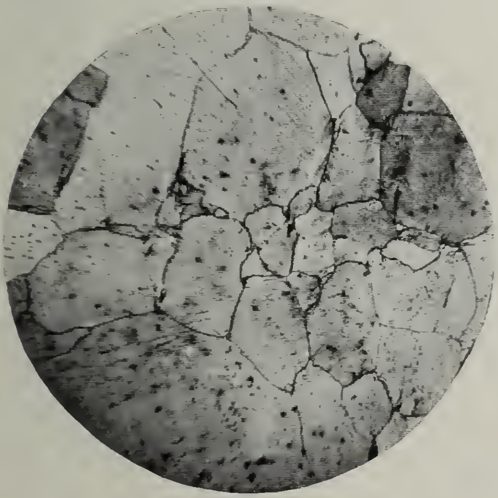
Alloy No. 3Si25 (Spec. 1) 3.40% Si.



**As Forged.**

Fig. 33. 40 Diam. Nitric Acid.

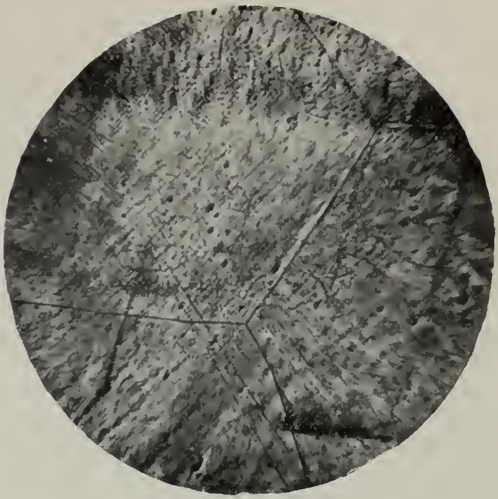
Alloy No. 3Si20. 3.58% Si.



**As Forged.**

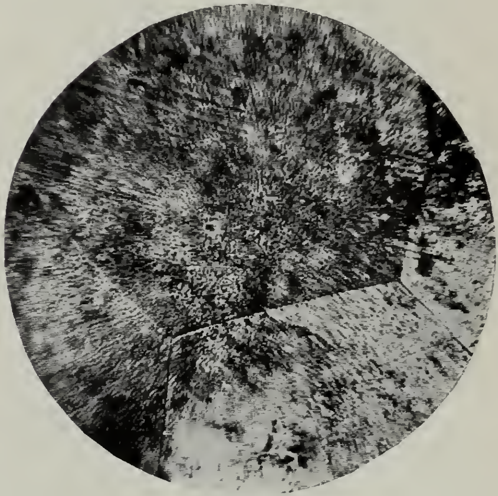
Fig. 34. 40 Diam. Nitric Acid.

Alloy No. 3Si36. 3.55% Si.



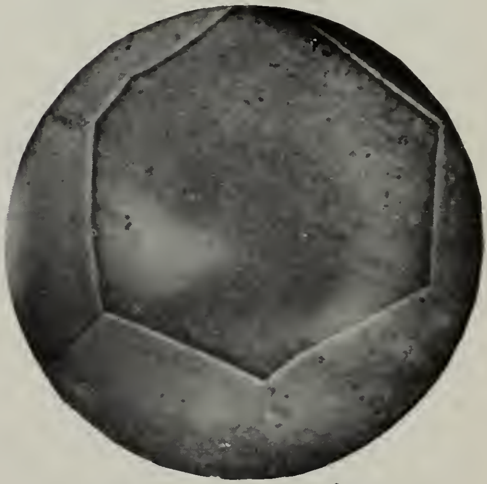
**As Forged.**

Fig. 35a. 10 Diam. Picric Acid.



**Annealed at 900° C.**

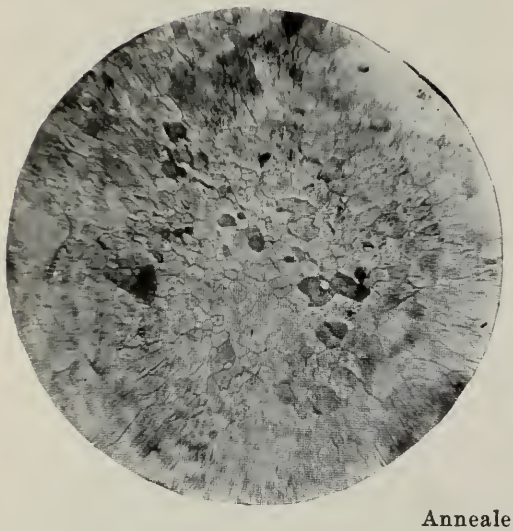
Fig. 35b. 10 Diam. Picric Acid.

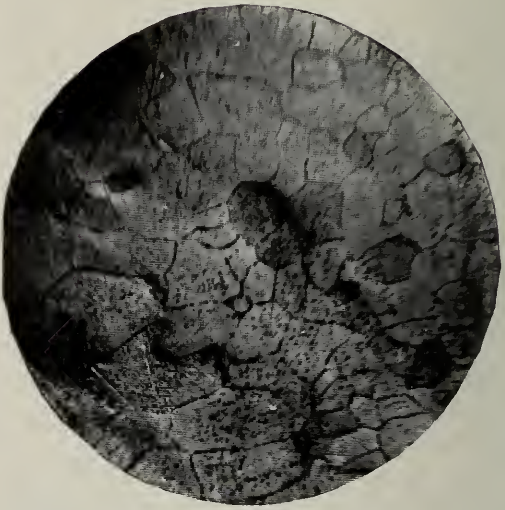


**Annealed at 1100° C.**

Fig. 35c. 7 Diam. Picric Acid.

Alloy No. 3Si25 (Spec. 2). 3.40% Si.

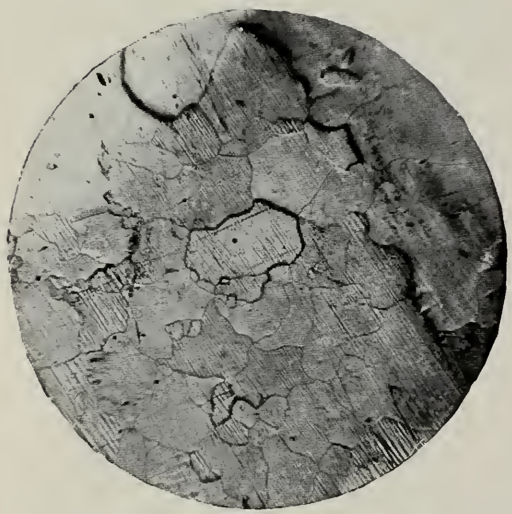


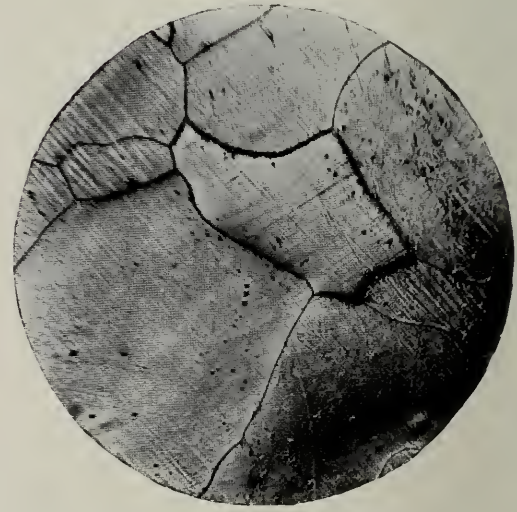


Annealed at 900° C.

Fig. $36b_1$. 10 Diam. Picric Acid.

Fig. $36b_2$. 40 Diam. Picric Acid.



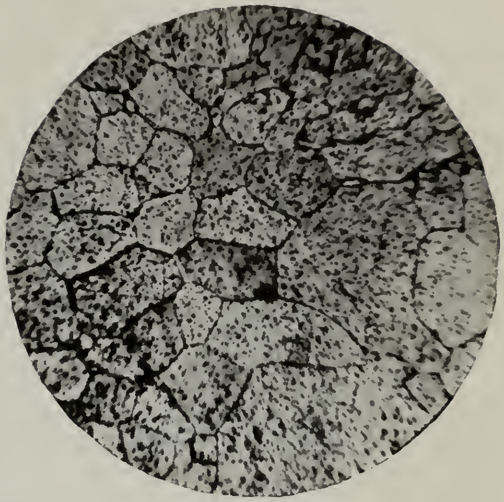


Annealed at 1100° C.

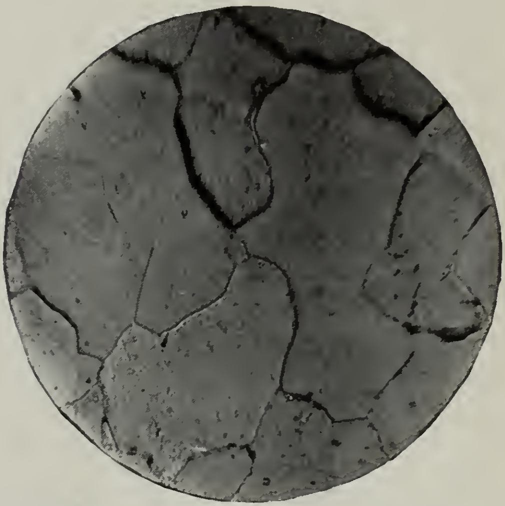
Fig. $36c_1$. 10 Diam. Picric Acid.

Fig. $36c_2$. 40 Diam. Picric Acid.

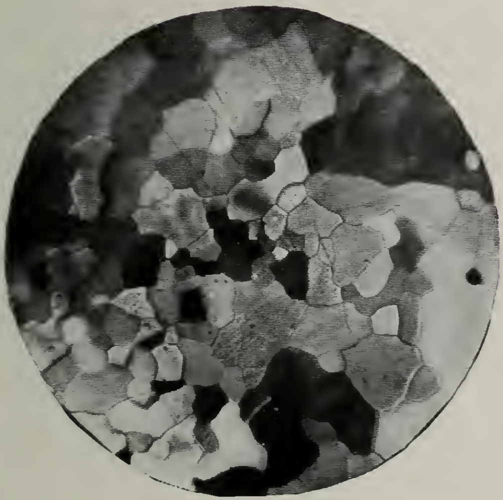
Alloy No. 3Si28. 4.44% Si.

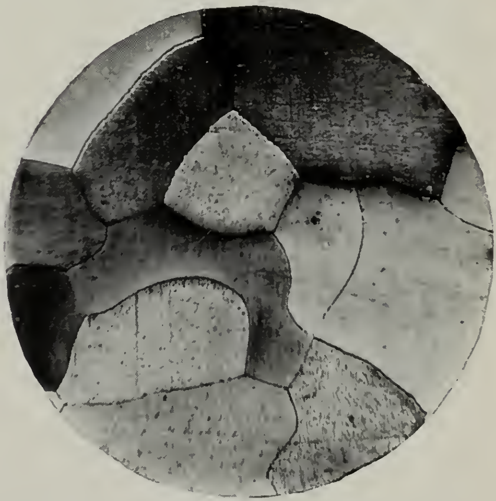


As Forged.
FIG. 37a. 40 DIAM. PICRIC ACID.



Annealed at 900° C.
FIG. 37b. 40 DIAM. PICRIC ACID.



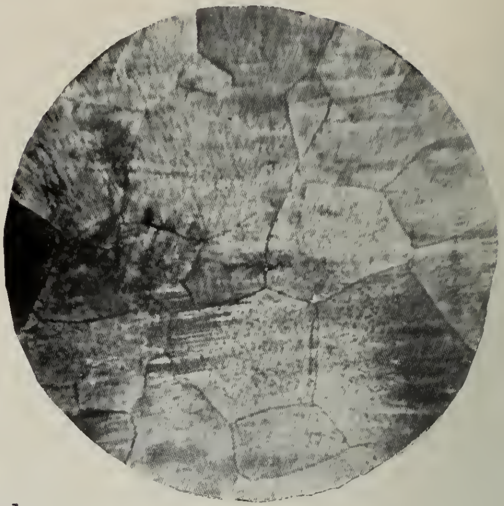


Annealed at 1100° C.

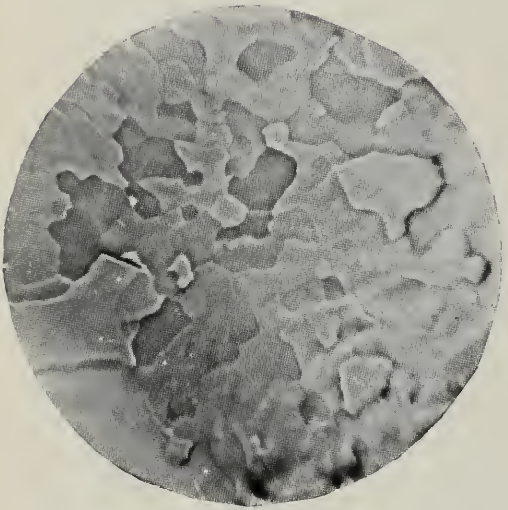
FIG. $37c_1$. 10 DIAM. PICRIC ACID.
Washed with 10% HF Sol.

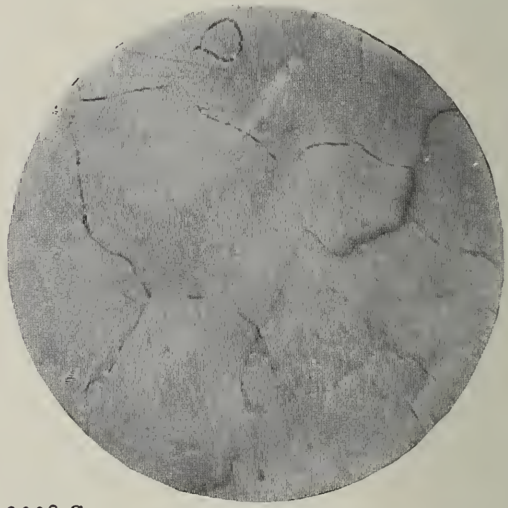
FIG. $37c_2$. 40 DIAM. PICRIC ACID.
Washed with 10% HF Sol.

ALLOY NO. 3SI29. 4.92% SI.



As Forged.
FIG. 38a. 40 DIAM. PICRIC ACID.

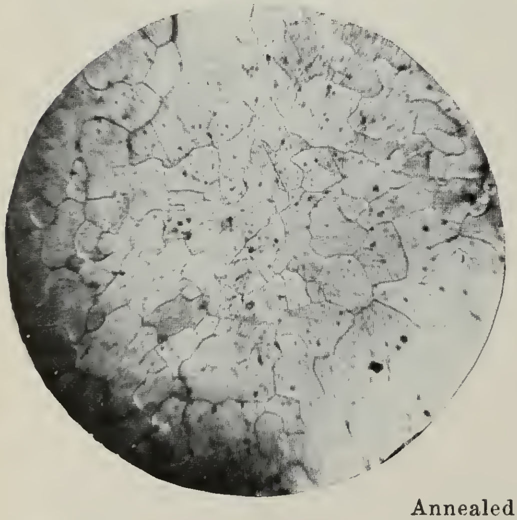


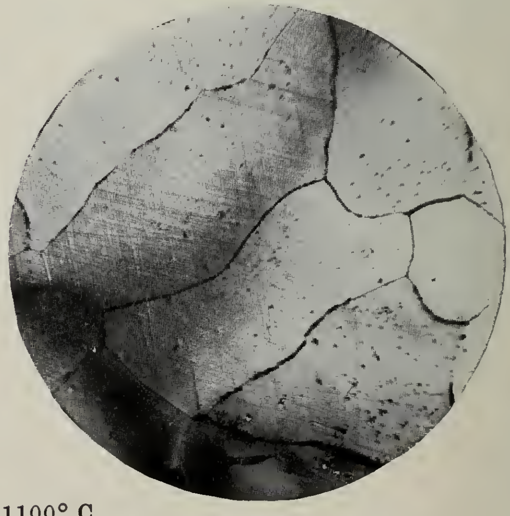


Annealed at 900° C.

FIG. $38b_1$. 10 DIAM. PICRIC ACID.
Slightly repolished.

FIG. $38b_2$. 40 DIAM. PICRIC ACID.
Slightly repolished.



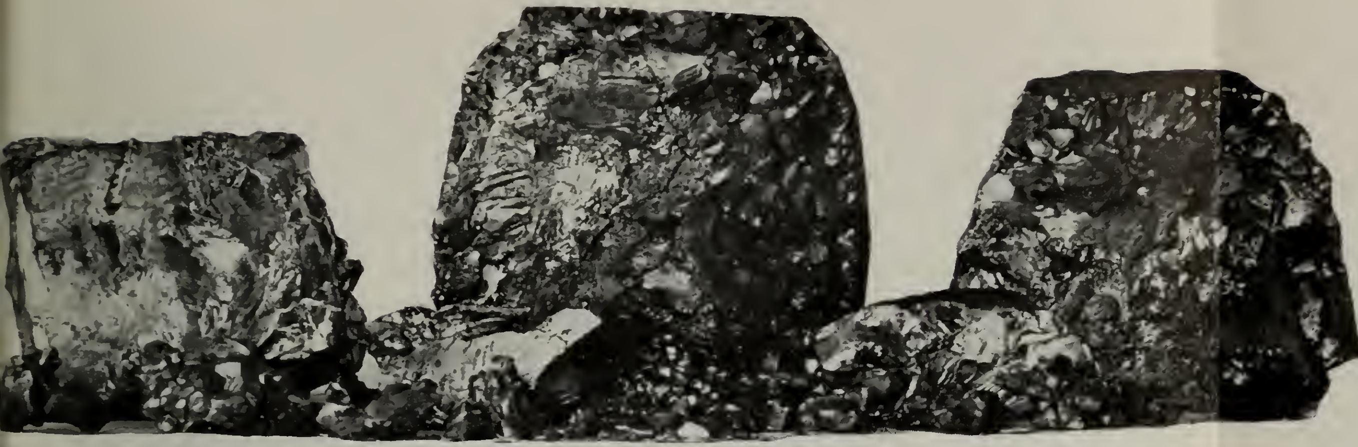


Annealed at 1100° C.

FIG. $38c_1$. 10 DIAM. PICRIC ACID.
Washed in 10% HF Sol.

FIG. $38c_2$. 40 DIAM. PICRIC ACID.
Washed with 10% HF Sol.

ALLOY NO. 3SI32. 6.57% SI.



The Non-forgeable Alloy 3Si30. 8.55% Si.
Fig. 39. Nearly Full Size.

## V. Summary and Conclusions.

The results recorded in the previous pages may be summarized as follows:

(a). By means of the vacuum method of melting it is possible to obtain a decidedly purer product than has thus far been obtained in any other manner. Consequently by the use of this method more definite conclusions can be drawn with regard to the effect of silicon upon iron, than has hitherto been possible.

(b). Silicon, like boron, has a double effect upon iron. Part of it combines with the iron and remains in solid solution throughout the cooling of the alloy, while a smaller part reduces the iron oxide present.

(c). The tensile strength of the vacuum product follows in general the same law as alloys made under ordinary conditions, but the ductility of the former is much greater, particularly below 2 per cent and above 3 per cent, probably due to the absence of carbon. The maximum tensile strength of 105000 pounds per square inch occurs with a silicon content of 4.5 per cent.

(d). The limit of forgeability lies between 7 and 8 per cent silicon. A critical range occurs between 2.55 and 2.60 per cent, in which the alloys are exceedingly brittle, in some cases being not even forgeable.

(e). With regard to the magnetic properties the vacuum alloys exhibit most remarkable characteristics. The best alloys are obtained with about 0.15 per cent and 3.40 per cent silicon after annealing at 1100°C. The maximum permeability for both of these alloys is above 50,000, and the hysteresis loss for $B_{max} = 10{,}000$ and 15,000 is about 300 and 1000 ergs per cu. cm. per cycle respectively. This hysteresis loss is ⅛ and ⅓ of the corresponding loss for commercial silicon steel. The most favorable annealing temperature is in every case 1100°C.

(f). The specific electrical resistance increases about 13 microhms for the first per cent silicon added. For each additional per cent added the increase is about 11 microhms. Consequently, the 3.40 per cent alloy mentioned under 5 has a resistance nearly 5 times that of the 0.15 per cent alloy.

By the vacuum process two silicon alloys have thus been produced that have very valuable characteristics; one, low in silicon, not very strong, but extremely ductile, of high permeability, low hysteresis loss, and of low electrical resistance; the other high in silicon, very strong, moderately tough, of high permeability, low hysteresis loss and of high electrical resistance. The properties of these two alloys are summarized in Table 12. The first is evidently suitable for use in places where high permeability and low hysteresis loss are the chief requirements, while the second alloy is suitable for electromagnetic machinery, principally transformers, where a low eddy current loss is an additional requirement. The mechanical properties of the second alloy makes possible its use also in dynamo machinery, where the present commercial silicon steel can not be used on account of its brittleness.

TABLE 12.
PROPERTIES OF THE TWO BEST IRON-SILICON VACUUM ALLOYS.

| Silicon Content % | Stress at Yield Point lb. per sq. in. | Ultimate Strength lb. per sq. in. | Elongation % | Reduction of Area % | Maximum Permeability | Density for Max. Permeability | Hysteresis Loss ergs ergs per cc. per cycle | | Spec. Elect. Resistance microhms |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | for $B_{\max} =$ 10000 | for $B_{\max} =$ 15000 | |
| 0.15 | 18500 | 37000 | 56 | 90 | 66500 | 6500 | 286 | 916 | 11.80 |
| 3.40 | 58000 | 76500 | 21 | 28.5 | 63300 | 6500 | 280 | 1025 | 48.50 |

It should be pointed out, that electrolytic iron is not essential to the attainment of high magnetic quality. Any low carbon iron that is practically free from phosphorus, sulphur and manganese, *when melted in vacuo,* will give magnetic properties approaching very closely to those obtainable with electrolytic iron. There are a number of commercial grades of iron coming within these specifications, that are obtainable at the present time.

The writer fully realizes the difficulties that have to be overcome, before the vacuum iron can be used in the industries in competition with the materials available at the present time. Suggestions have, nevertheless, already come from several sources for its employment in places, where its high cost of production is not of vital importance, and there is no doubt but that its usefulness in limited fields will increase as time goes on. However, the writer would like to suggest that the results here given be partly considered as a new indication of the possibilities obtainable in the realm of magnetic properties, rather than as the final word regarding certain properties of iron-silicon alloys, that can be turned into commercial use at the present time.

## VII. Appendix.

### *Results obtained with Ring Specimens.*

In view of the unprecedented results obtained with regard to the magnetic properties of the alloys described in this bulletin, it was deemed desirable to test a few alloys in the form of rings. As the ring method is the old established method of magnetic testing, and as this method requires no compensating or other auxiliary windings to cause uncertainties, the evidence obtained with such testpieces would naturally be more convincing than results obtained by any other method. On the other hand, it has been shown by Richter* and Lloyd† that the magnetic induction in a ring specimen is not uniformly distributed, but that it is crowded towards the inside of the ring. The variation is greatest near the steepest part of the magnetization curve, where the maximum permeability occurs, and may here amount to as much as 100 per cent for high permeability material. Consequently it is impossible with ring specimens to measure the maximum permeability, and this should be borne in mind when comparing the results obtained by the two methods.

The dimensions of the rings used are shown in Fig. 40.

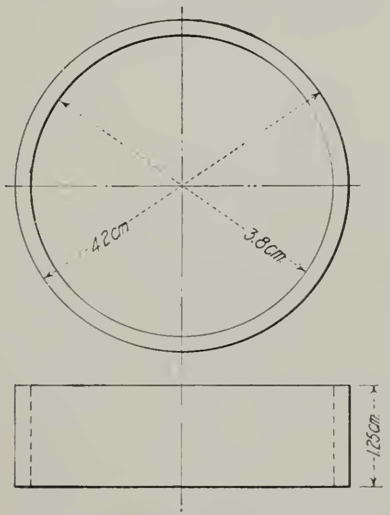


Fig. 40. Ring Specimen.

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*Electrotech. Zetischr. Vol. 24, p. 710; 1903.

†Bull. Bureau of Standards, Vol. 5, No. 3, Reprint No. 108, 1909.

With these dimensions the true magnetizing force, as shown by Lloyd and Richter is

$$H_0 = 1.0009 \frac{2NI}{10R_0}$$

Where $R_0$ = mean radius = 2.0 cm.
$N$ = total number of turns = 100
$I$ = magnetizing current in amperes
Hence by using $H_0 = 10\ I$
the error introduced is less than 0.1 per cent.

The secondary winding consisted of 100 turns of No. 30 B & S wire wound next to the ring and connected directly to the Grassot fluxmeter. With this arrangement

$$\triangle B = 400\, D$$

where $\triangle B$ = change of average flux density in ring in gausses
$D$ = deflection of fluxmeter.

Three rings were prepared all containing approximately 3 per cent silicon. From No. 3Si39, one ring was machined directly out of the ingot as it came from the melting furnace, while another was made from the remainder of the ingot after forging it. The latter was imperfect, however, and was discarded. The third ring was made from No. 3Si40 after forging the ingot, and the remainder of the latter was then forged into a rod to be tested by the Burrows method.

The specimens were first tested in the original state, unannealed. The windings were then removed and the rings and the rod placed in the annealing furnace and annealed in vacuo, the rod occupying the space along the axis of the tube. The maximum temperature to which the rod was subjected was 1100° C, but the rings were heated to a somewhat higher temperature as they occupied a space nearer the heating element, their axes coinciding with that of the rod. The specimens were cooled at the standard rate, namely 30° C per hour.

The results are shown in Table 13. In this table is included the results for rod No. 3Si25 from the main part of the investigation.

With regard to permeability the results show that in the unannealed state the two methods of testing agree very well. In the annealed state, however, the maximum permeability obtained for 3Si40 by the Burrows method is twice that obtained by the ring method. The latter method, however, as has been stated above, does not measure the maximum permeability, on account of the nonuniformity of the flux distribution, and in view of the results shown in Table 5, it is probable that the Burrows method gives too high a maximum permeability. Making allowance for possible differences due to material and heat treatment, as well as mechanical treatment, it seems probable from these results that the true maximum permeability is in the neighborhood of 50,000. That the permeability for B = 15,000 is so low for the ring specimens must be due to the fact that the rings were annealed at a higher temperature than was the rod. It was shown in Figs. 17 and 18, that with a silicon content of 3 per cent for H = 20 the flux density after annealing at 900° was 16,000, while after annealing at 1100° it was only 15,600. A corresponding decrease in this region may consequently be expected also for higher annealing temperatures. This point is further borne out by the results obtained with the ring made from the ingot 3Si39 without forging. This ring may be considered as having been annealed at the melting point of the alloy. Its permeability at B = 15,000 is only 583 as compared with 962 for the ring made from 3Si40 after forging.

Turning now to the hysteresis loss it is found that in the unannealed state the loss in ring No. 3Si40 is less than half the loss in the rod. As the two methods are known to be equally reliable in this case the difference must be attributed to the fact that the rod was forged much more than was the ring. That this is the cause becomes evident from the result obtained for ring No. 3Si39 that was not forged at all, as this shows a decidedly lower hysteresis loss even than ring No. 3Si40. After annealing, the loss for B = 10,000 is found to be less for the rod than for the ring, while for B = 15,000 the reverse is the case. The coercive force for B = 15,000 is 60 per cent larger for the rod than for the ring, while the retentivity is also lower for the ring.

These results seem to point towards the following conclusions: For such high quality material as that described in this bulletin the Burrows method gives too high maximum permeability and too high retentivity. For low densities the hysteresis loss obtained is low, while for medium and high densities the coercive force and consequently also the hysteresis loss is too high.

While these rings were prepared for the purpose of verifying the results given in the main part of the bulletin, the results can not be dismissed without calling especial attention to the results obtained with the ring made from the ingot No. 3Si39 without forging. Unannealed this ring has a retentivity of only 1000 and the hysteresis loss for B = 15,000 is only 1130 ergs per cc per cycle, not much above the value found for the best rod after annealing at 1100° C. After annealing the ring at 1100° the hysteresis loss for B = 15,000 drops to 577 ergs. This value is less than ⅕ of the corresponding value for the commercial 4 per cent silicon steel. The low hysteresis loss for this ring shows the advantage, as far as this characteristic is concerned, of annealing at as high a temperature as possible, as this ring may be considered as having been annealed at the melting point of the alloy. On the other hand, the results also show that the permeability for high magnetizing forces decreases with increasing annealing temperatures. That this is the case for temperatures between 900° and 1100° C has previously been established. That it is equally true for temperatures above 1100° C is now indicated by the results obtained for this ring.

## TABLE 13.

## COMPARISON BETWEEN THE RING METHOD AND THE BURROWS METHOD OF MAGNETIC TESTING.

| Test Piece | Silicon Content % | Maximum Permeability | Density for Max. Permeability gausses | Permeability | | Hysteresis Loss ergs per cc per cycle | | Coercive Force gilberts per cm | | Retentivity gausses | | Spec Elec. Resistance microhms | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | for B = 10000 | for B = 15000 | for $B_{max}$ 10000 | for $B_{max}$ 15000 | for $B_{max}$ 10000 | for $B_{max}$ 15000 | for $B_{max}$ 10000 | for $B_{max}$ 15000 | | |
| Unannealed | | | | | | | | | | | | | |
| 3Si39 Ring–Made from Ingot | 3.19 | 2000 | 4000 | 1600 | 405 | 955 | 1130 | .45 | .45 | 1000 | 1000 | | Tested by Ring Method |
| 3Si40 Ring–Made from Forging | 2.96 | 3000 | 3000 | 1470 | 485 | 1305 | 1540 | .50 | .50 | 2500 | 2500 | | '' '' '' '' |
| Rod–Made from Forging | 2.96 | 2350 | 4000 | 1210 | 545 | 3130 | 4450 | 1.00 | 1.00 | 4800 | 4800 | 44.75 | '' '' Burrow's '' |
| 3Si25 Rod–Made from Forging | 3.40 | 3000 | 3000 | 1065 | 476 | ...... | ...... | Not | Tested | ...... | ...... | 48.5 | '' '' '' '' |
| Annealed | | | | | | | | | | | | | |
| 3Si39 Ring–Made from Ingot | 3.19 | 32000 | 8000 | 28600 | 583 | 340 | 577 | .10 | .10 | 7320 | 9600 | | Tested by Ring Method |
| 3Si40 Ring–Made from Forging | 2.96 | 36200 | 8000 | 31200 | 962 | 337 | 757 | .10 | .10 | 7600 | 11000 | | '' '' '' '' |
| Rod–Made from Forging | 2.96 | 72600 | 9000 | 71400 | 2500 | 254 | 926 | .085 | .16 | 9400 | 13600 | 44.75 | '' '' Burrow's '' |
| 3Si25 Rod–Made from Forging | 3.40 | 63300 | 7000 | 46500 | 2030 | 280 | 1025 | .085 | .15 | 9000 | 12400 | 48.5 | '' '' '' '' |

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